

Tensor Decompositions for Multi-aspect Graph Analytics and Beyond

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Multi-View Social Networks



Social Network Matrix





What about the rest of the views??



S If we aggregate, we ignore important structure!!

Tensors

- Multi-dimensional matrices
- Can naturally model multi-aspect datasets
- Long list of applications: Psychometrics, Chemometrics, Signal Processing, Machine Learning, Data Mining



Survey Papers

3551

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 65, NO. 13, JULY 1, 2017

Tensor Decomposition for Signal Processing and Machine Learning

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Overview Article

Abstract-Tensors or multiway arrays are functions of three or more indices (i, j, k, ...)—similar to matrices (two-way arrays), which are functions of two indices (r, c) for (row, column). Tensors have a rich history, stretching over almost a century, and touching upon numerous disciplines; but they have only recently become ubiquitous in signal and data analytics at the confluence of signal processing, statistics, data mining, and machine learning. This overview article aims to provide a good starting point for researchers and practitioners interested in learning about and working with tensors. As such, it focuses on fundamentals and motivation (using various application examples), aiming to strike an appropriate balance of breadth and depth that will enable someone having taken first graduate courses in matrix algebra and probability to get started doing research and/or developing tensor algorithms and software. Some background in applied optimization is useful but not strictly required. The material covered includes tensor rank and rank decomposition; basic tensor factorization models and their relationships and properties (including fairly good coverage of identifiability); broad coverage of algorithms ranging from alternating optimization to stochastic gradient; statistical performance analysis; and applications ranging from source separation to collaborative filtering, mixture and topic modeling, classification, and multilinear subspace learning.

Index Terms-Tensor decomposition, tensor factorization, rank, canonical polyadic decomposition (CPD), parallel factor analysis (PARFAC), Tucker model, higher-order singular value decomposition (HIOSVD), multilinear singular value decomposition (MLSVD), uniqueness. NP-hard problems, alternating optimization, alternating direction method of multipliers, gradient descent, Gauss-Newton, stochastic gradient, Cramér-Rao bound, communications, source separation, harmonic retrieval, speech separation, collaborative filtering, mixture modeling, topic modeling, classification, subspace learning.

I. INTRODUCTION

T ENSORS¹ (of order higher than two) are arrays indexed by three or more indices, say (i, j, k, ...) – a generalization of matrices, which are indexed by two indices, say (r, c) for (row, column). Matrices are two-way arrays, and there are threeand higher-way arrays (or higher-order) tensors.

Tensor algebra has many similarities but also many striking differences with matrix algebra – e.g., low-rank tensor factorization is essentially unique under mild conditions; determining tensor rank is NP-hard, on the other hand, and the best low-rank approximation of a higher rank tensor may not even exist.

Geared towards theoretical & algorithmic understanding

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Tensors for Data Mining and Data Fusion: Models, Applications, and Scalable Algorithms

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Tensors and tensor decompositions are very powerful and versatile tools that can model a wide variety of heterogeneous, multiaspect data. As a result, tensor decompositions, which extract useful latent information out of multiaspect data tensors, have witnessed increasing popularity and adoption by the data mining community. In this survey, we present some of the most widely used tensor decompositions, providing the key insights behind them, and summarizing them from a practitioner's point of view. We then provide an overview of a very broad spectrum of applications where tensors have been instrumental in achiving stateof-the-art performance, ranging from social network analysis to brain data analysis, and from web mining to healthcare. Subsequently, we present recent algorithmic advances in scaling tensor decompositions up to today's big data, outlining the existing systems and summarizing the key ideas behind them. Finally, we conclude with a list of challenges and open problems that outline exciting future research directions.

