ROBUST TRANSMIT DESIGNS FOR AN ENERGY HARVESTING MULTICAST SYSTEM

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ABSTRACT

Recently, simultaneous wireless information and power transfer (SWIPT) has received considerable attention. In this paper, we consider a multicast SWIPT system, where a multi-antenna transmitter broadcasts common information to a group of single-antenna information receivers (IRs) and at the same time provides certain amount of energy transfer to a group of single-antenna energy receivers (ERs). Assuming imperfect channel state information (CSI) at the transmitter, two transmit schemes are proposed to maximize the IRs' outage-constrained multicast rate subject to a minimum provision of average energy transfer to ERs. In the first transmit scheme, we consider transmit beamforming and develop a safe approximation approach to obtain a conservative beamforming solution for maximizing the outage-constrained multicast rate. To further improve the performance of transmit beamforming, in the second transmit scheme, we consider a stochastic beamforming (SBF) approach, which allows the beamformer to randomly change over time according to some prescribed distribution. By doing so, the SBF scheme is able to fully exploit the temporal degree of freedom to achieve more balanced outage-constrained achievable rates among IRs. Simulation results demonstrated that the SBF scheme is generally better than the transmit beamforming scheme.

***Index Terms*—** Energy harvesting, chance-constrained optimization, semidefinite relaxation, stochastic beamforming.

1. INTRODUCTION

In wireless communication, radio-frequency (RF) wave is commonly used for conveying information from the transmitter to the information receiver. Recently, there is flourishing interest in using RF wave to transfer power, when some of the receivers in the system aim for receiving energy rather than information. The dual usage of RF wave naturally motivate a unified study on simultaneous wireless information and power transfer (SWIPT). Recent studies on SWIPT are mainly inspired by Varshney's work [1], where a non-trivial trade-off between information transfer and energy transfer is shown for a single-input single-output (SISO) AWGN channel. Extensions to more complex SWIPT scenarios have been considered in several endeavors; e.g., point-to-point parallel fading channels [2], multi-input multi-output (MIMO) broadcast channels with one information receiver (IR) and one energy receiver (ER) [3], three-node multi-input single-output (MISO) downlink channels with imperfect channel state information at the transmitter (CSIT) [4], MISO multicasting without CSIT [5], and MIMO two-hop relay channels [6].

In this paper, we consider transmit optimization for a multicast SWIPT system, where a multi-antenna transmitter broadcasts common information to a group of IRs and at the same time provides wireless energy transfer to a group of ERs. All the IRs and ERs are assumed to have a single antenna, i.e., MISO downlink. Assuming imperfect CSI at the transmitter, we propose two transmit schemes to maximize the IRs' outage-constrained multicast rate subject to a minimum provision of the average energy transfer to the ERs. Specifically, the first transmit scheme focuses on transmit beamforming, and the resulting beamformer design problem is formulated as a chance-constrained optimization problem. This kind of problem is in general hard to solve, and as a compromise we resort to a safe approximation approach. Specifically, the key to the safe approximation is to employ a recently-developed relaxation-restriction (RAR) approach [7], which consists of semidefinite relaxation (SDR) [8] and a Bernstein-type inequality-based conservative approximation of the probabilistic constraints [9]. In the second transmit scheme, we consider a stochastic beamforming (SBF) approach, which has been proposed in our recent work [10] for approaching the multicast capacity of an MISO downlink channel (without ERs). The idea of SBF is to allow the beamformer to randomly change over time according to some prescribed distribution. By doing so, we can intentionally and judiciously create fluctuated channels for the IRs, such that all the IRs' average (ergodic) rates are well balanced in a long-term average sense. Our numerical results demonstrated that the SBF scheme is generally better than the transmit beamforming scheme.

There are some related works that are worthwhile to mention. In [4], Xiang and Tao considered a robust beamformer design for a SWIPT system with one IR and one ER. Here we consider a more general setting — multiple IRs and ERs. Moreover, the CSI error model here is based on the Gaussian random model, which is different from the norm-bounded deterministic model in [4]. Also, [11] considered MISO unicast beamforming in a SWIPT system. However, each user therein aims at simultaneous information delivery and energy harvesting, and their design criterion is to split the power for both purposes. Another related work is [5], which considered an MISO multicast SWIPT system with co-located IR and ER; i.e., each receiver can work in either IR mode or ER mode. The focus of [5] is to study the mode switching scheme using random beamforming when no CSIT is available, while our work considers separate IRs and ERs, with an emphasis on the novel SBF transmit signal design when erroneous CSIT is known.

