Supplementary Appendix

Contingent Capital, Tail Risk, and Debt-Induced Collapse

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1 Treatment of CoCos in Bankruptcy

There are two possibilities for the treatment of CoCos when default occurs prior to conversion: (1) either CoCos are treated as equity if default happens before conversion, or (2) CoCos degenerate to junior straight debt in bankruptcy. In the main text of the paper, we consider case (1). Here we show that very similar results hold under (2).

It is important to understand that the evaluation of equity does not depend on this convention and all the main results of this paper do not depend on this choice. As the treatment of CoCo at bankruptcy only affects how the recovery value is distributed among Coco- and debt holders, it does not affect the combined value of CoCos and straight debt. In the case of default equity holders are wiped out and the remaining value is distributed among the debt holders. In an endogenous model with only straight debt, the equity value at default will be exactly zero. If CoCos are added to the capital structure it is possible that the shareholders choose an optimal default barrier where the recovery value is larger than the debt value at default. In the main text we make the economically sound assumption that equity holders are wiped out at default and debt holders receive the recovery value. In this supplementary appendix we will also solve the alternative model, in which the difference between recovery and debt value is paid out to the shareholders. In this case default could be interpreted as liquidating the company and paying off all debt. Under this formulation equity holders have a stronger incentive to default earlier. We show that the qualitative results are similar, but the problem of debt-induced collapse becomes more severe.

1.1 CoCos as junior straight debt at early default

Suppose, that if default occurs prior to conversion, CoCos degenerate to junior debt in bankruptcy. Upon default, CoCo holders are repaid from whatever assets remain after liquidation and payment of senior debt. Before default, the total market value of straight debt is

$$B(V;V_b) = P_1\left(\frac{m+c_1}{m+r}\right)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-(m+r)\tau_b}\right] + \mathbb{E}^{\mathbb{Q}}\left[e^{-(m+r)\tau_b}\left(\alpha V_{\tau_b}\wedge P_1\right)\right] \tag{1.1}$$

leaving a CoCo value of

$$D(V;V_b) = P_2\left(\frac{m+c_2}{m+r}\right)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-(m+r)\tau_b}\right] + \mathbb{E}^{\mathbb{Q}}\left[e^{-(m+r)\tau_b}\left(\alpha V_{\tau_b}-P_1\right)^+\right]. \tag{1.2}$$

Total firm value in this case is given by

$$F^{\mathsf{BC}}(V;V_b) = V + \left(\frac{\kappa_1 c_1 P_1}{r} + \frac{\kappa_2 c_2 P_2}{r}\right)\left(1-\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right) - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right]$$

As before the equity value follows from

$$E^{\mathsf{BC}}(V;V_b) = F^{\mathsf{BC}}(V;V_b) - B(V;V_b) - D(V;V_b). \tag{1.3}$$

Note that in the main text the total market value of CoCos before default is

$$D(V;V_b) = P_2\left(\frac{m+c_2}{m+r}\right)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-(m+r)\tau_b}\right]. \tag{1.4}$$

while the total firm value is

$$\begin{aligned} F^{\mathsf{BC}}(V;V_b) &= V + \left(\frac{\kappa_1 c_1 P_1}{r} + \frac{\kappa_2 c_2 P_2}{r}\right)\left(1-\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right) - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right] \\ &\quad -\mathbb{E}^{\mathbb{Q}}\left[e^{-(r+m)\tau_b}(\alpha V_{\tau_b}-P_1)^+\right]. \end{aligned}$$

The only difference is how the residual of the recovery value after paying out the debt holders at default is distributed. In the main text we assume that any potential recovery value at default after debt holders have been paid out is lost. In the alternative scenario, where CoCos are treated as junior straight debt at default, the CoCo holders receive this residual value. Obviously, the total market value of the equity is not affected by this assumption. Hence, all results concerning endogenous default are not affected.

1.2 CoCos as equity at early default

The standard case where CoCos are treated as equity in the case of default can be extended to allow for a positive equity value at default. In this case default could also be interpreted as liquidation, where first debt is repaid and the shareholders walk away with the remaining value. The qualitative results are very similar, but the problem of debt-induced collapse becomes more severe.

