Some Derivations and Further Simulation Results for "Semidefinite Relaxation and Approximation Analysis of a Beamformed Alamouti Scheme for Relay Beamforming Networks"

Technical Report
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Abstract— This is a companion technical report of the manuscript "Semidefinite Relaxation and Approximation Analysis of a Beamformed Alamouti Scheme for Relay Beamforming Networks". This report serves to give detailed derivations of the system model for the BF Alamouti AF scheme and provide more simulation results to verify the viability of the BF Alamouti AF schemes developed in the aforementioned manuscript.

In the main paper [1], we propose the BF Alamouti AF scheme for the two-hop one-way relay networks. Specifically, the new AF scheme aims at exploring 2 DoFs in the relay AF structure for improving users' SINRs. In this technical report, we give a detailed explanation of the system model and provide supplementary simulation results in Sections 1 and 2.

1 Exact Expression of the Received Signal at User-(k, i)

To help the readers understand the BF Alamouti AF structure, we write the receive signals at user-(k, i) as follows:

$$\begin{aligned}
\mathbf{y}_{k,i}(m) =& [\ y_{k,i}(2m),\ y_{k,i}(2m+1)\ ] \qquad (1)\\
=& \sum_{\ell=1}^{L} (g_{k,i}^{\ell})^{*}[w_{1,\ell}, w_{2,\ell}]\mathbf{C}(\mathbf{r}_{\ell}(m)) + [v_{k,i}(2m),\ v_{k,i}(2m+1)],\\
=& \sum_{\ell=1}^{L} (g_{k}^{\ell})^{*}[w_{1,\ell}, w_{2,\ell}] \begin{bmatrix} r^{\ell}(2m) & r^{\ell}(2m+1) \\ -r^{\ell}(2m+1)^{*} & r^{\ell}(2m)^{*} \end{bmatrix} + [v_{k,i}(2m),\ v_{k,i}(2m+1)],\\
=& \sum_{\ell=1}^{L} [(g_{k,i}^{\ell})^{*} w_{1,\ell} f_{k}^{\ell}, (g_{k,i}^{\ell})^{*} w_{2,\ell} f_{k}^{\ell *}] \begin{bmatrix} s_k(2m) & s_k(2m+1) \\ -s_k(2m+1)^{*} & s_k(2m)^{*} \end{bmatrix}\\
&+ \sum_{\ell=1}^{L} [(g_{k,i}^{\ell})^{*} w_{1,\ell}, (g_{k,i}^{\ell})^{*} w_{2,\ell}] \begin{bmatrix} n^{\ell}(2m) & n^{\ell}(2m+1) \\ -n^{\ell}(2m+1)^{*} & n^{\ell}(2m)^{*} \end{bmatrix} + [v_{k,i}(2m),\ v_{k,i}(2m+1)].
\end{aligned}$$

This will lead to the SINR expression in the main paper [1].

2 Further Simulation Results

In the main manuscript, we have already present the numerical results for the distributed relay network. In this section, we focus on an MIMO relay network and compare the performance of different AF schemes. Again, our numerical results will demonstrate the superiority of the proposed BF Alamouti AF scheme. We assume without loss of generality that each multicast group has an equal number of users (i.e., $m_k = M/G$ for $k = 1, \ldots, G$). The channels $\boldsymbol{f}_k, \boldsymbol{g}_{k,i}$, where $k = 1, \ldots, G$ and $i = 1, \ldots, m_k$, are identically and independently distributed (i.i.d.) according to $\mathcal{CN}(\mathbf{0}, \boldsymbol{I})$. The transmitted signal at each transmitter is with power 0dB (i.e., $P_j = 0$dB for $j = 1, \ldots, G$). Each single-antenna relay has the same noise power (i.e., $\sigma_\ell^2 = \sigma_{\mathsf{ant}}^2$, where $\ell = 1, \ldots, L$), and all users have the same noise power (i.e., $\sigma_{k,i}^2 = \sigma_{\mathsf{user}}^2$ for $k = 1, \ldots, G$ and $i = 1, \ldots, m_k$). We assume that $\sigma_{\mathsf{ant}}^2 > 0$ and $\sigma_{\mathsf{user}}^2 > 0$. The total power threshold for all the relays is $\bar{P}_0$; the power threshold at ℓth relay is $\bar{P}_\ell$, where $\ell = 1, \ldots, L$. For each AF scheme, 100 channel realizations were averaged to get the plots, and 1,000 trials were made in the Gaussian randomization algorithm to generate the BF AF and BF Alamouti weights.

