Lecture 4
Equivalence Calculations under Inflation
Part I – Review of the Last Lecture
Some Terminologies

- Nominal Interest Rates (Annual Percentage Rate)
- Effective Interest Rates (Annual Effective Yield)
- Discrete Compounding
- Continuous Compounding
Part II – Inflation
Measure of Inflation

• Inflation
  – A loss in the purchasing power of money over time.
  – the cost of an item tends to increase over time
    • The same dollar amount buys less of an item over time

• Deflation
  – Opposite of inflation
  – Prices decrease over time
  – A specified dollar amount gains purchasing power

• As inflation is more common, we focused on inflation in economic analyses in this lecture.
Measure of Inflation

- Suppose that your salary is $35,000, and you are promised a salary raise 6%/year. Suppose that the inflation is 8%. What is the real situation?
  - Is that you are really “rising” salary?
Consumer Price Index

• Consumer Price Index (CPI)
  – Measures prices of typical purchases made by consumers
  – Based on a typical market basket of goods and services required by the average consumers
    • Food & alcoholic beverages
    • Housing
    • Apparel
    • Transportation
    • Medical care
    • Entertainment
    • Personal care
    • Other goods and services
Consumer Price Index

- Compares the cost of the typical market basket of goods and services in a current month with its cost at a previous time (base period), e.g. 1 month ago, 1 year ago, etc.
- Not a cost-of-living index
- Good measure of the general price increase of consumer products
- Not a good measure of industrial price increases
  - → Producer Price Index
Inflation Rate

- With the price indexes,

\[
\text{Inflation rate} = \frac{\text{Change in price index}}{\text{Initial price index}} \times 100
\]

- For example, CPI in 2001 = 177.1, CPI in 2000 = 172.2,

\[
\text{Inflation rate in 2001} = \frac{177.1 - 172.2}{172.2} \times 100 = 2.8\%
\]
Average Inflation Rate $(f)$

- **Average Inflation Rate**
  - Account for the effect of varying yearly inflation rates over a period of several years

- **Compounding effect**
  - Each year’s inflation rate is based on the previous year’s rate
Average Inflation Rate ($f$)

- Example:
  - 1\textsuperscript{st} year’s inflation rate = 4\%, 2\textsuperscript{nd} year’s inflation rate = 8\%, with a base price of $100$.
  - The average inflation rate for the two years:
  - Step 1: Find the price at the end of the second year, with the process of compounding:
    $$100 \times (1 + 0.04\%) \times (1 + 0.08\%) = 112.32$$
  - Step 2: Find the average inflation rate, $f$, with the following equivalence equation:
    $$100(1 + f)^2 = 112.32 \quad \text{or} \quad 100\left(\frac{F}{P}, f,2\right) = 112.32$$
    $$f = 5.98\%$$
    The price increases in the last two years are equivalent to an average annual percentage rate of 5.98\% per year.
An Example

<table>
<thead>
<tr>
<th>Category</th>
<th>2003 Price</th>
<th>2000 Price</th>
<th>Average Inflation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postage</td>
<td>$0.37</td>
<td>$0.33</td>
<td>3.89%</td>
</tr>
<tr>
<td>Homeowners insurance (per year)</td>
<td>$603.00</td>
<td>$500.00</td>
<td>6.44%</td>
</tr>
<tr>
<td>Auto insurance (per year)</td>
<td>$855.00</td>
<td>$687.00</td>
<td>7.56%</td>
</tr>
<tr>
<td>Private college tuition and fees</td>
<td>$18,273.00</td>
<td>$15,518.00</td>
<td>5%</td>
</tr>
<tr>
<td>Gasoline (per gallon)</td>
<td>$1.65</td>
<td>$1.56</td>
<td>1.89%</td>
</tr>
<tr>
<td>Haircut</td>
<td>$12.00</td>
<td>$10.50</td>
<td>4.55%</td>
</tr>
<tr>
<td>Car (Toyota Camry)</td>
<td>$22,000.00</td>
<td>$21,000.00</td>
<td>1.56%</td>
</tr>
<tr>
<td>Natural gas (per million BTUs)</td>
<td>$5.67</td>
<td>$3.17</td>
<td>21.38%</td>
</tr>
<tr>
<td>Baseball tickets (family of four)</td>
<td>$148.66</td>
<td>$132.44</td>
<td>3.92%</td>
</tr>
<tr>
<td>Cable TV (per month)</td>
<td>$47.97</td>
<td>$36.97</td>
<td>9.07%</td>
</tr>
<tr>
<td>Movies (average ticket)</td>
<td>$5.80</td>
<td>$5.39</td>
<td>2.47%</td>
</tr>
<tr>
<td>Movies (concessions)</td>
<td>$2.17</td>
<td>$1.98</td>
<td>3.10%</td>
</tr>
<tr>
<td>Health care (per year)</td>
<td>$2,088.00</td>
<td>$1,656.00</td>
<td>8.30%</td>
</tr>
<tr>
<td>Consumer price index (CPI) Base period: 1982—84=100</td>
<td>184.20</td>
<td>171.20</td>
<td>2.47%</td>
</tr>
</tbody>
</table>
An Example