Before default, the total market value of straight debt is

$$B(V;V_b) = P_1\left(\frac{m+c_1}{m+r}\right)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-(m+r)\tau_b}\right] + \mathbb{E}^{\mathbb{Q}}\left[e^{-(m+r)\tau_b}(\alpha V_{\tau_b}\wedge P_1)\right] \tag{1.5}$$

Under the condition that CoCos are converted to equity at default and receive a fraction of the recovery value after paying out the debt holders, the total market value of CoCos before default is

$$D(V;V_b) = P_2\left(\frac{m+c_2}{m+r}\right)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-(m+r)\tau_b}\right] + \frac{\Delta P_2}{1+\Delta P_2}\mathbb{E}^{\mathbb{Q}}\left[e^{-(r+m)\tau_b}(\alpha V_{\tau_b}-P_1)^+\right]. \tag{1.6}$$

At default CoCo holders have a claim on the residual asset value in proportion to the number of shares that they obtain. Essentially in the case of default CoCos are immediately converted and treated as equity. Total firm value in this case is given by

$$F^{\mathsf{BC}}(V;V_b) = V + \left(\frac{\kappa_1 c_1 P_1}{r} + \frac{\kappa_2 c_2 P_2}{r}\right)\left(1-\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right) - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right]$$

In the main text we have introduced a hypothetical firm, the NC (no-conversion) firm whose equity value is give by

$$
\begin{aligned}
EQ^{NC} =& V + \left(\frac{\kappa_1 c_1 P_1}{r} + \frac{\kappa_2 c_2 P_2}{r} \right) \left(1 - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right) - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right] \\
&- \left(P_1 \left(\frac{m+c_1}{m+r} \right) + P_2 \left(\frac{m+c_2}{m+r} \right) \right) \mathbb{E}^{\mathbb{Q}}\left[1 - e^{-(m+r)\tau_b}\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-(m+r)\tau_b}\left(\alpha V_{\tau_b}\right)\right].
\end{aligned}
$$

It is the equity value of a firm where default precedes conversion and CoCos are either treated as equity and equity holders are wiped out at default or where CoCos degenerate to a form of junior straight debt. We have derived the optimal default barrier V_b^{NC} for a NC firm in the main text.

We introduce now a second hypothetical firm, which we label AR (asset remaining after default) firm. Its equity value is identical to a firm where default happens before conversion, but equity holders are not wiped out at default. Its equity value is given by

$$
EQ^{AR} = EQ^{NC} + \frac{1}{1+\Delta P_2} \mathbb{E}^{\mathbb{Q}}\left[e^{-(r+m)\tau_b}(\alpha V_{\tau_b} - P_1)^+\right].
$$

At default equity holders have a claim on the residual asset value in proportion to the number of shares that they own. For the NC firm there are no assets remaining after default and the equity holders and CoCo bondholders are completely wiped out at default. For the AR firm the equity and CoCo bondholder receive a positive payment at default. Note, that there is only a difference between a NC and an AR firm if $\alpha V_{\tau_b} > P_1$.

Tedious, but straightforward calculations show that the optimal default barrier for an AR firm equals:

$$
V_b^{AR} = V_b^{NC} + [\alpha V_b^{AR} - P_1]^+ \frac{(\gamma_{1,m+r}+1)(\gamma_{2,r+m}+1)}{(1-\alpha)(\gamma_{1,r}+1)(\gamma_{2,r}+1) + \alpha(\gamma_{1,r+m}+1)(\gamma_{2,r+m}+1)}
$$

The main text introduced the optimal default barrier V_b^{PC} for a firm with only straight debt labeled PC (post conversion) firm. There is a clear ordering between the different candidate default barriers.

Lemma 1. *The relationship between the optimal default barriers for different firms is*

$$
V_b^{PC} \leq V_b^{NC} \leq V_b^{AC}.
$$

Therefore the default risk is higher for an AC firm than for a NC or PC firm with the same parameter values.

Proposition 1. *For a firm with straight debt and with CoCos that convert at V_c, the optimal default barrier V_b^* has the following property: Either*

$$V_b^* = V_b^{PC} \leq V_c \qquad or \qquad V_b^* = V_b^{NC} \geq V_c \qquad or \qquad V_b^* = V_b^{AR} \geq \max(V_b^{NC}, P_1/\alpha).$$

In the case of debt-induced collapse the optimal default barrier is either V_b^{NC} or V_b^{AR}. V_b^{AR} is only different from V_b^{NC} if it leads to default barrier larger than P_1/α.

Allowing for a residual value for the shareholders at default adds this additional complication to the problem. However, if the optimal default barrier is V_b^{NC} for debt-induced collapse all the results derived in main text continue to hold. On the other hand, it is now possible to have debt-induced collapse with an even higher default barrier. Hence, the derivations in the main text assume a more conservative scenario.