A POLYNOMIAL OPTIMIZATION APPROACH FOR ROBUST BEAMFORMING DESIGN IN A DEVICE-TO-DEVICE TWO-HOP ONE-WAY RELAY NETWORK

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ABSTRACT

In this paper, we consider the robust beamforming design in a device-to-device (D2D) two-hop one-way relay network. Specifically, we study the amplify-and-forward (AF) scheme in the scenario where both the transmitter-to-relays link and relays-to-receiver link are subject to estimation errors. Assuming that those errors lie in a ball with bounded radius, the resulting design problem can be formulated as a semi-infinite program (SIP) that involves high-degree polynomial inequality constraints, which is difficult to deal with in general. In this paper, we employ the semidefinite relaxation (SDR) technique and tools from polynomial optimization to construct a safe approximation of the aforementioned SIP. Furthermore, we propose an alternating algorithm to tackle the safe approximation. To the best of our knowledge, our work is the first to provide an efficient algorithmic approach to the aforementioned robust beamforming design problem. In addition, our numerical results show that the proposed robust beamforming design is more reliable than the non-robust counterpart, and it can achieve better signal-to-noise ratios (SNRs) than the existing linear approximation approach, which ignores error terms with degree higher than one.

***Index Terms*—** device-to-device (D2D) relay network, amplify-and-forward (AF), quartic polynomial optimization, semidefinite relaxation (SDR).

1. INTRODUCTION

Nowadays, the newly-arising applications such as content distribution and location-aware advertisement motivate the device-to-device (D2D) communications in cellular networks, wherein multi-hop relays are the key technology in facilitating information delivery by amplifying-and-forwading (AF) received signals [1–3]. A critical issue in the D2D network is that normally there is no reference signaling specific for D2D users and sometimes the quantized channel state information (CSI) has to be passed from the base-station. Hence, it is generally impossible for the D2D users to have perfect CSI [2]. Taking into account the channel uncertainty, the robust relay beamforming design in D2D networks [4] becomes much more involved than the non-robust counterpart [5], as it contains infinitely many constraints. In a two-hop one-way relay network, when only one of the channel links has errors, it is well-known that the S-lemma can be applied to turn the semi-infinite program (SIP) [6, 7] to a tractable semidefinite program (SDP) [8, 9]. However, if both the transmitter-to-relays link and relays-to-receiver link have errors, then the SIP is in general difficult to deal with. So far, we are not aware of any work that provides a reliable robust design for this scenario.

In this paper, we address the aforementioned robust problem under the setting where the design is performed at a central unit in the network. Specifically, we aim at maximizing signal-to-noise (SNR) for the D2D transceiver link with a given power budget at the relays. The resulting SIP involves high-degree polynomial inequality constraints on the beamforming vector and the two channel error vectors and is non-convex in general. To tackle this problem, we employ the semidefinite relaxation (SDR) technique and tools from polynomial optimization to derive a safe approximation of it. The safe approximation just mentioned can be solved by alternately optimizing over the channel error vectors and the beamforming vector. The former amounts to solving a non-convex inhomogeneous quartic program, which can be tackled by homogenization and SDR techniques. The latter boils down to checking the feasibility of a sequence of SDPs, and a candidate beamforming vector can be extracted via the Gaussian randomization procedure. The two optimization problems mentioned above constitute one iteration in the alternating process, and an approximate solution to the original problem is obtained when the whole process converges.

Our contribution in this work is threefold. First, we consider and formulate a hard robust design problem that has not been well addressed in the literature. Second, we obtain, for the first time, a safe approximation of the aforementioned problem. Third, we propose an efficient alternating algorithm for finding an approximate solution to our target problem. It is worth mentioning that the authors in [4] considered almost the same scenario as ours, but they ignored the high-degree errors by using a linear approximation of the polynomials. This may cause serious stability and reliability issues. Our design, however, considers a safe approximation to overcome such issues. Our numerical results show that the proposed safe approximation approach is more reliable than the non-robust counterpart. Moreover, a significant SNR gain is achieved over the linear approximation design in [4].