- The average inflation rate of the private college tuition is calculated as follows:
- Given: \( P = $15,518 \), \( F = $18,273 \), \( N = 2003-2000 = 3 \)

\[
18,273 = 15,518 (1 + f)^3
\]

\[
f = \sqrt[3]{1.1775} - 1
\]

\[
= 5.6\%
\]
General Inflation Rate vs Specific Inflation Rate

- General Inflation Rate ($\bar{f}$)
  - The average inflation rate based on the CPI for all items in the market basket. The market interest rate is expected to respond to this general inflation rate.

- Specific Inflation Rate ($f_i$)
  - This rate is based on an index specific to segment $j$ of the economy.
The general inflation, in terms of CPI, is defined as:

\[ CPI_n = CPI_0 (1 + \bar{f})^n \]

or

\[ \bar{f} = \left[ \frac{CPI_n}{CPI_0} \right]^{1/n} - 1 \]

where
- \( \bar{f} \) = the general inflation rate,
- \( CPI_n \) = the consumer price index at the end period \( n \), and
- \( CPI_0 \) = the consumer price index for the base period.
The general inflation, given the CPI values for two consecutive years, is defined as:

\[ \bar{f}_n = \frac{CPI_n - CPI_{n-1}}{CPI_{n-1}} \]

where

- \( \bar{f}_n \) = the general inflation rate for period \( n \).

E.g. The general inflation rate for the year 2002, where \( CPI_{2001} = 530.4 \), and \( CPI_{2002} = 538.8 \),

\[ \frac{538.8 - 530.4}{530.4} = 0.0158 = 1.58\% \]
An Example

- For a utility company’s cost to supply a fixed amount of power to a new housing development

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$504,000</td>
</tr>
<tr>
<td>1</td>
<td>$538,400</td>
</tr>
<tr>
<td>2</td>
<td>$577,000</td>
</tr>
<tr>
<td>3</td>
<td>$629,500</td>
</tr>
</tbody>
</table>

- Assume that year 0 is the base period. Determine the inflation rate for each period, and the average inflation rate over the three years.

Answer: \( f = 7.69\% \)
Actual versus Constant Dollars

- Actual (current) dollars ($A_n$)
  - The dollar value that is “influenced” by inflation.
    - Estimates of future cash flows for year $n$ that take into account any anticipated changes in amount caused by inflationary or deflationary effects.

- Constant (real) dollars ($A'_n$)
  - The real purchasing power
    - Reflect constant purchasing power independent of the passage of time.
Conversion from Constant to Actual Dollars

- For cash flows estimated with inflationary effects were assumed, they can be converted to constant dollars (base-year dollars), using the general inflation rate.

\[ A_n = A'_n \left(1 + \bar{f}\right)^n = A'_n \left(\frac{F}{P}, \bar{f}, n\right) \]

where
- \( A'_n \) = the constant-dollar expression for the cash flow occurring at the end of year \( n \), and
- \( A_n \) = the actual-dollar expression for the cash flow occurring at the end of year \( n \).
An Example

Assume the general inflation rate is 5%. Convert the following cash flows into equivalent actual dollars.

