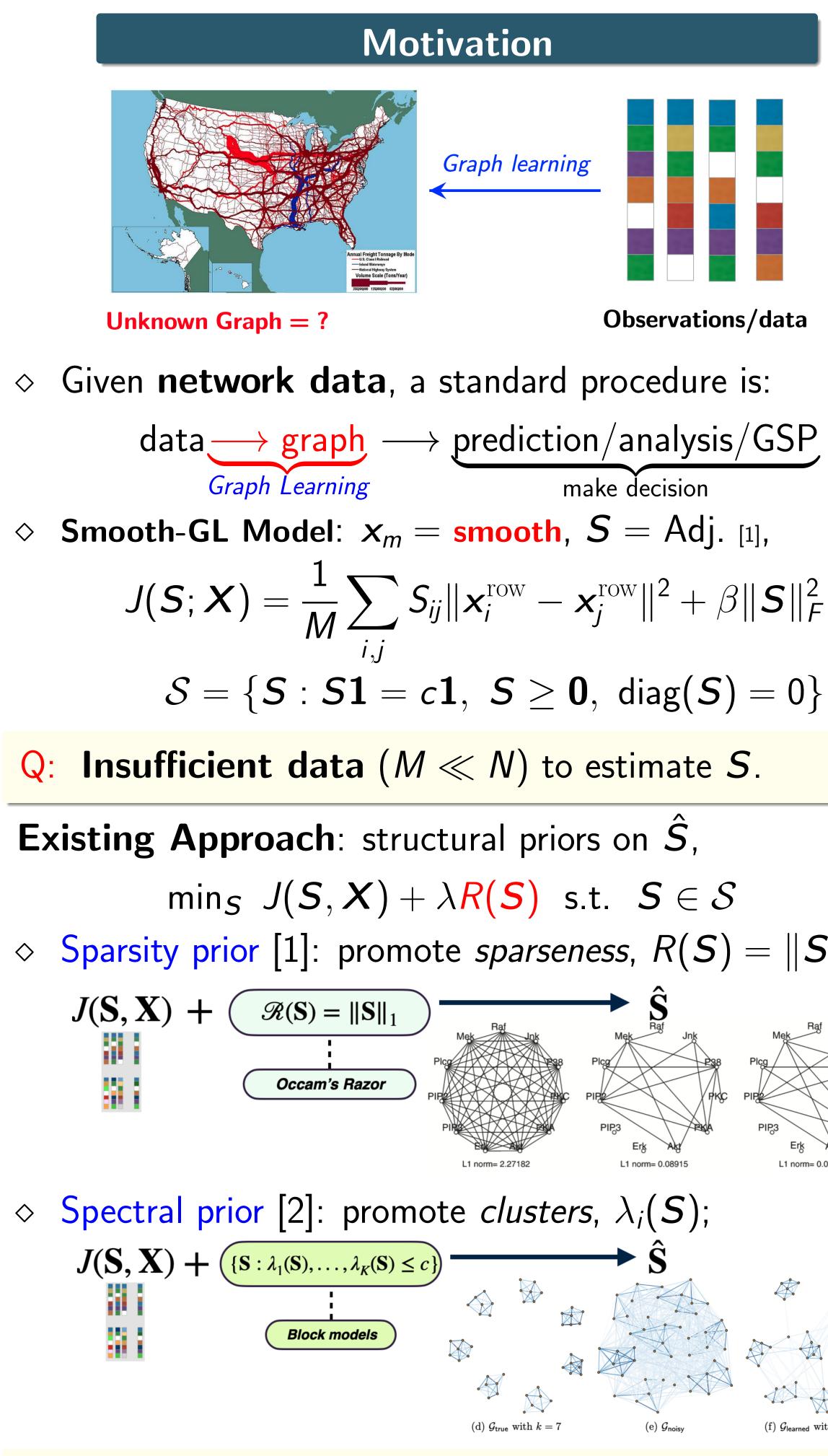
Network Games I

Chenyue ZHANG (CUHK), Shangyi



Observation: Structural priors are explicit on S and only capture macroscopic properties.