2.1 Worst User's SINR versus Total Power Threshold

In this simulation, we vary the total power budget at relays to see the worst user's SINR performance. For ease of exposition, we consider the scenario where only the total power constraint is present. For the MIMO relay case in Figure 1, we assume that there are $L = 4$ single-antenna relays and $G = 2$ multicast groups with a total of $M = 12$ users; i.e., each multicast group has 6 users. We set $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$. From the figure, we see that the objective values (obj.) of (R1SDR) and (R2SDR) are the same and they serve as upper bounds for the SDR-based BF AF scheme and the SDR-based BF Alamouti AF scheme, respectively. Moreover, we see that the BF Alamouti AF scheme has significantly better SINR performance than the BF AF scheme in all power regimes.

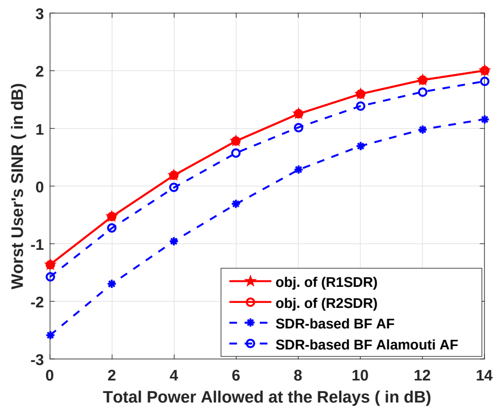

Figure 1: Worst user's SINR versus total power threshold at the MIMO relay: $L = 4$, $G = 2$, $M = 16$, $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$.

2.2 Worst User's SINR versus Number of Per-relay Power Constraints

In this simulation, we consider the scenario where both the total power constraint and per-antenna power constraints are present and the primary users are absent. Our purpose is to see how the worst user's SINR scales with the number of per-relay power constraints. Specifically, Figure 2 shows the MIMO relay case with $L = 4$, $G = 2$, $M = 16$, where the total power threshold is $\bar{P}_0 = 4$dB and the per-relay power threshold is -5dB for all relays (i.e., $\bar{P}_1 = \cdots = \bar{P}_L = -5$dB). We set $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$ and vary the number of per-relay power constraints from 0 to L to compare the SINR performance of different AF schemes. It shows that the BF Alamouti AF scheme outperforms the BF AF scheme. As the number of per-relay power constraints increases, the SINRs diverge from their SDR upper bounds, and both the BF AF and BF Alamouti AF schemes exhibit the same scaling with L, which is consistent with the approximation bounds in terms of J in Proposition 1 and Theorem 1.

2.3 Worst User's SINR versus Number of Primary Users

Similar to previous simulations, here we show the worst user's SINR scaling with the number of primary users. To set up the problem, we consider the scenario where the total power constraint and the primary users' interference constraints are present. We assume that $L = 4$, $G = 2$, and $M = 12$ in the MIMO relay network. We set $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$, the total power budget $\bar{P}_0 = 10$dB, and the noise power at all primary users to be $\sigma^2_{\mathsf{u}} = 0.25$. Moreover, we assume that the primary users are subject to an interference power threshold of $b_u = 3$dB. Figure 3 shows the worst user's SINR as the number of primary users in the network increses. From the figure, we see that as the number

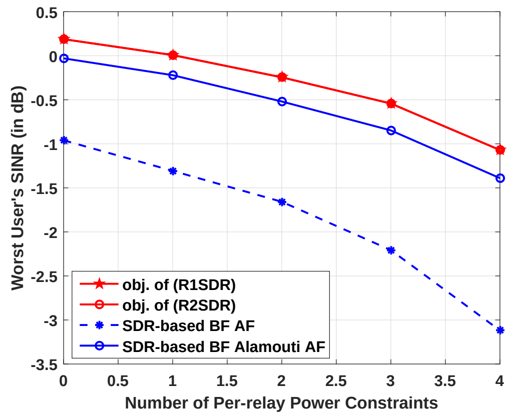

Figure 2: Worst user's SINR versus number of per-relay power constraints in the MIMO relay network: $L = 4$, $G = 2$, $M = 16$, $\bar{P}_0 = 4$dB, $\bar{P}_\ell = -5$dB for $\ell = 1, \ldots, L$, $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$.