 $\label{eq:ccs} CCS\ Concepts: \bullet\ General\ and\ reference \rightarrow Document\ types; \bullet\ Information\ systems \rightarrow Information\ systems\ applications;\ Data\ mining; \bullet\ Computing\ methodologies \rightarrow Factorization\ methods$

Additional Key Words and Phrases: Tensors, tensor decomposition, tensor factorization, multi-aspect data, multi-way analysis

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1. INTRODUCTION

Tensors are multidimensional extensions of matrices. Because of their ability to express multimodal or multiaspect data, they are very powerful tools in applications that inherently create such data. For instance, in online social networks, people tend to interact with each other in a variety of ways: they message each other, they post on each other's pages, and so on. All these different means of interaction are different

Geared towards applications & practitioners

What are we looking for?



Blocks within the data Subsets / co-clusters of: 1)Users ("senders") 2)Users ("receivers") 3)Means of communication

Blocks are rank-one tensors



Direct extension of matrix case!

CP/PARAFAC Decomposition



CP/PARAFAC Decomposition



Intuitive interpretation

1) Each triplet of vectors co-clusters: (users, users, means of communication) 2) Rows of matrix **A** (or **B**) matrix are **embeddings** to **A**(\bigcirc ,:)= "community space"

DBLP Multi-View Graph







(a) citation



- Take the arg max of the r-dim embedding per node for community assignment
- Baselines
 - Spectral clustering on sum of matrices / views
 - Linked Matrix Factorization
 - [Tang et al. ICDM 2009]
- Outperforms "2D"/matrix baselines wrt NMI (Normalized Mutual Information)

[Papalexakis, Akoglu, Ienco Fusion 2013]





DETOW Ord2Vec: Word Embeddings [1]

Task: Given a target word predict which words are more likely context Fortuitous by-product: The learned NN weights provide a vector representation for each word aka word embedding



img: https://www.tensorflow.org/versions/r0.11/tutorials/word2vec/index.html

Cool fact: Instead of a skipgram, we can factorize a matrix that holds Pointwise Mutual Information [2]

- Embedding space respects contextual relationships
- Can add and subtract the vector embeddings for different words
- Some interesting results:
 - King Man + Woman = Queen
 - Human Animal = Ethics
 - Library Books = Hall
 - President Power = Prime Minister

[1] Mikolov et al. "Distributed Representations of Words and Phrases and their Compositionality", NeurIPS'13

[2] Levy et al. "Neural Word Embeddings as Implicit Matrix Factorization", NeurIPS 2014

Node Embeddings

- Inspired by Word Embeddings
- Identifies the context by random walks
- Uses skipgram to learn node representation
 Variants: DeepWalk [KDD15], node2vec [KDD16]
- What if we also have node features?



Yang et al. Network Representation Learning with Rich Text Information, IJCAI'15





(a) Random walk generation.

Tensor-based Context-Aware Node Embeddings



t-PNE - Step 1



ASONAM 2018 w/ Saba Al Sayouri, Ekta Gujral, Danai Koutra, Sarah Lam

Tensor-based Context-Aware Node Embeddings



Tensor-based Context-Aware Node Embeddings

Algorithm	Wikipedia $(K = 8)$		W	WebKB ($K = 40$)		CiteSeer ($K = 15$)			
	10%	50%	90%	10%	50%	90%	10%	50%	90%
DeepWalk	59.04	68.25	69.75	42.82	45.49	45.57	54.22	61.91	0.62.11
node2vec	58.73	66.98	70.12	43.20	44.87	44.43	52.66	60.22	60.87
Walklets	58.17	65.61	66.68	42.16	46.83	49.09	52.57	59.25	60.96
TADW	19.25	32.69	46.27	48.10	49.25	48.98	25.52	56.51	67.92
t-PNE *	61.64	66.16	74.00	73.53	82.95	85.73	66.00	70.00	75.00
Gain over DeepWalk	4.4	_	6.1	71.7	82.4	88.1	21.7	13.1	20.8
Gain over node2vec	4.9	_	5.5	70.2	84.9	92.9	25.3	16.2	23.2
Gain over Walklets	6.0	0.8	11.0	74.4	77.1	74.6	25.5	18.1	23.0
Gain over TADW	220.2	102.4	59.9	52.9	68.4	75.0	158.6	23.9	10.4

Outperforms "traditional" node embeddings



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Rank-1 can be restrictive



Can only "see" full clique



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What if we have richer structure?