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a D2D transceiver pair in a cellular system, where the system consists of D2D user equipment (DUE) transmitters (DTxs), DUE relays and DUE receivers (DRxs), each of which is equipped with a single antenna. To avoid conflicts between different DUEs, we assume that there is a coordinator in the system to schedule only one DTx-DRx pair to be active at a time [1, 2]. Moreover, since the DTx and DRx are far apart, we assume that there is no direct transmitter-receiver link, and information delivery is enabled by the DUE relays, which AF received signals from the transmitter to the receiver. Specifically, the information is transmitted through two links [4, 5]. One is the *transmitter-to-relays link*, through which the transmitter sends information to relays. We assume that there are L DUE relays in the network. Then, the receive model is given by

$$\mathbf{r}(t) = \boldsymbol{f} s(t) + \mathbf{n}(t), \tag{1}$$

where $s(t)$ is the common information with $\mathbb{E}[|s(t)|^2] = P_t$ and P_t is the transmit power at the transmitter; $\boldsymbol{f} \in \mathbb{C}^L$ is the channel from the transmitter to the relays; $\mathbf{n}(t) = [n_1(t), \ldots, n_\ell(t), \ldots, n_L(t)]^T$ and $n_\ell(t)$ is the white noise at relay-ℓ with variance σ_ℓ^2. The other is the *relays-to-receiver link*, through which relays amplify and forward the received signal to the receiver. In this paper, we target at the relay beamforming scheme for which the AF process at the relay side is given by

$$\mathbf{x}(t) = \mathrm{Diag}(\mathbf{w})\mathbf{r}(t), \text{[1]} \tag{2}$$

where $\mathbf{w} = [w_1, \ldots, w_\ell, \ldots, w_L]^T$ and w_ℓ is the AF weight at relay ℓ. Under this model, the received signal can be expressed as

$$y(t) = \boldsymbol{g}^H \mathbf{x}(t) + v(t), \tag{3}$$

where $\boldsymbol{g} \in \mathbb{C}^L$ is the channel from the relays to the receiver; $v(t)$ is the white noise at the receiver with variance σ_v^2. Then, the SNR at the receiver can be expressed as

$$\mathrm{SNR} = \frac{\mathbf{w}^H P_t (\boldsymbol{f} \odot \boldsymbol{g}^*)(\boldsymbol{f} \odot \boldsymbol{g}^*)^H \mathbf{w}}{\mathbf{w}^H \mathrm{Diag}([|g^1|^2\sigma_1^2, |g^2|^2\sigma_2^2, \ldots, |g^L|^2\sigma_L^2])\mathbf{w} + \sigma_v^2}.$$

In this work, we model the uncertain channels for the transmitter-to-relays link and relays-to-receiver link as $\boldsymbol{f} = \bar{\boldsymbol{f}} + \epsilon\mathbf{x}$ and $\boldsymbol{g} = \bar{\boldsymbol{g}} + \eta\mathbf{y}$, respectively, where $\bar{\boldsymbol{f}}$ and $\bar{\boldsymbol{g}}$ are the estimated channels, $\mathbf{x}$ and $\mathbf{y}$ are error vectors such that $\|\mathbf{x}\| \le 1$, $\|\mathbf{y}\| \le 1$, and ϵ, η are known scalars to bound the error magnitudes [4, 5]. Our goal here is to design the AF weight $\mathbf{w}$ under a given power budget at the relays such that the receive SNR of the D2D transceiver pair is maximized while accounting for the channel errors in both two-way links. Specifically, the formulation of the robust relay beamforming design problem is similar to that in [4], where the worst-case SNR at the user is maximized subject to a power constraint; i.e.,

$$\begin{aligned}
(\mathrm{BF}) \quad & \max_{\mathbf{w} \in \mathbb{C}^L,\, \gamma} \quad \gamma \\
\text{s.t.} \quad & \max_{\|\mathbf{x}\| \le 1} p_1(\mathbf{x}, \mathbf{w}) = \mathbf{w}^H \left(\mathbf{I} \odot (\boldsymbol{f}\boldsymbol{f}^H) + \boldsymbol{\Sigma} \right) \mathbf{w} \le P, && \text{(4a)} \\
& \min_{\|\mathbf{x}\| \le 1, \|\mathbf{y}\| \le 1} \quad p_2(\mathbf{x}, \mathbf{y}, \mathbf{w}, \gamma) \ge 0, && \text{(4b)}
\end{aligned}$$

where $\boldsymbol{\Sigma} = \mathrm{Diag}([\sigma_1^2, \ldots, \sigma_L^2])$,

$$\begin{aligned}
p_1(\mathbf{x}, \mathbf{w}) \triangleq & \sum_{i=1}^{L} P_t w_i w_i^* (\epsilon^2 x_i x_i^* + \epsilon \bar{f}_i x_i^* + \epsilon \bar{f}_i^* x_i) \\
& + \sum_{i=1}^{L} w_i w_i^* (P_t \bar{f}_i \bar{f}_i^* + \sigma_i^2),
\end{aligned} \tag{5}$$