2. SYSTEM MODEL FOR TRANSMIT BEAMFORMING

Consider a simultaneous wireless information and power transfer

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(SWIPT) system, where a multi-antenna transmitter attempts to send a common message to a group of information receivers (IRs), denoted as $\mathcal{G}_I$, and at the same time provides wireless energy transfer to a group of energy receivers (ERs), denoted as $\mathcal{G}_E$. We assume that all the IRs and ERs are single-antenna and $\mathcal{G}_I \cap \mathcal{G}_E = \emptyset$; i.e., each receiver is either IR or ER, but not both. Suppose that all channels are frequency-flat and block fading, the received signal $y_i(t)$ at the ith receiver can be expressed as

$$y_i(t) = \mathbf{h}_i^H \mathbf{x}(t) + n_i(t), \quad \forall\, i \in \mathcal{G}_I \cup \mathcal{G}_E,\ t = 1, \ldots, T, \quad (1)$$

where $\mathbf{h}_i \in \mathbb{C}^N$ is the channel from the transmitter to the ith receiver, T is the frame length during which the channel $\mathbf{h}_i$ is invariant, $n_i(t)$ is a complex Gaussian noise with mean zero and variance σ_i^2, and $\mathbf{x}(t) \in \mathbb{C}^N$ is the transmit signal. For the time being, we consider the transmit beamforming scheme, for which the transmit signal $\mathbf{x}(t)$ takes the form

$$\mathbf{x}(t) = \sqrt{P}\mathbf{w}s(t), \quad t = 1, \ldots, T, \quad (2)$$

where $P > 0$ is the average transmit power of $\mathbf{x}(t)$, $\mathbf{w} \in \mathbb{C}^N$ is the transmit beamforming vector satisfying $\|\mathbf{w}\|^2 \le 1$, and $s(t)$ is a stream of data symbols with unit power, i.e., $\mathbb{E}[|s(t)|^2] = 1$. We should mention that the beamfomer $\mathbf{w}$ in (2) is assumed to be fixed during the whole frame length T, just like most of the existing transmit beamforming studies, e.g., [12–14].

According to (1) and (2), an achievable rate at IR i and the harvested energy (normalized by baseband symbol duration) at ER j may respectively be calculated as [1–3]

$$R_i(\mathbf{w}, \mathbf{h}_i) = \log(1 + P|\mathbf{h}_i^H \mathbf{w}|^2/\sigma_i^2), \quad \forall\, i \in \mathcal{G}_I,$$
$$Q_j(\mathbf{w}, \mathbf{h}_j) = \mu_j P |\mathbf{h}_j^H \mathbf{w}|^2, \quad \forall\, j \in \mathcal{G}_E,$$

where $\log(\cdot)$ is the natural logarithm (and thus R_i is in units of nats/s/Hz); $0 < \mu_j < 1$ denotes the energy harvesting efficiency at the ER j [15, 16]. Clearly, with knowledge of $\mathbf{h}_i$, we can appropriately design the beamformer $\mathbf{w}$ to achieve good multicast rates for the IRs and energy transfer for the ERs. However, in practice, due to imperfect channel estimation and/or outdated CSI feedback, the transmitter usually has only a rough knowledge of $\mathbf{h}_i$. To account for imperfect CSI, we consider the following random CSI error model:

$$\mathbf{h}_i = \bar{\mathbf{h}}_i + \mathbf{e}_i, \quad \forall\, i \in \mathcal{G}_I \cup \mathcal{G}_E, \quad (3)$$

where $\bar{\mathbf{h}}_i$ is an estimate of the actual CSI $\mathbf{h}_i$ at the transmitter, and $\mathbf{e}_i$ is the associated random estimation error, whose distribution follows

$$\mathbf{e}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega}_i). \quad (4)$$

Here, $\mathbf{\Omega}_i \in \mathbb{H}_+^N$ is given for all $i \in \mathcal{G}_I \cup \mathcal{G}_E$; $\mathbb{H}_+^N$ denotes the set of all $N \times N$ Hermitian positive semidefinite matrices.