Network Games in Social Network

- ◇ Generalized linear quadratic game [3] w/ utility fund $U(y_i; y_{-i}; S) := -y_i^2/2 + y_i \left(\sum_{j=1, j \neq i}^n S_{ij}f(y_j) - \right)$
 - b_i : marginal benefit, $f(\cdot)$ models interactions.
- \diamond Nash equilibrium (NE) = state where no agent ha intention to change,

 $NE(S) := \{ y \in \mathbb{R}^N : y_i = rg\max_{v \geq 0} U(y_i; y_{-i}; S), \}$

 \diamond If |NE(S)| = 1, $y^{NE}(S)$ denotes the unique NE.

| Image: symmetry of the symmet | | | | | | | |
|--|------------------------------------|---|--|--|---|---|--|
| Max-Welfare Induced GraSuppose $ NE(S) = 1$, denote Wel $(S) := 1^{T}y$ Empirical Study: Is intelligent network S local(a) WikiWe(b) Kente Club(c) WikiWe(b) Kente Club(c) WikiWe(c) DolphinsRewiring10%20%30%WikiWe(c) DolphinsRewiring10%96.89%95.33%98.80%84.72%81.17%78.30%75Conjecture: Human-made networks are self-or \rightarrow Utilize Wel (S) to inform graph learning. $J(S, X)$ \checkmark (S) = 1 ^T y*(S)Graph Learning with Functional Prior (G $min_{S,y \in \mathbb{R}^N} J(S, X) - \lambda 1^T y 8.1. y$ Q: What structural insights can be derivProposition: An approximation of (GLFP) ad $S_{ij}^{i} = \frac{1}{2j} max \{0, \eta_i + \lambda b_j - D_{ij}\}$, \circ Implication: let $\lambda \gg 1$, optimal S* admits a \diamond Empirical evidence: human-made network coUrimescales Gradient AlgorChallenge: took network dynamics as constrain \diamond Upper level: regularized graph learning. Loc \rightarrow Two-timescales algo:: η (Lower-level) $\varphi(S) \Phi(S^k, y^k)$ estimates hypergradient — free $\forall i$ $mi_{k=1,,K} \gamma^{-1}(S^k - \operatorname{Proj}_S(S^k - \gamma \nabla \Phi))$ \Rightarrow finds a stationary point of (GLFP). | nduc | ed F | ^D rio | r for | Gr | aph | Tc |
| Suppose $ NE(S) = 1$, denote $Wel(S) := 1$ 'y Empirical Study: Is intelligent network S local (a) WileVote (b) Karate Club: (c) Dolphine. Rewiring 10% 20% 30% 40% 50% WileVote 96.29% 93.07% 93.03% 88.01% 67.19% Natate 96.69% 97.37% 88.17% 78.30% Dolphine 98.15% 96.48% 95.13% 93.97% 93.08% Conjecture: Human-made networks are self-or \Rightarrow Utilize Wel(S) to inform graph learning. (1) S _1. (1) S _1. (1) S _1. (1) S _2. (1) S _2. (1) S _2. (1) S _2. (1) S _2. (1) S _2. (2) Graph Learning with Functional Prior (G min _{S,Y} $\subset \mathbb{R}^N J(S, X) = \lambda 1^{+} y$ S.t. y Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad $S_{ij}^{*} = \frac{1}{23} \max \{0, \eta_i + \lambda b_j - D_{ij}\},$ \diamond Implication: let $\lambda \gg 1$, optimal S* admits as \diamond Empirical evidence: human-made network con the tion: \Rightarrow Two-timescales algo:: η (Lower-lew $y_i^{k+1} = y_i^k + \eta \nabla_{y_i} U(y_i^k, y_i^k; S^{k+1} = \operatorname{Proj}_S(S^k - \gamma \nabla \varphi)(X_i^k, y_i^k; S^{k+1} = \operatorname{Proj}_S(S^k$ | uan LIU | (CUHK), | Hoi-To | WAI (| cuhk), A | Anthon | y Ma |