<table>
<thead>
<tr>
<th>Period</th>
<th>Net Cash Flows in Constant $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$250,000</td>
</tr>
<tr>
<td>1</td>
<td>$100,000</td>
</tr>
<tr>
<td>2</td>
<td>$110,000</td>
</tr>
<tr>
<td>3</td>
<td>$120,000</td>
</tr>
<tr>
<td>4</td>
<td>$130,000</td>
</tr>
<tr>
<td>5</td>
<td>$120,000</td>
</tr>
</tbody>
</table>

Answer:

<table>
<thead>
<tr>
<th>Period</th>
<th>Cash Flow in Actual $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$250,000</td>
</tr>
<tr>
<td>1</td>
<td>$105,000</td>
</tr>
<tr>
<td>2</td>
<td>$121,275</td>
</tr>
<tr>
<td>3</td>
<td>$138,915</td>
</tr>
<tr>
<td>4</td>
<td>$158,016</td>
</tr>
<tr>
<td>5</td>
<td>$153,154</td>
</tr>
</tbody>
</table>
Conversion from Actual to Constant Dollars

- The reverse of converting from constant to actual dollars.

\[ A_n' = A_n (1 + \bar{f})^{-n} = A_n' (P/F, \bar{f}, n) \]

where
- \( A_n' \) = the constant-dollar expression for the cash flow occurring at the end of year \( n \), and
- \( A_n \) = the actual-dollar expression for the cash flow occurring at the end of year \( n \).
An Example

- Assume the general inflation rate is 5%. A company has negotiated a five-year lease on 20 acres of land. The annual cost stated in the lease is $20,000 to be paid at the beginning of each of the five years.
- The equivalent cost in constant dollars in each period:

Answer:

<table>
<thead>
<tr>
<th>Cash Flow in Constant $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20,000</td>
</tr>
<tr>
<td>$19,048</td>
</tr>
<tr>
<td>$18,141</td>
</tr>
<tr>
<td>$17,277</td>
</tr>
<tr>
<td>$16,454</td>
</tr>
</tbody>
</table>
Equivalence Calculations under Inflation

- **Interest effects**
  - Factors in changes in earning power of money

- **Inflation**
  - Factors in changes in purchasing power of money

- **Market and Inflation-Free interest rates**
  - Market interest rate \( (i) \)
    - Nominal interest rate
      - Takes into account the combined effects of the earning value of capital (earning power) and any anticipated inflation or deflation (purchasing power).
      - All interest rates stated by financial institutions for loans and saving accounts are market interest rates.
  - Inflation-free interest rate \( (i') \)
    - An estimate of the true earning power of money when the effects of inflation have been removed.
    - Real interest rate
      - Can be computed from the market interest rate and the inflation rate.
      - In absence of inflation, the market interest rate is the same as the inflation-free interest rate.
Equivalence Calculations under Inflation

- Equivalence Calculations
  - Case 1
    - All cash flow elements are estimated in constant dollars.
  - Case 2
    - All cash flow elements are estimated in actual dollars.
  - Case 3
    - Some of the cash flow elements are estimated in constant dollars, and others are in actual dollars.
Constant-Dollar Analysis

- Since all cash flow elements are already in constant dollars.
- Absence of an inflationary effect
- Use $i'$ to account for only the earning power of the money
- To find the present-worth equivalent

\[ P_n = \frac{A_n'}{(1 + i')^n} \]

- As governments do not pay income taxed, constant-dollar analysis is common in long-term public projects.
An Example

- Given the following cash flows in constant dollars, if the inflation-free interest rate is 12% before tax, what is the present worth?