Knowledge Graph:

Relations of the sort: <Trump, is-president, USA> <Merkel, is-chancellor, Germany> <Mitsotakis, is-primeminister, Greece>

Social Graph:

. . .

Social media "influencer" Telemarketer/spammer Near cliques/bipartite cores





The Web Conference (WWW) 2020 w/ Ekta Gujral

Related work



(a) Original Wikipedia Controversy graph (with 'spring embedded' layout [15]). No structure stands out.





(b) VOG: 8 out of the 10 most (c) VOG: The most informative informative structures are stars bipartite graph - 'edit war' - war-(their centers in red - Wikipedia ring factions (one of them, in editors, heavy contributors etc.). the top-left red circle), changing each-other's edits.



(d) VOG: the second most informative bipartite graph - another 'edit war', between vandals (bottom left circle of red points) vs responsible editors (in white).

- Koutra et al. "VoG: Summarizing and Understanding Large Graphs", SDM'14
- Uses a vocabulary of graph structures and tries to compress the graph by using it
- Follow-up work by Shah et al. "TimeCrunch", KDD'15 stitches graph snapshots over time
- Can we automatically extract that rich structure?

We need higher-rank blocks!

Rank 1 Term



Can express richer structure Still has the nice interpretability of CP/PARAFAC



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Block-Term Decomposition



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E. Papalexakis @ OneWorldSP'20

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Beyond Rank-1: Discovering Rich Structure in Multi-Aspect Graphs



Structure Detection and Visualization



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Beyond Rank-1: Discovering Rich Structure in Multi-Aspect Graphs









Detect anomalies / real-life events

Papalexakis, Evangelos E., Nicholas D. Sidiropoulos, and Rasmus Bro. "From k-means to higher-way co-clustering: Multilinear decomposition with sparse latent factors." *IEEE transactions on signal processing* 61, no. 2 (2012): 493-506.

Gorovits, Alexander, Ekta Gujral, Evangelos E. Papalexakis, and Petko Bogdanov. "LARC: Learning activity-regularized overlapping communities across time." KDD 2018

Shen, Yanning, Brian Baingana, and Georgios B. Giannakis.

"Tensor decompositions for identifying directed graph topologies and tracking dynamic networks." IEEE Transactions on Signal Processing 2017

Time-evolving communities



Incremental Decomposition of Streaming Tensors



- Tensor updated in streaming fashion
- New slices arrive
 - New snapshots on a temporal graph
 - In general, new slices (or batches) over time

How can we *incrementally* update the decomposition?

- Nion & Sidiropoulos, Adaptive Algorithms to Track the PARAFAC Decomposition of a Third-Order Tensor, IEEE TSP 2009
- Mardani, Morteza, Gonzalo Mateos, and Georgios B. Giannakis. "Subspace learning and imputation for streaming big data matrices and tensors." IEEE Transactions on Signal Processing, 2015
- Baskaran et al, Accelerated Low-Rank Updates to Tensor Decompositions, IEEE HPEC 2016
- Zhou et al, Accelerating Online CP Decompositions for Higher Order Tensors, ACM KDD 2016
- Sun et al., Beyond Streams and Graphs: Dynamic Tensor Analysis, ACM KDD 2006
 E. Papalexakis @ OneWorldSP'20
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A Tale of Two Sketches