Based on the above uncertainty model, our problem of interest is formulated as follows:

$$\begin{aligned}
&\max_{\mathbf{w}, R}\ R && (5a)\\
&\text{s.t. } \mathrm{Prob}_{\mathbf{e}_i}\left\{R_i(\mathbf{w}, \bar{\mathbf{h}}_i + \mathbf{e}_i) \ge R\right\} \ge 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I, && (5b)\\
&\qquad \mathbb{E}\left[Q_j(\mathbf{w}, \bar{\mathbf{h}}_j + \mathbf{e}_j)\right] \ge \eta_j,\ \forall\, j \in \mathcal{G}_E, && (5c)\\
&\qquad \|\mathbf{w}\|^2 \le 1, && (5d)
\end{aligned}$$

where $0 < \rho_i < 1, \forall\, i \in \mathcal{G}_I$ and $\eta_j > 0, \forall\, j \in \mathcal{G}_E$ are given constants that specify the rate outage probability (i.e., the chance of transmission rate falling below a prescribed value) at IR i and the minimum average harvested energy at ER j, respectively. In words, the goal of problem (5) is to optimize the beamformer $\mathbf{w}$ such that the outage-constrained multicast rate at the IRs is maximized while the average harvested energy at each ER is kept above a certain level.

Problem (5) is a chance-constrained optimization problem, which in general could be difficult to deal with. Actually, even for the perfect CSI case, it has been shown in [12] that problem (5) (without (5c)) is $\mathcal{NP}$-hard in general. As such, in the ensuing section, we will focus on developing an approximate safe[1] solution for (5). The key to our approach is to employ a recently-developed relaxation-restriction (RAR) methodology [7], as we detail below.

3. RELAXATION-RESTRICTION APPROACH TO (5)

The relaxation-restriction (RAR) approach consists of two key ingredients — semidefinite relaxation (SDR) and a Bernstein-type inequality-based conservative approximation of the probabilistic constraints (5b). We now describe the RAR approach in two steps.

Step 1: Relaxation. For ease of exposition, let us define $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and

$$\mathbf{Q}_i = P\mathbf{\Omega}_i^{\frac{1}{2}}\mathbf{W}\mathbf{\Omega}_i^{\frac{1}{2}}/\sigma_i^2,\ \mathbf{r}_i = P\mathbf{\Omega}_i^{\frac{1}{2}}\mathbf{W}\bar{\mathbf{h}}_i/\sigma_i^2,\ s_i = P\bar{\mathbf{h}}_i^H\mathbf{W}\bar{\mathbf{h}}_i/\sigma_i^2$$

for all $i \in \mathcal{G}_I$. Note that

$$\mathbf{W} = \mathbf{w}\mathbf{w}^H \Longleftrightarrow \mathbf{W} \succeq \mathbf{0} \text{ and } \mathrm{rank}(\mathbf{W}) \le 1.$$

After some algebraic manipulations, it can be verified that problem (5) amounts to the following problem:

$$\begin{aligned}
&\max_{\mathbf{W}, \gamma}\ \gamma && (6a)\\
&\text{s.t. } \mathrm{Prob}_{\bar{\mathbf{e}}_i}\{f_i(\mathbf{W}, \bar{\mathbf{e}}_i) \ge \gamma\} \ge 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I, && (6b)\\
&\qquad \mathrm{Tr}((\bar{\mathbf{h}}_j\bar{\mathbf{h}}_j^H + \mathbf{\Omega}_j)\mathbf{W}) \ge \frac{\eta_j}{P\mu_j},\ \forall\, j \in \mathcal{G}_E, && (6c)\\
&\qquad \mathrm{Tr}(\mathbf{W}) \le 1, \quad \mathbf{W} \succeq \mathbf{0}, && (6d)\\
&\qquad \mathrm{rank}(\mathbf{W}) \le 1, && (6e)
\end{aligned}$$

where $\bar{\mathbf{e}}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, $\gamma = 2^R - 1$ and $f_i(\mathbf{W}, \bar{\mathbf{e}}_i) \triangleq \bar{\mathbf{e}}_i^H\mathbf{Q}_i\bar{\mathbf{e}}_i + 2\mathfrak{Re}(\bar{\mathbf{e}}_i^H\mathbf{r}_i) + s_i$. As a standard procedure of SDR, we drop the hard rank constraint (6e) to obtain a relaxed problem of (6), i.e.,