and $p_2(\mathbf{x}, \mathbf{y}, \mathbf{w}, \gamma) = \mathrm{SNR} - \gamma$ is defined in a similar fashion as (5) (we omit the details due to the page limit). Note that herein maximizing the worst-case SNR is equivalent to maximizing γ. It can be verified that p_2 is an inhomogeneous quartic polynomial in $(\mathbf{x}, \mathbf{y})$ when $\mathbf{w}$ and γ are fixed, and is a quadratic polynomial in $\mathbf{w}$ when $\mathbf{x}$, $\mathbf{y}$, and γ are fixed. Problem (BF) is an SIP [6,7], which is generally difficult to deal with. In fact, it is NP-hard even when $\mathbf{x} = \mathbf{y} = \mathbf{0}$. When accounting for channel uncertainty, the problem becomes even more complicated, as verifying the constraint (4b) for a given pair $(\mathbf{w}, \gamma)$ involves solving a non-convex quartic optimization problem. To the best of our knowledge, there has been no previous attempt to tackle Problem (BF). In the sequel, we shall develop the first tractable safe approximation of Problem (BF) by employing the SDR technique and tools from polynomial optimization.

[1] The operator $\mathrm{Diag}(\mathbf{v})$ will output a diagonal matrix with the elements of the vector $\mathbf{v}$ on the diagonal.

3. A SAFE APPROXIMATION OF PROBLEM (BF)

3.1. Homogenization of the Polynomials

To begin, we first homogenize $p_1(\mathbf{x}, \mathbf{w})$ and $p_2(\mathbf{x}, \mathbf{y}, \mathbf{w}, \gamma)$ by introducing $\hat{\mathbf{x}} = \begin{pmatrix} \mathbf{x} \\ x_{L+1} \end{pmatrix}$ and $\hat{\mathbf{y}} = \begin{pmatrix} \mathbf{y} \\ y_{L+1} \end{pmatrix}$ [10, 11], where $x_{L+1} = 1$ and $y_{L+1} = 1$. Then, $p_1(\mathbf{x}, \mathbf{w})$ can be rewritten as

$$\begin{aligned}
\hat{p}_1(\hat{\mathbf{x}}, \mathbf{w}) \triangleq & \sum_{i=1}^{L} P_t w_i w_i^* (\epsilon^2 x_i x_i^* + \epsilon \bar{f}_i x_i^* x_{L+1} + \epsilon \bar{f}_i^* x_{L+1}^* x_i) \\
& + \sum_{i=1}^{L} w_i w_i^* (P_t \bar{f}_i \bar{f}_i^* + \sigma_i^2).
\end{aligned} \tag{6}$$

By letting $\mathbf{W} = (\mathbf{w}^*)(\mathbf{w}^*)^H$ and defining

$$\mathbf{J} = \begin{bmatrix} \epsilon^2 P_t (\mathbf{I} \odot \mathbf{W}) & \epsilon P_t (\mathbf{I} \odot \mathbf{W}) \bar{\boldsymbol{f}} \\ \epsilon P_t \bar{\boldsymbol{f}}^H (\mathbf{I} \odot \mathbf{W}) & \mathrm{Tr}((\mathbf{I} \odot \mathbf{W})(P_t \bar{\boldsymbol{f}} \bar{\boldsymbol{f}}^H + \boldsymbol{\Sigma})) \end{bmatrix}, \tag{7}$$

we have $\hat{p}_1(\hat{\mathbf{x}}, \mathbf{w}) = \hat{\mathbf{x}}^H \mathbf{J} \hat{\mathbf{x}} = \mathrm{Tr}(\mathbf{J}\mathbf{X}^*)$ with $\mathbf{X} = (\hat{\mathbf{x}}^*)(\hat{\mathbf{x}}^*)^H$. Moreover, since $\|\mathbf{x}\| \le 1$ and $x_{L+1} = 1$, by applying SDR, we obtain the following conservative reformulation of constraint (4a):[2]

$$\max_{\mathrm{Tr}(\mathbf{X}) \le 2} \mathrm{Tr}(\mathbf{J}\mathbf{X}^*) \le P. \tag{8}$$

Similarly, a conservative reformulation for constraint (4b) is given by

$$\min_{\|\hat{\mathbf{x}}\| \le \sqrt{2}, \|\hat{\mathbf{y}}\| \le \sqrt{2}} \hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma) \ge 0, \tag{9}$$

where $\hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma)$ is derived in a similar fashion as $\hat{p}_1(\hat{\mathbf{x}}, \mathbf{w})$ (we omit the details due to the page limit). For a given pair $(\mathbf{w}, \gamma)$, we may decompose $\hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma)$ as

$$\hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma) = s_4(\mathbf{X}, \mathbf{Y}) + s_2(\mathbf{X}, \mathbf{Y}) + s_0(\mathbf{X}, \mathbf{Y}),$$

