Stochastic Amplify-and-Forward Schemes for Multigroup Multicast Transmission in a Distributed Relay Network

Sissi Xiaoxiao Wu*, Qiang Li†, Wing-Kin Ma‡, and Anthony Man-Cho So*
*Department of System Engineering & Engineering Management, The Chinese University of Hong Kong
†School of Communication & Information Engineering, University of Electronic Science & Technology of China
‡Department of Electronic Engineering, The Chinese University of Hong Kong
Email: {xxwu, manchoso}@ee.cuhk.edu.hk, lq@uestc.edu.cn, wkma@ieee.org

***Abstract*— In this paper, we study amplify-and-forward (AF) schemes to for multigroup multicast information delivery between long-distance users. The target scenario is a two-hop distributed one-way relay network where the transmitters, relays and receivers are all equipped with a single antenna. Assuming that channel state information (CSI) is perfectly known, our goal here is to design the AF weights at the relays so that the system rate performance can be optimized. A classic AF scheme in this context is to employ a rank-one beamformed AF (BF-AF) strategy with a max-min-fair (MMF) achievable rate objective. In this way, the semidefinite relaxation (SDR) technique is widely used to provide an (approximate) solution for the MMF problem. It is known that the achievable rate performance of the SDR-based BF-AF scheme tends to degrade seriously with the number of users served in the relay network. This motivates us to propose stochastic beamformed AF (SBF-AF) schemes to improve the achievable rate performance. The salient feature of the SBF-AF schemes is that it employs time-varying AF weights and bypass some inherent issues in the SDR-based BF-AF scheme. Our theoretical analysis and numerical results both show that the SBF-AF schemes can outperform the BF-AF scheme.**

***Index Terms*—one-way relay, stochastic beamforming, amplify-and-forward (AF), multigroup multicast, SDR.**

I. Introduction

It is well known that for facilitating information delivery between long-distance users, one can introduce relay nodes in the wireless network. In this work, we are particularly interested in the scenario where amplify-and-forward (AF) schemes are used, and the transmitters, relays and receivers are all with a single antenna. Such a scenario setting is common in military communication or in device-to-device (D2D) communication in a cellular network, where the two ends of the link are usually limited by the apparatus and power. It is worth mentioning that the relay network we considered here can be seen as a special example of the *cloud relay network (C-RN)* in [1], where channel state information (CSI) is shared in processing units (PUs) pool and data information is isolated among different relays due to the limited fronthaul-backhaul link capacity; see an example of the C-RN in Figure 1. This results in the so called distributed (or cooperative) relay network in the literature.

In this paper, we consider physical-layer multigroup multicasting in the aforementioned relay scenario. Prior to this work, there are many works that study AF designs in distributed relay networks [2]–[7]. A popular approach therein is to let relays perform a beamformed amplify-and-forward (BF-AF) scheme [2], [5]. Then, a fractional quadratically constrained quadratic problem (QCQP) is formulated from a max-min-fair (MMF) quality-of-service (QoS) perspective, which is NP-hard in general [8]–[10]. By applying the semidefinite relaxation (SDR) technique [9], the MMF problem can be approximated by a fractional SDR. Our provable result in this work reveals that the achievable rate of the BF-AF scheme degrades at a rate of $\log M$, where M is the number of users served in the network. This motivates us to develop the stochastic beamformed AF (SBF-AF) schemes to improve the achievable rate performance. Specifically, instead of applying a fixed rank-one AF weight, SBF-AF schemes adopt time-varying random AF weights, and exploit the temporal degree of freedom to capture the non-rank-one nature of the SDR solution. In particular, we introduce two types of SBF-AF schemes, which are guided by Gaussian and elliptic distributions, respectively. Theoretically, we prove that the SBF-AF schemes can yield SBF rates that are at most 0.83 bits/s/Hz worse than the rate associated with the SDR optimal solution. Our simulation results further demonstrate the superiority of the SBF. It is worth mentioning that the idea of SBF first appears in [11] for single-group multicasting (without relaying) [11], [12]. This paper is the first attempt to apply SBF in relay networks, and we extend the scope to multigroup multicasting.