$$\begin{aligned}
&\max_{\mathbf{W}, \gamma}\ \gamma && (7a)\\
&\text{s.t. } \mathrm{Prob}_{\bar{\mathbf{e}}_i}\{f_i(\mathbf{W}, \bar{\mathbf{e}}_i) \ge \gamma\} \ge 1 - \rho_i,\ \forall i \in \mathcal{G}_I, && (7b)\\
&\qquad (6c) - (6d) \text{ satisfied.} && (7c)
\end{aligned}$$

Step 2: Restriction. Note that the inequality $f_i(\mathbf{W}, \bar{\mathbf{e}}_i) \ge \gamma$ is quadratic with respect to (w.r.t.) the Gaussian random vector $\bar{\mathbf{e}}_i$. As such, we can use a Bernstein-type inequality [7, 9] to construct an efficiently computable restricted or safe approximation of the probabilistic constraint (7b). The crux of the construction is the following lemma, which has been previously established in [7] to tackle a different chance-constrained optimization problem.

Lemma 1 ([7]). *For any* $(\mathbf{Q}, \mathbf{r}, c) \in \mathbb{H}_+^n \times \mathbb{C}^n \times \mathbb{R}$, $\mathbf{e} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$ *and* $\rho \in (0, 1]$, *the following implication holds true:*

$$\mathrm{Prob}_{\mathbf{e}}\left\{\mathbf{e}^H\mathbf{Q}\mathbf{e} + 2\mathfrak{Re}\{\mathbf{e}^H\mathbf{r}\} + c \ge 0\right\} \ge 1 - \rho$$
$$\Longleftarrow \left\{ \begin{array}{l} \mathrm{Tr}(\mathbf{Q}) - \sqrt{-2\log(\rho)} \cdot x + c \ge 0, \\ \left\| \begin{bmatrix} \mathrm{vec}(\mathbf{Q}) \\ \sqrt{2}\mathbf{r} \end{bmatrix} \right\|_2 \le x, \end{array} \right. \quad (8)$$

[1] Here "safe" means that the obtained solution must fulfill the probabilistic constraints (5b).

where x is a slack variable. Moreover, the system (8) *is convex in* $(\mathbf{Q}, \mathbf{r}, c, x)$.

By applying Lemma 1 to (7b), we get the following safe approximation of (7):

$$\max_{\mathbf{W},\gamma,\{x_i\}_{i\in\mathcal{G}_I}} \ \gamma \tag{9a}$$

$$\text{s.t. } \mathrm{Tr}(\mathbf{Q}_i) - \sqrt{-2\log(\rho_i)}\cdot x_i + s_i - \gamma \geq 0, \ \forall\, i \in \mathcal{G}_I, \tag{9b}$$

$$\left\| \begin{bmatrix} \mathrm{vec}(\mathbf{Q}_i) \\ \sqrt{2}\mathbf{r}_i \end{bmatrix} \right\|_2 \leq x_i, \quad \forall\, i \in \mathcal{G}_I, \tag{9c}$$

$$\mathrm{Tr}((\bar{\mathbf{h}}_j\bar{\mathbf{h}}_j^H + \mathbf{\Omega}_j)\mathbf{W}) \geq \frac{\eta_j}{P\mu_j}, \ \forall\, j \in \mathcal{G}_E, \tag{9d}$$

$$\mathrm{Tr}(\mathbf{W}) \leq 1, \quad \mathbf{W} \succeq \mathbf{0}. \tag{9e}$$

Problem (9) is a convex conic optimization problem, which can be solved using some general-purpose conic optimization solvers, e.g., CVX [17]. Let $\mathbf{W}^\star$ be an optimal solution of (9). If $\mathrm{rank}(\mathbf{W}^\star) = 1$, then one can perform eigen-decomposition $\mathbf{W}^\star = \mathbf{w}^\star\mathbf{w}^{\star H}$ to obtain a beamforming solution $\mathbf{w}^\star$. It can be checked that $\mathbf{w}^\star$ fulfills (5b)-(5d) (cf. Lemma 1), and thus is a safe approximate solution for problem (5). If $\mathrm{rank}(\mathbf{W}^\star) > 1$, then a Gaussian randomization procedure can be employed to yield a safe approximate solution for problem (5); see Algorithm 1.