| Empirical Study: Is intelligent network S local (a) WikWoll: (b) Karate Club. (c) Dupleme. Rewring 10% 20% 30% 40% 50% WikiYong 96.29% 90.33% 88.86% 64.72% 81.17% 78.30% polphins 98.15% 96.48% 95.13% 93.97% 93.08% Conjecture: Human-made networks are self-or $R(S) = 1^{T}y^{*}(S)$ Conjecture: Human-made networks are self-or $R(S) = 1^{T}y^{*}(S)$ Graph Learning with Functional Prior (G min $_{S,y \in \mathbb{R}^{N}} J(S, X) - \lambda 1^{T}y$ s.t. y Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad $S_{ij}^{ij} = \frac{1}{2ij} \max\{0, \eta_{i} + \lambda b_{j} - D_{ij}\},$ \diamond Implication: let $\lambda \gg 1$, optimal S^{*} admits a \diamond Empirical evidence: human-made network con- trone \rightarrow Two-timescales algo: η (Lower-lew $y_{i}^{k-1} = y_{i}^{k} + \eta \nabla_{y_{i}} U(y_{i}^{k}, y_{i}^{k};$ $S^{k-1} - \Prroj_{S}(S^{k} - \gamma \nabla \phi)(x_{i})$ $\forall i\}$ $\hat{\nabla} fnds a stationary point of (GLFP).$ | | | | Max-V | Velfare | Induced | d Gra |
| (a) WikiVece. (b) Karate Club. (c) Delphines. Rewiring 10% 20% 30% 40% 50% WikiVece. 94.06% 93.07% 90.33% 88.01% 66.119 Karate 94.06% 93.07% 90.33% 93.07% 93.08% Dolphins 98.15% 96.48% 95.13% 93.97% 93.08% Conjecture: Human-made networks are self-of $R(S) = 1^{T}y^{+}(S)$ Conjecture: Human-made networks are self-of $R(S) = 1^{T}y^{+}(S)$ Graph Learning with Functional Prior (G min _{S,y} $\in \mathbb{R}^{N} J(S, X) - \lambda 1^{T}y$ s.t. y Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad $S_{ij}^{*} = \frac{1}{23} \max \{0, \eta_{i} + \lambda b_{j} - D_{ij}\},$ \circ Implication: let $\lambda \gg 1$, optimal S^{*} admits a \diamond Empirical evidence: human-made network co Two-timescales Gradient Algor Challenge: took network dynamics as constrain \diamond Upper level: regularized graph learning. Lo \Rightarrow Two-timescales algo.: η (Lower-lew $\begin{bmatrix} y_{i}^{k-1} = y_{i}^{k} + \eta \nabla_{y_{i}} U(y_{i}^{k}, y^{k}; S^{k-1} = \operatorname{Proj}_{S}(S^{k} - \gamma \nabla_{S} \Phi)(X)$ as the $\hat{\nabla}_{S} \Phi(S^{k}, y^{k})$ estimates hypergradient — free \diamond Theorem: With $\eta = \frac{1-c}{1-c_{2}}, \gamma \ll \eta$ (+ additi min_{k-1,,K} $\ \gamma^{-1}(S^{k} - \operatorname{Proj}_{S}(S^{k} - \gamma \nabla \Phi)(X)$ \Rightarrow finds a stationary point of (GLFP). | | Suppose | NE(S) |)ert=1, d | lenote W | /el(<i>S</i>) := | $= 1^{	op} \mathbf{y}$ |