SamBaTen: Sampling-based Batch Incremental Tensor Decomposition

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Abstract

Tensor decompositions are invaluable tools in analyzing multimodal datasets. In many real-world scenarios, such datasets are far from being static, to the contrary they tend to grow over time. For instance, in an online social network setting, as we observe new interactions over time, our dataset gets updated in its "time" mode. How can we maintain a valid and accurate tensor decomposition of such a dynamically evolving multimodal dataset, without having to re-compute the entire decomposition after every single update? In this paper we introduce SAMBATEN, a Sampling-based Batch Incremental Tensor Decomposition algorithm, which incrementally maintains the decomposition given new updates to the tensor dataset. SAMBATEN is able to scale to datasets that the state-of-the-art in incremental tensor decomposition is unable to operate on, due to its ability to effectively summarize the existing tensor and the incoming updates, and perform all computations in the reduced summary space. We extensively evaluate SAMBATEN using synthetic and real datasets. Indicatively, SAMBATEN achieves comparable accuracy to state-of-the-art incremental and non-incremental techniques, while being up to 25-30 times faster. Furthermore, SAMBATEN scales to very large sparse and dense dynamically evolving tensors of dimensions up to $100K \times 100K \times 100K$ where state-of-the-art incremental approaches were not able to operate.



Figure 1: SamBaTen outperforms state-of-the-art baselines while maintaining competitive accuracy.

In a wide array of modern real-world applications, data are far from being static. To the contrary, data get updated dynamically. For instance, in an online social network, new interactions occur every second and new friendships are formed at a similar pace. In the tensor realm, we may view a large proportion of these dynamic updates as an introduction of new "slices" in the tensor: in the social network example, new interactions that happen as time evolves imply the introduction of new snapshots of the network, which grow the tensor in the "time" mode. A tensor decomposition in that tensor can discover *communities* and their evolution over

Gujral et al, SIAM SDM 2018 Randomized index sampling

OCTEN: ONLINE COMPRESSION-BASED TENSOR DECOMPOSITION

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ABSTRACT

Tensor decompositions are powerful tools for large data analytics, as they jointly model multiple aspects of data into one framework and enable the discovery of the latent structures and higher-order correlations within the data. One of the most widely studied and used decompositions, especially in data mining and machine learning, is the Canonical Polyadic or PARAFAC decomposition. However, today's datasets are not static and often grow and change over time. To operate on such large dynamic data, we present OCTEN, the first ever compression-based online parallel implementation for the CP/PARAFAC decomposition. We conduct an extensive empirical analysis of the algorithms in terms of fitness, memory used and CPU time and in order to demonstrate the compression and scalability of the method, we apply OCTEN to big tensor data. Indicatively, OCTEN performs on-par or better than state-of-the-art online and offline methods in terms of decomposition accuracy and efficiency, while achieving memory savings ranging in 40-200%.



Fig. 1: Framework. Compressed tensor summaries $\underline{Y}_{\mathbf{D}}$ and $\underline{Z}_{\mathbf{D}}$ are obtained by applying randomly generated compression matrices $(\mathbf{U}_p, \mathbf{V}_p, \mathbf{W}_p)$ and $(\mathbf{U}'_p, \mathbf{V}'_p, \mathbf{W}'_p)$ to \underline{X}_{old} and \underline{X}_{new} respectively. The updated summaries are computed by $\underline{X}_p = \underline{Y}_p + \underline{Z}_p$. Each \underline{X}_p is independently decomposed in parallel. The update step anchors all compression and factor matrices to a single reference i.e. (P_a, P_b, P_c) and $(\mathbf{A}_s, \mathbf{B}_s, \mathbf{C}_s)$, and solves a linear equation for the overall A, B, and C.

tensors. In this paper, we fill that gap. Our contributions are summarized as follows:

• Novel Parallel Algorithm We introduce OCTEN, a

Gujral et al, IEEE CAMSAP 2019 Randomized compression

Central limiting assumption

• Most streaming work (incl. our work 🙂) assumes:





What if this doesn't hold?