Algorithm 1 Gaussian Randomization Procedure

1: Input: an optimal solution $\mathbf{W}^\star$ of (9), number of randomizations $L \geq 1$;
2: **for** $\ell = 1$ to L **do**
3: generate an independent random vector $\boldsymbol{\xi}^\ell \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}^\star)$;
4: let $\hat{\mathbf{w}}_\ell = \boldsymbol{\xi}^\ell / \|\boldsymbol{\xi}^\ell\|$ and $\mathbf{Q}_i = P\mathbf{\Omega}_i^{\frac{1}{2}}\hat{\mathbf{w}}_\ell\hat{\mathbf{w}}_\ell^H\mathbf{\Omega}_i^{\frac{1}{2}}/\sigma_i^2$, $\mathbf{r}_i = P\mathbf{\Omega}_i^{\frac{1}{2}}\hat{\mathbf{w}}_\ell\hat{\mathbf{w}}_\ell^H\bar{\mathbf{h}}_i/\sigma_i^2$, $s_i = P\bar{\mathbf{h}}_i^H\hat{\mathbf{w}}_\ell\hat{\mathbf{w}}_\ell^H\bar{\mathbf{h}}_i/\sigma_i^2$, $\forall\, i \in \mathcal{G}_I$;
5: **if** $\mathrm{Tr}(\bar{\mathbf{h}}_j\bar{\mathbf{h}}_j^H\hat{\mathbf{w}}_\ell\hat{\mathbf{w}}_\ell^H + \mathbf{\Omega}_j\hat{\mathbf{w}}_\ell\hat{\mathbf{w}}_\ell^H) < \frac{\eta_j}{P\mu_j}$ for some $j \in \mathcal{G}_E$ **then**
6: set $p_\ell = 0$;
7: **else**
8: let $p_\ell = \min_{i\in\mathcal{G}_I}\{\mathrm{Tr}(\mathbf{Q}_i) - \sqrt{-2\log(\rho_i)}\cdot x_i + s_i\}$, where $x_i = \left\| \begin{bmatrix} \mathrm{vec}(\mathbf{Q}_i) \\ \sqrt{2}\mathbf{r}_i \end{bmatrix} \right\|_2$, $\forall\, i \in \mathcal{G}_I$;
9: **end if**
10: **end for**
11: **if** $\max_{\ell=1,\ldots,L} p_\ell = 0$ **then**
12: Output: Fail.
13: **else**
14: set $\ell^\star := \arg\max_{\ell=1,\ldots,L} p_\ell$ and output: $\mathbf{w}^\star = \hat{\mathbf{w}}_{\ell^\star}$.
15: **end if**

Remark 1. Generally speaking, the SDR-based transmit beamforming scheme can work well when there are not too many IRs. However, as revealed in [10, 12], the performance of transmit beamforming could deteriorate as the number of IRs increases. According to our numerical experience, for a large number of IRs, it is possible that problem (9) is feasible, but Algorithm 1 fails to output a feasible solution for problem (5) (cf. line 12 of Algorithm 1). Part of the reason is that when $\mathbf{W}^\star$ has a higher rank, it may be difficult to use a *single* beamformer $\mathbf{w}$ to well approximate $\mathbf{W}^\star$. In light of this, we will propose in the next section another transmit scheme — stochastic beamforming [10], which is demonstrated to have better performance than the transmit beamforming scheme, especially for a large number of the IRs.

4. STOCHASTIC BEAMFORMING FOR THE MULTICAST SWIPT SYSTEM

Stochastic beamforming (SBF) was proposed in our recent work [10] for approaching the multicast capacity of a multiuser MISO downlink system (without ERs). The main idea of SBF is to allow the beamformer to randomly change over time according to a certain distribution as determined by $\mathbf{W}^\star$. By doing so, we can exploit the additional temporal degree of freedom (d.o.f.) to better approximate $\mathbf{W}^\star$, especially for higher rank $\mathbf{W}^\star$. Readers are referred to [10] for a more detailed description of SBF; here we provide only some of its key aspects, with an emphasis on its outage-rate performance analysis for the multicast SWIPT system.