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| Rewiring10%20%30%40%50%WikiYote96.29%93.07%90.33%88.01%86.11%78.30%Polphins99.15%96.48%95.13%93.97%93.08%SDolphins99.15%96.48%95.13%93.97%93.08%(1)S _1.Image: Conjecture: Human-made networks are self-or R(S) = 1 ^T y*(S)Image: Conjecture: Human-made networks are self-or Proposition: An approximation of (GLFP) ad S''_i = $\frac{1}{23}$ max $\{0, \eta_i + \lambda b_j - D_{ij}\}$,(1)S _1.Image: Conjecture: Human-made networks are self-or Proposition: An approximation of (GLFP) ad S''_i = $\frac{1}{23}$ max $\{0, \eta_i + \lambda b_j - D_{ij}\}$,(1)SImage: Conjecture: An approximation of (GLFP) ad S''_i = $\frac{1}{23}$ max $\{0, \eta_i + \lambda b_j - D_{ij}\}$,(2)What structural insights can be derive Proposition: An approximation of (GLFP) ad S''_i = $\frac{1}{23}$ max $\{0, \eta_i + \lambda b_j - D_{ij}\}$,(2)Implication: let $\lambda \gg 1$, optimal S* admits a \diamond Empirical evidence: human-made network co Two-timescales algo.: η (Lower-lew $y_i^{k-1} = y_i^k + \eta \nabla_{y_i} U(y_i^k, y_i^k;$ $S^{k+1} = \operatorname{Proj}_S(S^k - \gamma \widehat{\nabla}_S \Phi)$ (as the $i \forall i$ $\widehat{\nabla}_S \Phi(S^k, y^k)$ estimates hypergradient — free $i = 1 + c_1^{-2}$, $\gamma \ll \eta$ (+ additi $mi_{k-1,,K} \gamma^{-1}(S^k - \operatorname{Proj}_S(S^k - \gamma \nabla \Phi)$ \Rightarrow finds a stationary point of (GLFP). | | | | 25 26 28 29 30 34 31 33 10 10 | 11 6 7 5 12 22 18 | 56 5 24 1 60 37 40 8 2 20 49 58 42 5 5 28 27 6 10 7 18 26 33 57 | $\begin{array}{c} & 36 \\ & 13 \\ & 59 \\ 6 \\ 46 \\ 9 \\ 9 \\ 41 \\ 21 \\ 48 \\ 29 \\ 31 \end{array}$ |
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| $\begin{array}{c} \begin{array}{c} & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ $ | - | • | | | | | |
| $\Rightarrow Utilize Wel(S) to inform graph learning.$ $J(S, X) = J^{T}y^{*}(S)$ $Graph Learning with Functional Prior (Gmin_{S,y \in \mathbb{R}^{N}} J(S, X) - \lambda 1^{T}y \text{ s.t. } y$ $Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad S_{ij}^{*} = \frac{1}{2g} \max\{0, \eta_{i} + \lambda b_{j} - D_{ij}\}, \Rightarrow Implication: let \lambda \gg 1, optimal S^{*} admits at \Rightarrow Empirical evidence: human-made network cond Two-timescales Gradient Algor Challenge: took network dynamics as constrain \Rightarrow Upper level: regularized graph learning. Loc J_{i}^{k+1} = y_{i}^{k} + \eta \nabla_{y_{i}} U(y_{i}^{k}, y_{i}^{k}; S^{k-1} = \operatorname{Proj}_{S}(S^{k} - \gamma \widehat{\nabla}_{S} \Phi(S^{k}, y^{k}) estimates hypergradient - from \forall i \} \widehat{\nabla}_{S} \Phi(S^{k}, y^{k}) estimates hypergradient - from \Rightarrow \operatorname{Theorem}: With \eta = \frac{1-c}{(1+c)^{2}}, \gamma \ll \eta (+ addition) \lim_{k=1,,K} \gamma^{-1}(S^{k} - \operatorname{Proj}_{S}(S^{k} - \gamma \nabla \Phi(S^{k}) - \nabla \Phi(S^{k}) - \nabla \nabla \Phi($ | 2 F, | Karate | 94.06% | 88.86% | 84.72% | 81.17% | 78.30% |