of primary users increases, the SINRs of both the BF AF and BF Alamouti AF schemes diverge from their SDR upper bounds. Moreover, the BF Alamouti AF scheme shows a significantly better performance than the BF AF scheme. These results further validate Proposition 1 and Theorem 1 in terms of the scaling of J.

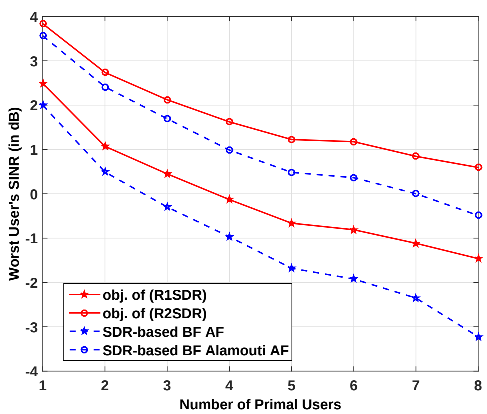

Figure 3: Worst user's SINR versus number of primary users in the MIMO CR relay network: $L = 4$, $G = 2$, $M = 12$, $\bar{P}_0 = 10$dB, $b_u = 3$dB for $u = 1, \ldots, U$, $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$, $\sigma^2_{\mathsf{u}} = 0.25$.

2.4 Actual Bit Error Rate (BER) Performance

To further demonstrate the efficacy of the proposed AF scheme, we study the actual coded bit error rate (BER) performance of the scenario setting in Figure 1. The resulting BERs are shown in Figure 4. To simulate the SDR bound in the BER plots, we assume that there exists an SISO channel whose SINR is equal to $\gamma(\boldsymbol{W}^\star)$ or $\theta(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$. In our simulations, we adopt a gray-coded QPSK modulation scheme and a rate-1/3 turbo code in [2] with a codelength of 2,880 bits. We simulate 100 code blocks for each channel realization and thus the BER reliability level is 10e−4. We see that the actual BER performance of the proposed BF Alamouti AF scheme indeed outperforms the BF AF scheme at almost all power thresholds. The results are consistent with those SINR results in Figure 1 and show that BF Alamouti AF can achieve good performance in practice.

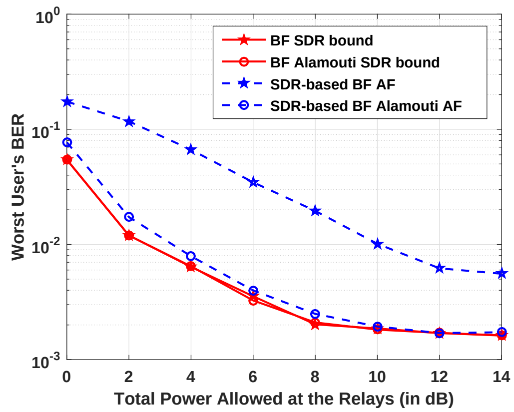

Figure 4: Worst user's BER achieved by different AF schemes versus total power threshold at the MIMO relay: $L = 4$, $G = 2$, $M = 16$, $\sigma^2_{\mathsf{ant}} = \sigma^2_{\mathsf{user}} = 0.25$. A rate-$\frac{1}{3}$ turbo code with codelength 2,880 is used.

References

[1] S. X. Wu, A. M.-C. So, J. Pan, and W.-K. Ma, "Semidefinite relaxation and approximation analysis of a beamformed Alamouti scheme for relay beamforming networks," *submitted IEEE Trans. Signal Process.*, 2016, available online at `http://arxiv.org/abs/1603.05680`.

[2] *IEEE Standard for Local and Metropolitan Area Networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems*, IEEE Std. 802.16e, 2005.