Generalized convexity and separation theorems

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In 1937 E. F. Beckenbach generalized the notion of convex functions in the following way: Let $\mathcal{F}$ be a family of continuous real functions defined on an interval $I \subseteq \mathbb{R}$ such that for any two points $(x_1, y_1), (x_2, y_2) \in I \times \mathbb{R}$ with $x_1 \neq x_2$ there exists exactly one $\varphi = \varphi_{(x_1,y_1)(x_2,y_2)} \in \mathcal{F}$ such that $\varphi(x_i) = y_i$ for $i = 1, 2$. A function $f : I \to \mathbb{R}$ is said to be $\mathcal{F}$-*convex* if for any $x_1, x_2 \in I$, $x_1 < x_2$

$$f(x) \leqslant \varphi_{(x_1,f(x_1))(x_2,f(x_2))}(x), \quad x_1 \leqslant x \leqslant x_2.$$

In a similar way $\mathcal{F}$-convex sets are defined. Many properties of $\mathcal{F}$-convex functions and $\mathcal{F}$-convex sets are similar to those known for the classical convexity. In the talk some new results of this type will be presented. First, the Kakutani theorem (stating that two disjoint convex sets can be separated by complementary convex sets) will be extended to $\mathcal{F}$-convex sets. Then some characterizations of pairs of functions that can be separated by an $\mathcal{F}$-convex function or by a function belonging to $\mathcal{F}$ will be given.