| (1) $S _{1}$ $R(S) = 1^{T}y^{*}(S)$ $Graph Learning with Functional Prior (Gmin_{S,y \in \mathbb{R}^{N}} J(S, X) - \lambda 1^{T}y \text{ s.t. } y$ $Q: What structural insights can be derived by the structural insights by the structural insight by the st$ | Ĵ | Conjec | ture: H | luman-m | ade netv | vorks are | e self-o |
| (1) $S _{1}$ $R(S) = 1^{\top} \mathbf{y}^{*}(S)$ Graph Learning with Functional Prior (Gmin _{S,y \in \mathbb{R}^{N} J(S, X) - \lambda 1^{\top} \mathbf{y} \text{ s.t. } \mathbf{y} Graph Learning with Functional Prior (Gmin_{S,y \in \mathbb{R}^{N} J(S, X) - \lambda 1^{\top} \mathbf{y} \text{ s.t. } \mathbf{y} Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad $S_{ij}^{*} = \frac{1}{2\beta} \max \{0, \eta_{i} + \lambda b_{j} - D_{ij}\},$ \Rightarrow Implication: let $\lambda \gg 1$, optimal S^{*} admits at \Rightarrow Empirical evidence: human-made network conditions \Rightarrow Two-timescales Gradient Algor Challenge: took network dynamics as constraint \diamond Upper level: regularized graph learning. Low $\mathbf{y}_{i}^{k+1} = \mathbf{y}_{i}^{k} + \eta \nabla_{y_{i}} U(\mathbf{y}_{i}^{k}, \mathbf{y}_{i}^{k};$ $S^{k+1} = \operatorname{Proj}_{S}(S^{k} - \gamma \widehat{\nabla}_{S} \Phi(\mathbf{x}, \mathbf{y}^{k}))$ estimates hypergradient — from \Rightarrow Theorem: With $\eta = \frac{1-c}{(1+c)^{2}}, \gamma \ll \eta$ (+ addition $\min_{k=1,,K} \gamma^{-1}(S^{k} - \operatorname{Proj}_{S}(S^{k} - \gamma \nabla \Phi(\mathbf{x}, \mathbf{y}^{k}))$ \Rightarrow finds a stationary point of (GLFP).}} | J , | $\implies Uti$ | <i>lize</i> Wel | (\boldsymbol{S}) to in | nform gr | aph learr | ning. |
| $\min_{S,y \in \mathbb{R}^{N}} J(S, X) - \lambda 1^{\top} y \text{ s.t. } y$ Q: What structural insights can be derive Proposition: An approximation of (GLFP) ad $S_{ij}^{*} = \frac{1}{2\beta} \max \{0, \eta_{i} + \lambda b_{j} - D_{ij}\},$ \diamond Implication: let $\lambda \gg 1$, optimal S^{*} admits at \diamond Empirical evidence: human-made network constrain \diamond Empirical evidence: human-made network constrain \diamond Upper level: regularized graph learning. Loc \rightarrow Two-timescales algo.: η (Lower-level) $g_{i}^{k+1} = y_{i}^{k} + \eta \nabla_{y_{i}} U(y_{i}^{k}, y_{i}^{k}),$ $S^{k+1} = \operatorname{Proj}_{S}(S^{k} - \gamma \widehat{\nabla}_{S} \Phi)(S^{k}, y^{k})$ estimates hypergradient — from \diamond Theorem: With $\eta = \frac{1-c}{(1+c)^{2}}, \gamma \ll \eta$ (+ addition $\min_{k=1,,K} \gamma^{-1}(S^{k} - \operatorname{Proj}_{S}(S^{k} - \gamma \nabla \Phi)(X_{i}) $ | (1) $S\ _1$. | | | | | | |