where $\mathbf{X} = (\hat{\mathbf{x}}^*)(\hat{\mathbf{x}}^*)^H$, $\mathbf{Y} = (\hat{\mathbf{y}}^*)(\hat{\mathbf{y}}^*)^H$, and $s_4(\mathbf{X}, \mathbf{Y})$, $s_2(\mathbf{X}, \mathbf{Y})$, $s_0(\mathbf{X}, \mathbf{Y})$ represent the quartic, quadratic, and constant terms in $\hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma)$, respectively. In the next section, we shall derive explicit expressions for s_4, s_2, and s_0. For ease of presentation, let X_{ij} denote the (i, j)th element of the matrix $\mathbf{X}$ and define the operator $\mathrm{vecb}(\cdot)$ by

$$\mathrm{vecb}\left(\begin{bmatrix} \mathbf{Z} & \mathbf{z}_1 \\ \mathbf{z}_2^H & z \end{bmatrix} \right) = [\mathrm{vec}(\mathbf{Z}); \mathrm{vec}(\mathbf{z}_1); \mathrm{vec}(\mathbf{z}_2^H); z], \tag{10}$$

where $\mathrm{vec}(\cdot)$ is the conventional column vectorization operator.

3.2. Explicit Expression for $\hat{p}_2$

3.2.1. *Quartic Term s_4*

The quartic term $s_4(\mathbf{X}, \mathbf{Y})$ is bi-linear in $\mathbf{X}$ and $\mathbf{Y}$ and we can rewrite it as

$$s_4(\mathbf{X}, \mathbf{Y}) = \mathrm{vecb}(\mathbf{X})^H \bar{\mathbf{B}} \mathrm{vecb}(\mathbf{Y}),$$

[2] The reader may notice that constraint (4a) can be exactly reformulated as a linear matrix inequality using the S-lemma. The reason we chose to present an alternative, conservative reformulation of (4a) is to illustrate the homogenization and SDR techniques, which can be used to derive a conservative reformulation of constraint (4b). We remark that constraint (4b) does not admit an exact convex reformulation.

where the folded matrix $\bar{\mathbf{B}}$ has the following block structure:

$$\bar{\mathbf{B}} = \left[\begin{array}{c|c|c|c} \mathbf{B}_1 & \mathbf{B}_5 & \mathbf{B}_4 & \mathbf{0} \\ \hline \mathbf{B}_3 & \mathbf{B}_7 & \mathbf{B}_8 & \mathbf{0} \\ \hline \mathbf{B}_2 & \mathbf{B}_9 & \mathbf{B}_6 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \end{array}\right]. \tag{11}$$

Recall that $\mathbf{W} = (\mathbf{w}^*)(\mathbf{w}^*)^H$. Then, we have the following:

- $\mathbf{B}_1 \in \mathbb{C}^{L^2\times L^2} = \epsilon^2\eta^2\frac{P_t}{\sigma_v^2}\mathrm{Diag}(\mathrm{vec}(\mathbf{W}))$.
- $\mathbf{B}_2$ is an $L \times L^2$ matrix, is a horizontal alignment of L block matrices; i.e., $\mathbf{B}_2 = [\mathbf{B}_2^{(1)}, \mathbf{B}_2^{(2)}, \ldots, \mathbf{B}_2^{(\ell)}, \ldots, \mathbf{B}_2^{(L)}]$. It follows that all rows in $\mathbf{B}_2^{(\ell)}$ are zeroes except that the ℓth row is $\epsilon\eta^2\frac{Pt}{\sigma_v^2}[\bar{f}_1W_{1\ell}, \bar{f}_2W_{2\ell}, \ldots, \bar{f}_LW_{L\ell}]$.
- $\mathbf{B}_3 \in \mathbb{C}^{L\times L^2}$ is a horizontal alignment of L block diagonal matrices; i.e., $\mathbf{B}_3 = [\mathbf{B}_3^{(1)}, \mathbf{B}_3^{(2)}, \ldots, \mathbf{B}_3^{(\ell)}, \ldots, \mathbf{B}_3^{(L)}]$, with $\mathbf{B}_3^{(\ell)} = \epsilon\eta^2\frac{P_t}{\sigma_v^2}\mathrm{Diag}([\bar{f}_\ell^*W_{1\ell}, \bar{f}_\ell^*W_{2\ell}, \ldots, \bar{f}_\ell^*W_{L\ell}])$.
- $\mathbf{B}_4$ is an $L^2 \times L$ matrix and is a vertical alignment of L block matrices; i.e., $\mathbf{B}_4 = [\mathbf{B}_4^{(1)}; \mathbf{B}_4^{(2)}; \ldots; \mathbf{B}_4^{(\ell)}; \ldots; \mathbf{B}_4^{(L)}]$, where all columns in $\mathbf{B}_4^{(\ell)}$ are zeroes except that the ℓth column is $\epsilon^2\eta\frac{P_t}{\sigma_v^2}[\bar{g}_1^*W_{1\ell}, \bar{g}_2^*W_{2\ell}, \ldots, \bar{g}_L^*W_{L\ell}]^T$.
- $\mathbf{B}_5 \in \mathbb{C}^{L^2\times L}$ is a vertical alignment of L block diagonal matrices; i.e., $\mathbf{B}_5 = [\mathbf{B}_5^{(1)}; \mathbf{B}_5^{(2)}; \ldots; \mathbf{B}_5^{(\ell)}; \ldots; \mathbf{B}_5^{(L)}]$, with $\mathbf{B}_5^{(\ell)} = \epsilon^2\eta\frac{P_t}{\sigma_v^2}\mathrm{Diag}([\bar{g}_\ell W_{1\ell}, \bar{g}_\ell W_{2\ell}, \ldots, \bar{g}_\ell W_{L\ell}])$.
- $\mathbf{B}_6 = \epsilon\eta\frac{P_t}{\sigma_v^2}\mathrm{Diag}((\bar{\boldsymbol{f}}^*\odot\bar{\boldsymbol{g}})^H\mathbf{W})$, $\mathbf{B}_7 = \epsilon\eta\frac{P_t}{\sigma_v^2}\mathrm{Diag}(\mathbf{W}(\bar{\boldsymbol{f}}^*\odot\bar{\boldsymbol{g}}))$, $\mathbf{B}_8 = \epsilon\eta\frac{P_t}{\sigma_v^2}(\bar{\boldsymbol{g}}^*\bar{\boldsymbol{f}}^H)\odot\mathbf{W}$, and $\mathbf{B}_9 = \epsilon\eta\frac{P_t}{\sigma_v^2}(\bar{\boldsymbol{g}}\bar{\boldsymbol{f}}^T)\odot\mathbf{W}^T$.