For SBF, the transmit signal $\mathbf{x}(t)$ and the received signal $y_i(t)$ at the ith receiver take the form

$$\mathbf{x}(t) = \sqrt{P}\mathbf{w}(t)s(t), \quad t = 1, \ldots, T,$$
$$y_i(t) = \sqrt{P}\mathbf{h}_i^H\mathbf{w}(t)s(t) + n_i(t), \quad t = 1, \ldots, T.$$

In contrast to (2), SBF replaces the *fixed* beamformer $\mathbf{w}$ with a *time-varying* random beamformer $\mathbf{w}(t)$, thereby resulting in a virtual fast fading SISO channel $\mathbf{h}_i^H\mathbf{w}(t)$ (though the physical channel $\mathbf{h}_i$ is still invariant over the whole frame). As demonstrated in [10], by judiciously choosing the distribution for $\mathbf{w}(t)$, every IR is able to enjoy a good average (ergodic) rate (assuming ideal channel coding over sufficiently large T). By contrast, when a fixed beamformer is used, each IR undergoes distinct slow fading channel, and thus the overall multicast rate performance will be dominated by the worst user's achievable rate.

From the above discussion, it is clear that choosing the distribution for $\mathbf{w}(t)$ is crucial for SBF. However, finding an optimal distribution for $\mathbf{w}(t)$ seems to be a challenging task. For simplicity, we focus on the complex Gaussian distribution[2]; i.e.,

$$\mathbf{w}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}^\star)$$

for $t = 1, \ldots, T$, where $\mathbf{W}^\star$ is an optimal solution of problem (9). Denote by $\boldsymbol{w}$ a random variable for $\mathbf{w}(t)$. Then, the achieved ergodic rate at IR i and the harvested energy at ER j by SBF are respectively given by

$$R_i^{\mathsf{SBF}}(\mathbf{h}_i) = \mathbb{E}_{\boldsymbol{w}}[\log(1 + P|\mathbf{h}_i^H\boldsymbol{w}|^2/\sigma_i^2)], \ \forall\, i \in \mathcal{G}_I,$$
$$Q_j^{\mathsf{SBF}}(\mathbf{h}_j) = \mathbb{E}_{\boldsymbol{w}}[\mu_j P|\mathbf{h}_j^H\boldsymbol{w}|^2] = \mu_j P\mathbf{h}_j^H\mathbf{W}^\star\mathbf{h}_j, \ \forall\, j \in \mathcal{G}_E.$$

The description of SBF is now complete. Next, we analyze its rate-energy performance under the random CSI error model (3)-(4).

First of all, let us verify that SBF satisfies the total power constraint (5d) and the average energy harvesting constraints (5c). The total transmit power of SBF is

$$\mathbb{E}_{\boldsymbol{w}}[\|\boldsymbol{w}\|^2] = \mathrm{Tr}(\mathbb{E}_{\boldsymbol{w}}[\boldsymbol{w}\boldsymbol{w}^H]) = \mathrm{Tr}(\mathbf{W}^\star) \leq 1,$$

where the last inequality follows from (9e), and the average harvested energy is

$$\mathbb{E}_{\mathbf{e}_j}[Q_j^{\mathsf{SBF}}(\bar{\mathbf{h}}_j + \mathbf{e}_j)] = \mu_j P\mathrm{Tr}((\bar{\mathbf{h}}_j\bar{\mathbf{h}}_j^H + \mathbf{\Omega}_j)\mathbf{W}^\star) \geq \eta_j, \ \forall\, j \in \mathcal{G}_E,$$

where the last inequality is due to (9d). Next, we identify the outage-constrained achievable rate performance of SBF. With a slight abuse

[2]There are other more sophisticated distributions that one can choose, such as elliptic and Bingham [10].

of notations, we denote by

$$R_i^{\mathsf{SDR}}(\mathbf{W}^\star, \mathbf{h}_i) \triangleq \log(1 + P\mathbf{h}_i^H \mathbf{W}^\star \mathbf{h}_i / \sigma_i^2), \forall\, i \in \mathcal{G}_I$$

the ith IR's "rate" associated with the SDR solution $\mathbf{W}^\star$, and by

$$R^{\mathsf{SDR}}(\mathbf{W}^\star) \triangleq$$
$$\sup_v \{ v \mid \mathrm{Prob}_{\mathbf{e}_i}\{ R_i^{\mathsf{SDR}}(\mathbf{W}^\star, \bar{\mathbf{h}}_i + \mathbf{e}_i) \geq v \} \geq 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I \} \tag{10}$$

the "outage-constrained multicast rate" associated with $\mathbf{W}^\star$. We are now ready to state our main result on the Gaussian SBF outage-constrained achievable rate.