New concept appears

Concept is missing



Identifying and Alleviating Concept Drift in Streaming Tensor Decomposition



 Algorithm for detecting and alleviating concept drift: SeekAndDestroy

ECML-PKDD 2018 w/ Ravdeep Pasricha & Ekta Gujral

R=4

<u>X</u>new

SeekAndDestroy in a nutshell

• At every step we have



- To determine drift:
 - We compute rank(Y) and compare with rank(X)
 - Even if rank(Y) = rank(X), we may have new components
 - We compute matching of components
 - If similarity>threshold, same component
 - Else this is a new component

Detection of Concept Drift



Fig. 4: SeekAndDestroy is able to successfully detect concept drift, which is manifested as changes in the rank throughout the stream.

DataSet	Dimension	Initial Rank	Full Rank
SDS1 SDS2	100 x 100 x 100	2	510
SDS3 SDS4	300 x 300 x 300	2	510
SDS5 SDS6	500 x 500 x 500	2	$5 \\ 10$

Synthetic data with simulated drift

Concept Drift Effects in Reconstruction

Reconstruction error

DataSet	OnlineCP	OnlineCP	SamBaTen	SamBaTen	SeekAndDestroy
	(Initial Rank)	(Full Rank)	(Initial	(Full Rank)	
			Rank)		
SDS1	$0.2782 {\pm} 0.0221$	$0.197{\pm}0.086$	$0.261{\pm}0.048$	$0.317 {\pm} 0.058$	$0.283 {\pm} 0.075$
SDS2	$0.2537 {\pm} 0.0125$	$0.168 {\pm} 0.507$	$0.244{\pm}0.028$	$0.480 {\pm} 0.051$	$0.253{\pm}0.0412$
SDS3	$0.2731 {\pm} 0.0207$	$0.205 {\pm} 0.164$	$0.385 {\pm} 0.021$	$0.445 {\pm} 0.164$	$0.266{\pm}0.081$
SDS4	$0.245 {\pm} 0.013$	$0.171 {\pm} 0.537$	$0.299 {\pm} 0.045$	$0.402{\pm}0.049$	$0.221{\pm}0.0423$
SDS5	$0.2719 {\pm} 0.0198$	$0.206 {\pm} 0.022$	$0.559 {\pm} 0.046$	$0.519 {\pm} 0.0219$	$0.256{\pm}0.105$
SDS6	$0.238 {\pm} 0.013$	$0.171 {\pm} 0.374$	$0.510 {\pm} 0.036$	$0.547 {\pm} 0.0276$	$0.208{\pm}0.0433$

DataSet	Dimension	Initial Rank	Full Rank
SDS1 SDS2	100 x 100 x 100	2	$5 \\ 10$
SDS3 SDS4	300 x 300 x 300	2	510
SDS5 SDS6	500 x 500 x 500	2	$5 \\ 10$

Synthetic data with simulated drift

If final/full rank is unknown:

SeekAndDestroy can detect drift and have lower error than SOTA

If final rank is known (*unrealistic***):** SOTA performs on par or better



Evidence of drift in real data

Running	Predicted	Batch	Approximation Error			
Rank	Full Rank	Size	SeekAndDestroy	SambaTen	OnlineCP	
7 ± 0.88	4 ± 0.57	22	$\boldsymbol{0.68} \pm \boldsymbol{0.002}$	0.759 ± 0.059	0.941 ± 0.001	

When streaming Enron, we encounter a number of drifting communities that other methods miss







What's happening here??



https://arxiv.org/pdf/1412.6572.pdf : fast gradient sign method

What's happening here??