3.2.2. *Quadratic Term s_2*

The quadratic term $s_2(\mathbf{X}, \mathbf{Y})$ can be expressed as

$$s_2(\mathbf{X}, \mathbf{Y}) = \mathrm{vecb}(\mathbf{X})^H\mathrm{vecb}(\mathbf{P}) + \mathrm{vecb}(\mathbf{R})^H\mathrm{vecb}(\mathbf{Y}), \tag{12}$$

where $\mathbf{P}$ and $\mathbf{R}$ are Hermitian matrices given by $\mathbf{P} = \begin{bmatrix}\mathbf{P}_1 & \mathbf{p}\\ \mathbf{p}^H & 0\end{bmatrix}$, $\mathbf{R} = \begin{bmatrix}\mathbf{R}_1 & \mathbf{r}\\ \mathbf{r}^H & 0\end{bmatrix}$ with $\mathbf{P}_1, \mathbf{R}_1 \in \mathbb{C}^{L\times L}$, $\mathbf{p}, \mathbf{r} \in \mathbb{C}^{L\times 1}$. In particular, we have $\mathbf{P}_1 = \frac{P_t\epsilon^2}{\sigma_v^2}((\bar{\boldsymbol{g}}^*)(\bar{\boldsymbol{g}}^*)^H)\odot\mathbf{W}$, $\mathbf{p} = \frac{P_t\epsilon}{\sigma_v^2}\Big(\mathbf{W}(\bar{\boldsymbol{f}}^*\odot\bar{\boldsymbol{g}})\Big)\odot\bar{\boldsymbol{g}}^*$, $\mathbf{R}_1 = \frac{P_t\eta^2}{\sigma_v^2}\mathbf{W}^T\odot\Big((\bar{\boldsymbol{f}}^*)(\bar{\boldsymbol{f}}^*)^H\Big) - \frac{\gamma\eta^2}{\sigma_v^2}(\mathbf{I}\odot\mathbf{W})\boldsymbol{\Sigma}$, and $\mathbf{r} = \frac{P_t\eta}{\sigma_v^2}\Big(\mathbf{W}^T(\bar{\boldsymbol{f}}\odot\bar{\boldsymbol{g}}^*)\Big)\odot\bar{\boldsymbol{f}}^* - \frac{\gamma\eta}{\sigma_v^2}\mathrm{diag}(\mathbf{W}\boldsymbol{\Sigma})\odot\bar{\boldsymbol{g}}^*$.[3]

3.2.3. *Constant Term s_0*

Lastly, the constant term $s_0(\mathbf{X}, \mathbf{Y})$ can be expressed as

$$s_0(\mathbf{X}, \mathbf{Y}) = \frac{P_t}{\sigma_v^2}\mathbf{1}\cdot\Big(\mathbf{W}\odot(\bar{\boldsymbol{f}}\bar{\boldsymbol{f}}^H)\odot((\bar{\boldsymbol{g}}^*)(\bar{\boldsymbol{g}}^*)^H)\Big) - \frac{\gamma}{\sigma_v^2}\mathbf{1}\cdot\Big(\mathrm{Diag}(\mathbf{W})\mathrm{Diag}(\bar{\boldsymbol{g}}\bar{\boldsymbol{g}}^H)\boldsymbol{\Sigma}\Big) - \gamma.$$

[3] The operator $\mathrm{diag}(\mathbf{X})$ will output a column vector containing the diagonal elements of a given square matrix $\mathbf{X}$.