Proposition 1. *Let* $R^{\mathsf{SBF}} \triangleq \sup_v \{ v \mid \mathrm{Prob}_{\mathbf{e}_i}\{ R_i^{\mathsf{SBF}}(\bar{\mathbf{h}}_i + \mathbf{e}_i) \geq v \} \geq 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I \}$. *Then, we have*

$$R^{\mathsf{SDR}}(\mathbf{W}^\star) - R^{\mathsf{SBF}} \leq 0.5772 \quad \textit{for all}\ \ P \geq 0.$$

The proof of Proposition 1 is given in the Appendix. Proposition 1 implies that the Gaussian SBF is at most 0.8314 bits/s/Hz ($0.5772/\log(2) = 0.8314$) away from the outage-constrained multicast rate specified by the SDR solution $\mathbf{W}^\star$. In the next section, we will verify this result by numerical simulations.

5. SIMULATION RESULTS AND CONCLUSION

In this section, numerical results are provided to compare the rate-energy performances of the transmit beamforming scheme (cf. Sec. 3) and the Gaussian SBF scheme (cf. Sec. 4). The simulation settings are as follows: The number of transmit antennas is $N = 8$. There are 16 IRs and 16 ERs. Each element of the channel vector $\mathbf{h}_i, \forall\, i \in \mathcal{G}_I$ (resp. $\mathbf{h}_j, \forall\, j \in \mathcal{G}_E$) is generated by complex Gaussian distribution with mean zero and variance -50dBm (resp. -10dBm). Each entry of the channel error $\mathbf{e}_i, \forall\, i \in \mathcal{G}_I$ (resp. $\mathbf{e}_j, \forall\, j \in \mathcal{G}_E$) follows an i.i.d. complex Gaussian distribution with mean zero and variance -77dBm (resp. -37dBm), and the receive noise at the IRs is white Gaussian with variance -70dBm. For simplicity, we assume that all the IRs have the same outage probability $\rho_i = 0.1, \forall\, i \in \mathcal{G}_I$, and that all the ERs have the same energy harvesting efficiency $\mu_j = 50\%, \forall\, j \in \mathcal{G}_E$.

Fig. 1 shows the IRs' outage-constrained multicast rate against the transmit power P by fixing the energy harvesting threshold $\eta_j = 0.05$mW for all $j \in \mathcal{G}_E$. In the legend, "SDR bound" and "Gaussian SBF" correspond to the outage-constrained multicast rates $R^{\mathsf{SDR}}(\mathbf{W}^\star)$ and R^{SBF}, respectively, which were evaluated via Monte-Carlo simulations. The transmit beamforming result is obtained in a similar manner. As seen, Gaussian SBF outperforms transmit beamforming over the whole range of powers. In particular, when P is small, there is a notable rate gap between Gaussian SBF and transmit beamforming. However, as P increases, transmit beamforming is able to approach Gaussian SBF. In addition, the performance of Gaussian SBF is quite close to that of the SDR bound (about 0.7 bits/s/Hz rate gap) over the whole power range, which further corroborates the result in Proposition 1. Fig. 2 plots the rate-energy region of the three methods for $P = 39$dBm. Again, Gaussian SBF outperforms transmit beamforming over the whole rate-energy region. Moreover, with the increase in the energy threshold, transmit beamforming drops quickly to zero. One reason is that when compared to (9), (5) has a higher probability to be infeasible; see Remark 1.

In this paper we have considered an MISO downlink multicast SWIPT system with imperfect CSI at the transmitter. Two transmit schemes, namely transmit beamforming and stochastic beamforming (SBF), were proposed to maximize the outage-constrained multicast rate for an SWIPT system. In particular, the transmit beamforming scheme employs a Bernstein-type inequality to obtain a safe beamforming solution, while SBF uses a randomize-in-time transmit strategy to further improve the performance of transmit beamforming. Simulation results demonstrated that SBF yields better rate-energy tradeoff than transmit beamforming.