| Proposition: An approximation of (GLFP) ad $S_{ij}^{\star} = \frac{1}{2\beta} \max \{0, \eta_i + \lambda b_j - D_{ij}\},$ \diamond Implication: let $\lambda \gg 1$, optimal S^{\star} admits a \diamond Empirical evidence: human-made network co Two-timescales Gradient Algor Challenge: took network dynamics as constrain \diamond Upper level: regularized graph learning. Lo \rightarrow Two-timescales algo.: η (Lower-lev $y_i^{k+1} = y_i^k + \eta \nabla_{y_i} U(y_i^k, y_i^k;$ $S^{k+1} = \operatorname{Proj}_{\mathcal{S}}(S^k - \gamma \widehat{\nabla}_{\mathcal{S}} \Phi(S^k, y^k))$ estimates hypergradient — from \diamond Theorem: With $\eta = \frac{1-c}{(1+c)^2}, \gamma \ll \eta$ (+ addition $\min_{k=1,,K} \gamma^{-1}(S^k - \operatorname{Proj}_{\mathcal{S}}(S^k - \gamma \nabla \Phi(S^k - \gamma \nabla $ | m= 0.04251 | | | | | | - |
| $\widehat{\nabla}_{s} \Phi(\mathbf{S}^{k}, \mathbf{y}^{k}) estimates hypergradient — freebigging for the stationary point of (GLFP).$ | i = 2 | Propos A state of the state of th | sition: A $S_{ij}^{\star} =$ cation: I | An <i>appro</i> $=rac{1}{2eta}$ maxes of the second se | ximation $<\{0,\eta_i+1, \text{ optim}\}$ | of (GLF $-\lambda b_j - L$) al S^* ac | FP) ad D _{ij} } , Imits a |
| Challenge: took network dynamics as constraint \diamond Upper level: regularized graph learning. Lower-level \rightarrow Two-timescales algo.: η (Lower-level) $y_i^{k+1} = y_i^k + \eta \nabla_{y_i} U(y_i^k, y^k;$ $S^{k+1} = \operatorname{Proj}_{\mathcal{S}}(S^k - \gamma \widehat{\nabla}_{\mathcal{S}} \Phi)$ as the $\widehat{\nabla}_{\mathcal{S}} \Phi(S^k, y^k)$ estimates hypergradient — from \diamond Theorem: With $\eta = \frac{1-c}{(1+c)^2}, \ \gamma \ll \eta$ (+ addition $\min_{k=1,,K} \gamma^{-1}(S^k - \operatorname{Proj}_{\mathcal{S}}(S^k - \gamma \nabla \Phi)) $ \Longrightarrow finds a stationary point of (GLFP). | | • Emp | | | | | |
| $\forall S \Psi(S^{\kappa}, Y^{\kappa}) \text{ estimates hypergradient} \longrightarrow \text{free} \\ \Rightarrow \text{ Theorem: With } \eta = \frac{1-c}{(1+c)^2}, \ \gamma \ll \eta \ (+ \text{ additives}) \\ \min_{k=1,\dots,K} \ \gamma^{-1}(S^k - \operatorname{Proj}_{\mathcal{S}}(S^k - \gamma \nabla \Phi(A^k)) - \gamma \nabla \Phi(A^k)) \ _{\mathcal{S}} \\ \longrightarrow \text{ finds a stationary point of (GLFP).} \end{aligned}$ | $-b_i$ | ♦ Uppe | ge: took er level: | network regulariz escales | dynamie ed graph algo.: = y _i ^k + y | cs as cor learning η (Low $\eta \nabla_{v_i} U(y)$ | nstrain g. Lo er-lev |
| ICASSP 2025, Hyderabad, India. Contact E-mail: {czhang,s | as the $\langle \forall i \rangle$ | $\begin{array}{c} \diamond \text{Theo} \\ \\ \\ \\ \end{array} \\ \end{array} \\ \end{array}$ | rem: Wi $n_{k=1,,K}$ finds a s | th $\eta = \overline{0}$ $\ \gamma^{-1}(S')$ tationary | $\frac{1-c}{1+c)^2}, \gamma$ | $\ll \eta (+$ $_{S}(\boldsymbol{S}^{k} - \boldsymbol{\gamma})$ f (GLFP) | additi γ∇Φ(.). |