3.3. A Safe Approximation of Constraint (4b)

Based on the previous discussion, we can rewrite

$$\begin{aligned}&\hat{p}_2(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{w}, \gamma) = \mathrm{vecb}(\mathbf{X})^H\bar{\mathbf{B}}\mathrm{vecb}(\mathbf{Y}) + \mathrm{vecb}(\mathbf{X})^H\mathrm{vecb}(\mathbf{P})\\ &+\mathrm{vecb}(\mathbf{R})^H\mathrm{vecb}(\mathbf{Y}) + \frac{P_t}{\sigma_v^2}\mathbf{1}\cdot\Big(\mathbf{W}\odot(\bar{\boldsymbol{f}}\bar{\boldsymbol{f}}^H)\odot((\bar{\boldsymbol{g}}^*)(\bar{\boldsymbol{g}}^*)^H)\Big)\\ &-\frac{\gamma}{\sigma_v^2}\mathbf{1}\cdot\Big((\mathbf{I}\odot\mathbf{W})(\mathbf{I}\odot\bar{\boldsymbol{g}}\bar{\boldsymbol{g}}^H)\boldsymbol{\Sigma}\Big) - \gamma. \end{aligned}\tag{13}$$

Now, we can apply SDR on $\mathbf{X}$ and $\mathbf{Y}$, and then follow Theorem 2.4 in [12] to further restrict constraint (4b) as

$$\begin{aligned}0 \le \min_{\mathbf{X},\mathbf{Y},\mathbf{U}}\ &(\mathcal{F}(\bar{\mathbf{B}}) + \alpha\mathbf{I})\cdot\mathbf{U} - 8\alpha + s_2(\mathbf{X}, \mathbf{Y}) + s_0(\mathbf{X}, \mathbf{Y})\\ \text{s.t.}\ &\begin{bmatrix} 1 & \mathrm{vecb}(\mathbf{X})^H & \mathrm{vecb}(\mathbf{Y})^H \\ \mathrm{vecb}(\mathbf{X}) & & \\ \mathrm{vecb}(\mathbf{Y}) & & \mathbf{U} \end{bmatrix} \succeq \mathbf{0}, &&(14a)\\ &\mathrm{Tr}(\mathbf{X}) \le 2,\ \mathrm{Tr}(\mathbf{Y}) \le 2, &&(14b)\\ &\mathbf{X} \succeq \mathbf{0},\ \mathbf{Y} \succeq \mathbf{0}, &&(14c)\end{aligned}$$

where $\mathcal{F}(\bar{\mathbf{B}}) = \frac{1}{2}\begin{bmatrix}\mathbf{0} & \bar{\mathbf{B}}\\ \bar{\mathbf{B}}^H & \mathbf{0}\end{bmatrix}$ and we choose $\alpha = \frac{1}{2}(\lambda_{\max}(\bar{\mathbf{B}}^H\bar{\mathbf{B}}))^{\frac{1}{2}}$ so that (14) is convex in $\mathbf{X}$ and $\mathbf{Y}$.

4. SAFE APPROXIMATION OF PROBLEM (BF) AND AN ALTERNATING ALGORITHM

Putting the above pieces together, we arrive at the following safe approximation of Problem (BF):

$$\begin{aligned}(\text{SABF})\quad &\max_{\mathbf{w}\in\mathbb{C}^L,\gamma}\quad \gamma\\ &\text{s.t}\quad \text{(8) and (14) satisfied,}\\ &\qquad\ \mathbf{W} = (\mathbf{w}^*)(\mathbf{w}^*)^H. \end{aligned}\tag{15}$$

Problem (SABF) can be tackled by an alternating algorithm together with the SDR technique. Specifically, we proceed in two steps:

Step 1. Given $\mathbf{W}^{(k)}$ and $\gamma^{(k)}$, we solve for $\hat{\mathbf{X}}^{(k+1)}$ and $(\mathbf{X}^{(k+1)}, \mathbf{Y}^{(k+1)})$ based on (8) and (14). In other words,

$$\begin{aligned}\hat{\mathbf{X}}^{(k+1)} = &\arg\max_{\mathbf{X}}\ \mathrm{Tr}(\mathbf{J}\mathbf{X}^*)\\ &\text{s.t}\ \mathrm{Tr}(\mathbf{X}) \le 2,\ \mathbf{X} \succeq \mathbf{0}\end{aligned}\tag{16}$$

and

$$\begin{aligned}&(\mathbf{X}^{(k+1)},\ \mathbf{Y}^{(k+1)})\\ &= \arg\min_{\mathbf{X},\mathbf{Y},\mathbf{U}}(\mathcal{F}(\bar{\mathbf{B}}) + \alpha\mathbf{I})\cdot\mathbf{U} - 8\alpha + s_2(\mathbf{X}, \mathbf{Y}) + s_0(\mathbf{X}, \mathbf{Y})\\ &\qquad \text{s.t.}\quad \text{(14a), (14b), and (14c) satisfied.}\end{aligned}\tag{17}$$

Step 2. Given $(\hat{\mathbf{X}}^{(k+1)}, \mathbf{X}^{(k+1)}, \mathbf{Y}^{(k+1)})$, we apply SDR by dropping constraint (15), and then use the Gaussian randomization procedure to find a feasible $\mathbf{w}$, if needed. Specifically, after applying SDR, the newly updated $\mathbf{W}^{(k+1)}$ can be obtained by the bisection method. In the nth bisection round with $\gamma = \gamma_{(n)}$, we check the feasibility of the following convex problem:

$$\begin{aligned}\text{find}\quad &\mathbf{W} \in \mathbb{H}_+^L\\ \text{s.t.}\quad &s(\mathbf{X}^{(k+1)}, \mathbf{Y}^{(k+1)}, \mathbf{W}, \gamma_{(n)}) \ge 0,\\ &\mathrm{Tr}(\mathbf{J}(\hat{\mathbf{X}}^{(k+1)})^*) \le P.\end{aligned}\tag{18}$$

Here, $s(\mathbf{X}, \mathbf{Y}, \mathbf{W}, \gamma)$ is given by the expression on the right-hand side of (13). The whole process of the alternating procedure is given in Algorithm 1. Note that $\mathbf{w}^\star$ is in general a sub-optimal solution to (BF), as we have restricted the feasible set.

Algorithm 1 The Alternating Algorithm for Problem (SABF)

Input: initial $\mathbf{W}^{(1)}$ and $\gamma^{(1)}$, the maximum number of alternations $K_{\max}$, the alternating accuracy ξ_1, bisection accuracy ξ_2, an initial $\gamma_L = 0$, a sufficient large γ_U.
for $k = 1, 2, \dots, K_{\max}$ **do**
  if $|\gamma^{(k)} - \gamma^{(k-1)}| < \xi_1$ **then**
    break;
  else
    Solve (16) and (17) with $\mathbf{W}^{(k)}$ and $\gamma^{(k)}$ to obtain $(\mathbf{X}^{(k+1)}, \mathbf{Y}^{(k+1)}, \hat{\mathbf{X}}^{(k+1)})$.
    Initialize $n = 1$, and set $\gamma_{(1)} = (\gamma_L + \gamma_U)/2$.
    while $|\gamma_{(n)} - \gamma_{(n-1)}| > \xi_2$ **do**
      Solve Problem (18) with $(\mathbf{X}^{(k+1)}, \mathbf{Y}^{(k+1)}, \hat{\mathbf{X}}^{(k+1)})$ and $\gamma_{(n)}$ to obtain $\mathbf{W}_{(n)}$. If it is feasible, set $\gamma = (\gamma_{(n)} + \gamma_U)/2$; otherwise, set $\gamma = (\gamma_L + \gamma_{(n)})/2$.
      Set $n = n + 1$ and $\gamma_{(n)} = \gamma$.
    end while
    Set $\gamma^{(k+1)} = \gamma_{(n)}$ and $\mathbf{W}^{(k+1)} = \mathbf{W}_{(n)}$.
  end if
end for
If $\text{rank}(\mathbf{W}^{(k)}) = 1$, output $\mathbf{w}^\star$ such that $(\mathbf{w}^\star)(\mathbf{w}^\star)^H = \mathbf{W}^{(k+1)}$. If $\text{rank}(\mathbf{W}^{(k+1)}) > 1$, apply the Gaussian randomization procedure to find $\mathbf{w}^\star$.