<table>
<thead>
<tr>
<th>Period</th>
<th>Net Cash Flows in Constant $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$250,000</td>
</tr>
<tr>
<td>1</td>
<td>$100,000</td>
</tr>
<tr>
<td>2</td>
<td>$110,000</td>
</tr>
<tr>
<td>3</td>
<td>$120,000</td>
</tr>
<tr>
<td>4</td>
<td>$130,000</td>
</tr>
<tr>
<td>5</td>
<td>$120,000</td>
</tr>
</tbody>
</table>

Answer: $P = $163,099$
An Example

- Suppose that your salary is $35,000, and you are promised a salary raise 6%/year. Suppose that the inflation is 8%. What is the real situation?
  - Is that you are really “rising” salary?

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary in Actual Dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$35,000</td>
</tr>
<tr>
<td>2</td>
<td>$37,100</td>
</tr>
<tr>
<td>3</td>
<td>$39,326</td>
</tr>
<tr>
<td>4</td>
<td>$41,685</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary in Real Dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$35,000</td>
</tr>
<tr>
<td>2</td>
<td>$34,351</td>
</tr>
<tr>
<td>3</td>
<td>$33,714</td>
</tr>
<tr>
<td>4</td>
<td>$33,090</td>
</tr>
</tbody>
</table>

You will receive this amount for sure
The real purchasing power you have
Actual-Dollar Analysis

- **Method 1: Deflation Method – two steps**
  1. Convert actual dollars to constant dollars by deflating with the general inflation rate of $\bar{f}$
  2. Calculate the present worth of constant dollars by discounting at $i'$.

- **Method 2: Adjusted-discount method – one step (use the market interest rate):**
  - Combine steps 1 & 2 into one step.

\[
P_n = \frac{A_n}{[(1+\bar{f})(1+i')]^n}
\]
\[
= \frac{A_n}{(1+i)^n}
\]

• where $i = i' + \bar{f} + i' \bar{f}$
Actual-Dollar Analysis

Step 1

$$P_n = \frac{A_n}{(1 + \bar{f})^n}$$

Step 2

$$P_n = \frac{A_n}{(1 + i')^n}$$

$$= \frac{A_n}{(1 + \bar{f})^n (1 + i')^n}$$

$$= \frac{A_n}{[(1 + \bar{f}) (1 + i')]^n}$$

$$P_n = \frac{A_n}{(1 + i)^n}$$

$$= \frac{A_n}{[(1 + \bar{f}) (1 + i')]^n}$$

$$(1 + i) = (1 + \bar{f})(1 + i')$$

$$= 1 + i' + \bar{f} + i' \bar{f}$$

$$i = i' + \bar{f} + i' \bar{f}$$
An Example

- A project of an electronic company is expected to generate the following cash flows in actual dollars:

<table>
<thead>
<tr>
<th>n</th>
<th>Net Cash Flows in Actual Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$75,000</td>
</tr>
<tr>
<td>1</td>
<td>$32,000</td>
</tr>
<tr>
<td>2</td>
<td>$35,700</td>
</tr>
<tr>
<td>3</td>
<td>$32,800</td>
</tr>
<tr>
<td>4</td>
<td>$29,000</td>
</tr>
<tr>
<td>5</td>
<td>$58,000</td>
</tr>
</tbody>
</table>

- What are the equivalent year-zero dollars (constant dollars) if the general inflation rate $f$ is 5% per year?
- Compute the present worth of the cash flows in constant dollars at $i' = 10\%$.

Answer: Total Equivalent Present Worth = $45,268
Mixed-Dollar Analysis

- Convert all cash flow elements into the same dollar units (either constant or actual).
- Case 1: converted to actual dollars
  - Use market interest rate $i$ in calculating the equivalence value
- Case 2: converted to constant dollars
  - Use the inflation-free interest rate $i'$ in calculating the equivalence value
An Example

- A couple wishes to establish a college fund at a bank for their five-year-old child. The college fund will earn 8% interest compounded quarterly. Assuming the child enters college at 18, the estimated expenses for the four years is $30,000 per year. The college expenses are estimated to increase at an annual rate of 6%.
- Determine the equal quarterly deposits the couple must make until they send their child to college.

Answer: $2,888.48
• References
  – Chan S. Park, Fundamentals of Engineering Economics. Prentice Hall
  – Lecture 9: Inflation, by Gabriel Fung