opology Learning

an-Cho SO (CUHK)

aph Learning

$V^{NE}(S)$ as the **social welfare**.

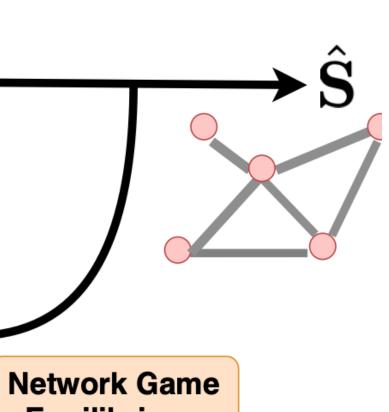
Ily optimized for max-welfare ?

- ♦ Random rewiring and evaluate the relative % of welfare loss due to rewiring.
- ♦ Human-made networks (Karate & WikiVote) are sensitive to rewiring;

%

non-human-made networks are not (Dolphins).

-optimized w.r.t. Wel(S).



Equilibrium

GLFP):

$\mathbf{y} \in \mathsf{EQ}(\mathbf{S}), \ \mathbf{S} \in \mathcal{S}.$

ved from (GLFP)?

dmits the optimal solution:

for some $\eta_i \in \mathbb{R}$.

a 'star' structure.

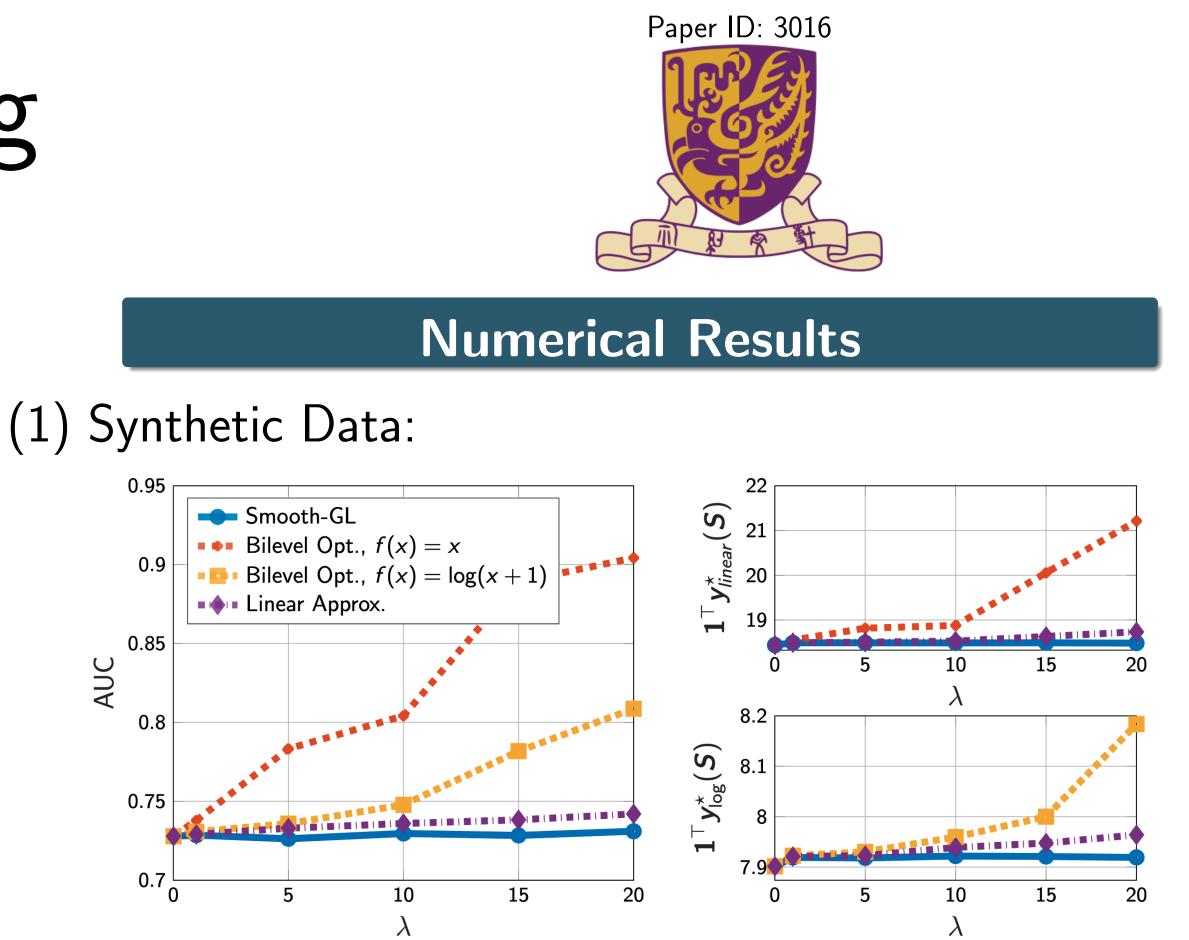
contains few hub nodes.

rithm for (GLFP)

 $nt \Rightarrow$ **Bilevel Optimization**: ower level: NE seeking. evel) > γ (Upper-Level) S^k), $\forall i \in [N]$ (2) $(S^k, y^k)),$

