On embedding of diffeomorphisms on Banach space in regular iteration semigroup

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Abstract

Let X be a real Banach space, $U \subset X$ be an open set and $F : U \to U$ be a diffeomorphism. A family of C^1 mappings $\{f_t : U \longrightarrow U,\ t \geq 0\}$ is said to be a regular iteration semigroup or semiflow of f if

$$f_s(f_t(x)) = f_{t+s}(x), \quad t, s \geq 0, \quad x \in U$$

and $f = f_1$.

Let $0 \in U$ be a unique globally attractive fixed point of f. Assume that there exists a linear bounded operator $A : X \longrightarrow X$ such that $f'(0) = expA$. We give some conditions which imply the existence of the following limit

$$f_t(x) := lim_{n\to\infty} f^{-n}((exp\ tA)f^n(x)), \quad x \in U, \quad t \geq 0$$

and the property that the mappings $\{f_t,\ t \geq 0\}$ yields a regular iteration semigroup of f.

An application for C^1 iterative roots of f is given, that is the functions g such that $g^n = f$. In particular we consider the case $X = \mathbb{R}^n$.

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