Distance/Angle	Subtle Poster	Subtle Poster Right Turn	Camouflage Graffiti	Camouflage Art (LISA-CNN)	Camouflage Art (GTSRB-CNN)
5′ 0°	STOP		STOP	STOP	STOP
5′ 15°	STOP		STOP	STOP	STOP
10′ 0°	STOP			STOP	STOP
10′ 30°				STOP	STOP
40′ 0°					
Targeted-Attack Success	100%	73.33%	66.67%	100%	80%

Table 1: Sample of physical adversarial examples against LISA-CNN and GTSRB-CNN.

https://arxiv.org/pdf/1707.08945.pdf

Not just an "academic curiosity"

MIT Technology Review

Computing / Cybersecurity

Hackers can trick a Tesla into accelerating by 50 miles per hour

A two inch piece of tape fooled the Tesla's cameras and made the car quickly and mistakenly speed up.

by Patrick Howell O'Neill

February 19, 2020

https://www.technologyreview.com/2020/02/19/868188/hackers-can-trick-a-tesla-into-accelerating-by-50-miles-per-hour/

"Defense" Problem Definition

For a given model *C*:

- x: clean instance, x': perturbed instance
- Goal of adversarial attack:

 $x' = x + \delta \Rightarrow C(x) \neq C(x')$ while $||x - x'|| < \tau$



• Goal of defense mechanism: apply a preprocessing operation g(.) that brings back x' closer to the clean instance x such that: C(g(x')) = C(x)

Attack on Graph Convolutional Networks



Nettack [Zugner et al. KDD18]

All you need is low (rank)





WSDM 2020 w/ Negin Entezari

All you need is low (rank)



	Method	CiteSeer	Cora-ML	PoliticalBlogs
CCN	Clean	0.83	0.82	0.90
GUN	Nettack	0.02	0.01	0.06
t-PINE	Clean	0.74	0.68	0.87
	Nettack	0.72	0.64	0.30

R

WSDM 2020 w/ Negin Entezari

Attack in images is of highfrequency!

SHIELD: Fast, Practical Defense and Vaccination for Deep Learning using JPEG Compression



JPEG compression removes adversarial perturbations

https://arxiv.org/pdf/1802.06816.pdf

Tensor-Based Defense Mechanism



On-going work & arXiv:2002.10252 w/ Negin Entezari

Choice of Tensor Model

• CP/PARAFAC:

- Pros: Interpretable latent factors
- Cons: Slow,

Restricted to have same ranks for all modes and super-diagonal core makes it not a suitable choice for image decomposition

• Tucker:

- Pros: No constraint on the core tensor Each mode can have a different rank
- Cons: Slow latent factors are not easily interpretable
- Tensor-Train:
 - Pros: No constraint on ranks of different modes Linearly scalable with respect to tensor dimension
 - Cons: latent factors are not easily interpretable







On-going work & arXiv:2002.10252 w/ Negin Entezari

How to Represent Batch of Images?

- Batching images in either
 - 4-mode tensor
 - Stack all slices into 3-mode tensor
- Through batching we
 - Amortize computational cost
 - Leverage patterns across images





On-going work & arXiv:2002.10252 w/ Negin Entezari

Experimental Results

Configurations	PGD	FGSM	i-FGSM	Runtime
	$(\epsilon = 4)$	$(\epsilon = 4)$	$(\epsilon = 4)$	(seconds)
No defense	11.10	18.40	7.49	
[Tensor-Train, 4-mode, 5, [5,90,3]]	51.53	43.59	50.46	675
[Tensor-Train, 4-mode, 10, [10,100,3]]	51.01	43.10	49.95	605
[Tensor-Train, 3-mode, 1, 40]	49.75	42.32	48.52	530
[Tucker, 3-mode-stacked, 30, [105,105,90]]	49.37	40.07	48.79	1050
[Parafac, 3-mode, 1, 60]	48.11	41.38	49.75	5500
SLQ	44.60	29.40	38.60	410



On-going work & arXiv:2002.10252 w/ Negin Entezari



Tensors Everywhere!

- Unsupervised exploratory analysis

 Challenges:
 - Is there structure in the data? What kind?
 - How many useful patterns in the data?
 - Which model should I use?
- Tensors in a Brave New World
 - Interplay of traditional tensor methods & deep learning
 - E.g., defending against adv. attacks

Thank you! Questions?

How to reach me: <u>http://www.cs.ucr.edu/~epapalex/</u>



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