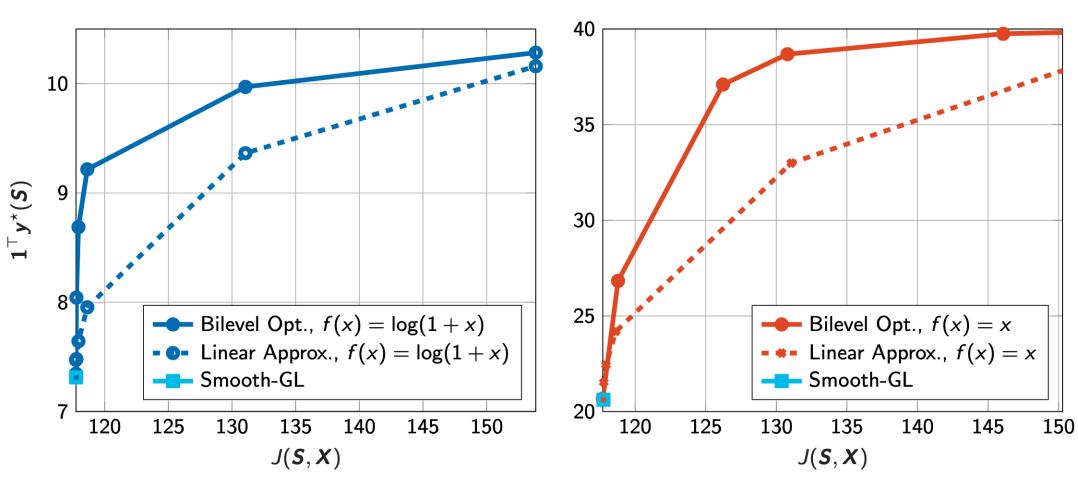
rom derivatives of $U(\cdot)$ tional assumptions),

$$old S^k,old y^\star(old S^k))))\|^2=\mathcal{O}(K^{-1}).$$



- \diamond For the network dynamics & (GLFP), we take b = TopEV(D).

(2) Karate Club:



- \diamond N = 34 from M = 50 smooth graph signals.
- \diamond Pareto Front by adjusting λ to get tradeoffs for h(y) vs J(S, X). ◇ Bilevel OPT by TT algorithm achieves better *Pareto optimality*.
- (3) Real Data: Indian village [5]

| | AUROC | Maximum | Average | Minimum |
|------|--|--------------|------------|----------------|
| | Bilevel - f(y) = y | 0.6345 | 0.5788 | 0.5271 |
| | Bilevel $- f(y) = \log(1+y)$ | 0.6570 | 0.5937 | 0.5490 |
| | Smooth-GL | 0.5075 | 0.4888 | 0.4777 |
| | $Welfare(\hat{\boldsymbol{S}};f) - Welfare(\boldsymbol{S}^{true};f)$ | Maximum | Average | Minimum |
| | Bilevel - f(y) = y | 4.2754 | 3.2077 | 2.1693 |
| | Smooth-GL – $f(y) = y$ | -2.3288 | -5.8141 | -8.6115 |
| | $\mathbf{Bilevel} - f(y) = \log(1+y)$ | 1.4464 | 1.1892 | 0.8461 |
| | Smooth-GL - $f(y) = \log(1+y)$ | -0.5440 | -2.8292 | -5.0419 |
| Da | ata : network sizes $N = 77$ | to 330, ea | ach w/ h | ⁄/ = 16 sa |
| lin | nited amount of data (/ | $M \ll N$). | | |
| | aph learning with <i>functional</i> | l prior max | imizing W | Velfare(S) |
| › Gr | 1 0 | ' | 0 | |

[2] Kumar, Sandeep, et al.

- [5] Kalofolias V. How to learn a graph from smooth signals. In AISTATS 2016 (pp. 920-929). PMLR

 \diamond **Setting**: $S \sim PA$ graph with N = 50 nodes & M = 10 stationary + smooth graph signals \Rightarrow limited amount of data ($M \ll N$). \diamond **Benchmark**: (i) linear approximation of (GLFP) with $\mathbf{1}^{\top}y^{\star}(S) \approx \mathbf{1}$

 $\mathbf{1}^{ op} Sb \Rightarrow$ single-level problem & (ii) Smooth-GL [4].

◇ Proposed methods has better AUROC than vanilla Smooth-GL.

 $ples \Rightarrow$

mproves

3):432-41. "A unified framework for structured graph learning via spectral constraints." JMLR 21.22 (2020): 1-60. [3] Cai J, Zhang C, Wai HT. Optimal pricing for linear-quadratic games with nonlinear interaction between agents. IEEE L-CSS. 2024 Jun 6. [4] Candogan O, Bimpikis K, Ozdaglar A. Optimal pricing in networks with externalities. Operations Research. 2012 Aug;60(4):883-905.

[6] Banerjee A, Chandrasekhar AG, Duflo E, Jackson MO. The diffusion of microfinance. Science. 2013 Jul 26;341(6144):1236498.