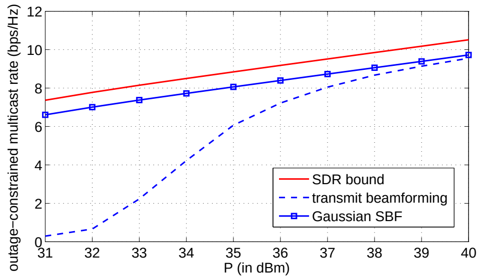

Fig. 1. IRs' outage-constrained multicast rate versus the transmit power P.

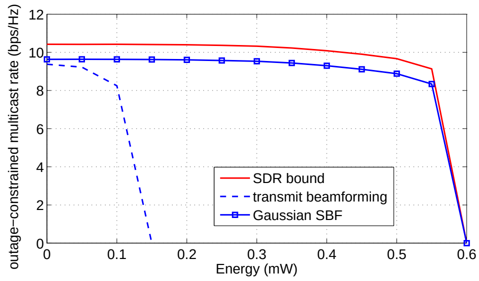

Fig. 2. Rate-energy region for $P = 39$dBm.

6. APPENDIX

Given $\mathbf{W}^\star$, we partition the ith IR's CSI errors $\mathbf{e}_i$ into the following two disjoint sets:

$$\mathcal{B}_i^0(\mathbf{W}^\star) = \{ \mathbf{e}_i \mid R_i^{\mathsf{SDR}}(\mathbf{W}^\star, \bar{\mathbf{h}}_i + \mathbf{e}_i) \geq R^{\mathsf{SDR}}(\mathbf{W}^\star) \},$$
$$\mathcal{B}_i^1(\mathbf{W}^\star) = \{ \mathbf{e}_i \mid R_i^{\mathsf{SDR}}(\mathbf{W}^\star, \bar{\mathbf{h}}_i + \mathbf{e}_i) < R^{\mathsf{SDR}}(\mathbf{W}^\star) \},$$

for all $i \in \mathcal{G}_I$. In words, $\mathcal{B}_i^1(\mathbf{W}^\star)[\mathcal{B}_i^0(\mathbf{W}^\star)]$ characterizes the set of errors that [does not] induce outage. According to the definition of $R^{\mathsf{SDR}}(\mathbf{W}^\star)$ [cf. Eq. (10)], we have

$$\mathrm{Prob}_{\mathbf{e}_i}\{ \mathbf{e}_i \in \mathcal{B}_i^0(\mathbf{W}^\star) \} \geq 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I. \tag{11}$$

To complete the proof, we need the following key lemma:

Lemma 2. *For any* $\bar{\mathbf{h}}_i$ *and* $\mathbf{e}_i \in \mathbb{C}^N$*, it holds true that*

$$R_i^{\mathsf{SDR}}(\mathbf{W}^\star, \bar{\mathbf{h}}_i + \mathbf{e}_i) - R_i^{\mathsf{SBF}}(\bar{\mathbf{h}}_i + \mathbf{e}_i) \leq 0.5772 \quad \textit{for all}\ \ P \geq 0.$$

The proof of Lemma 2 is almost identical to that of Theorem 1 in [10], and thus we omit it for brevity. It follows from Lemma 2 and the definition of $\mathcal{B}_i^0(\mathbf{W}^\star)$ that for all $P \geq 0$

$$R_i^{\mathsf{SBF}}(\bar{\mathbf{h}}_i + \mathbf{e}_i) \geq R^{\mathsf{SDR}}(\mathbf{W}^\star) - 0.5772$$

holds for all $\mathbf{e}_i \in \mathcal{B}_i^0(\mathbf{W}^\star)$ and all $i \in \mathcal{G}_I$, which together with (11) implies that

$$\mathrm{Prob}_{\mathbf{e}_i}\{ R_i^{\mathsf{SBF}}(\bar{\mathbf{h}}_i + \mathbf{e}_i) \geq R^{\mathsf{SDR}}(\mathbf{W}^\star) - 0.5772 \} \geq 1 - \rho_i,\ \forall\, i \in \mathcal{G}_I.$$

Therefore, $R^{\mathsf{SBF}} \geq R^{\mathsf{SDR}}(\mathbf{W}^\star) - 0.5772$ holds.

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