5. NUMERICAL SIMULATIONS AND CONCLUSIONS

In this section, we compare three relay beamforming designs, namely, the non-robust (NR) design, a linear approximation (LA) design, and the proposed safe approximation (SA) design. In the NR design, the channel errors are ignored and the resulting design problem is a fractional QCQP which can be solved by SDR [13, 14]. In the LA design, the error terms with order higher than one are dropped [4]. Thus, given any beamformer, the optimizers of $\max_{\|\mathbf{x}\|\le 1} p_1(\mathbf{x}, \mathbf{w})$ and $\min_{\|\mathbf{x}\|\le 1, \|\mathbf{y}\|\le 1} p_2(\mathbf{x}, \mathbf{y}, \mathbf{w}, \gamma)$ have closed-form expressions, and we can directly use the bisection method to find an approximate beamformer for Problem (BF). To set up the simulation, we assume that there are $L = 4$ relays in the D2D network; the channels are independently generated by $\boldsymbol{f}, \boldsymbol{g} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$; the noise power at relays and users are both set to be 0.25; the signal power at each transmitter is 0dB; we set $\xi_1 = 1e{-}2$, $\xi_2 = 1e{-}5$ and $\epsilon = \eta = 0.3$.

In Figure 1, we show the Monte Carlo (MC) test by uniformly generating a large number of error vectors $\mathbf{x}, \mathbf{y}$ such that $\|\mathbf{x}\| \le 1, \|\mathbf{y}\| \le 1$ and plot the averaged SNR for the transmitter-receiver link versus power allowed at relays when different beamformer design approaches are adopted. It shows that NR gives the best SNR performance, while the proposed SA exhibits better SNR performance than the LA design. In Figure 2, we show the satisfaction percentage of the SNR constraint (SNR $\ge \gamma$) and the power constraint ($p_1(\mathbf{x}, \mathbf{w}) \le P$) in the MC test. The results show that for the NR design, both the SNR and power constraints are satisfied around 50% of the time. For the LA design, the power constraint is always satisfied under our problem setting. However, the satisfaction of SNRs is less than 60%. For the proposed SA design, the power constraint is satisfied with probability 1, while the SNRs are satisfied when the power budget at the relays is less than 8dB and this

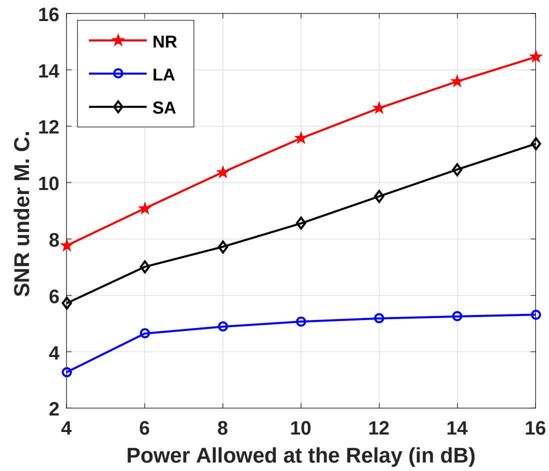

Fig. 1. SNR results for different schemes.

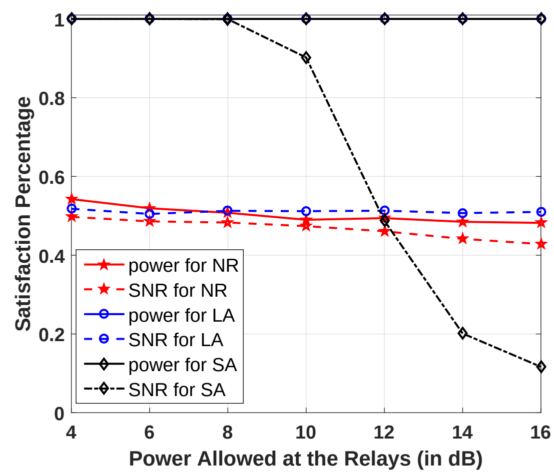

Fig. 2. Satisfaction probability of the constraints.

satisfaction percentage will decrease as the power increases. Another interesting observation is that in this simulation, all the $\mathbf{W}^{(k)}$'s are rank-one. Moreover, we have $\text{rank}(\mathbf{X}) = 1$ for all power region and $\text{rank}(\mathbf{Y}) = 1$ when the power budget is less than 8dB. This reveals that the SNR constraint is tight whenever the SDR approximation for $\mathbf{X}, \mathbf{Y}$ is tight, which is consistent with the theory. In all, under this problem setting, both LA and SA designs satisfy the power constraint, while SA can output a better SNR.

To conclude, in this paper we have studied the robust relay beamforming design for a D2D network. Specifically, we maximized the received SNR subject to a power budget at relays. The difficulty in our design problem lies in the fact that channel estimation errors are present in both the transmitter-to-relays link and relays-to-receiver link, which differentiates our work from others. We have provided the first tractable safe approximation of the target design problem and proposed an alternating algorithm for generating an approximate solution. Our numerical results further validate the superiority of the proposed design and the algorithm. In the future, many transceiver pairs could be considered as a non-trivial extension of this work.

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