II. The Multigroup Multicast Model and the Beamformed AF Scheme

We consider a multigroup multicast scenario in the distributed relay network as depicted in Figure 2, where there

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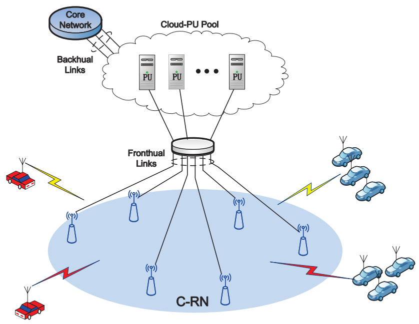

Fig. 1. An example of the cloud relay network.

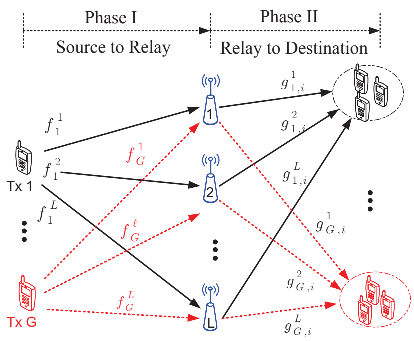

Fig. 2. The two-hop distributed one-way relay network.

are G single-antenna transmitters (sources) sending G independent information to G groups of single-antenna receivers (destinations, or users). Assume that in group-k, there are m_k users, who request the same information, while users in different groups request different information. In total, we have $\sum_{k=1}^{G} m_k = M$ users in the distributed relay network. There are L single-antenna relays distributively located in the network. We assume that there is no direct link between the transmitters and receivers, and the relays amplify and forward the received signals in a collaborative manner. The whole transmission consists of two phases:

1) Phase I: *sources send information to relays.* Assume that the channels from transmitters to relays are frequency-flat and quasi-static. The received signal at the relays can be modeled as

$$\mathbf{r}(t) = \sum_{j=1}^{G} \boldsymbol{f}_j s_j(t) + \mathbf{n}(t), \tag{1}$$

where $\boldsymbol{r}(t) = [r^1(t), ..., r^\ell(t), ..., r^L(t)]^T$ with $r^\ell(t) = \sum_{j=1}^{G} f_j^\ell s_j(t) + n^\ell(t)$ being the received signal at relay-ℓ; $s_j(t)$ is the common information for group-j ($j = 1, ..., G$) with $\mathbb{E}[|s_j(t)|^2] = P_j$, where P_j is the transmit power at the transmitter-j; $\boldsymbol{f}_j = [f_j^1, ..., f_j^\ell, ..., f_j^L]^T$ with f_j^ℓ being the channel from transmitter-j to relay-ℓ; $\mathbf{n}(t) = [n^1(t), ..., n^\ell(t), ..., n^L(t)]^T$ with $n^\ell(t)$ being the Gaussian noise at relay-ℓ with zero mean and variance σ_ℓ^2.

2) Phase II: *relays amplify and forward the received signals to destinations.* A popular approach under this model is to AF the received signals by beamforming [2]. In this approach, the AF process at the relay side is written as

$$\mathbf{x}(t) = \mathrm{Diag}(\mathbf{w})\mathbf{r}(t), \tag{2}$$

where $\mathbf{w} = [w_1, ..., w_\ell, ...w_L]^T$ is the AF weight with w_ℓ being the AF coefficient at relay-ℓ and $\mathrm{Diag}(\mathbf{w})$ is a diagonal matrix parametrized with the elements of $\mathbf{w}$. Note that the AF weight matrix here is a diagonal matrix, since each relay can only AF the signal received by its own. Based on the transmit model (2), and assuming frequency-flat and quasi-static channels from the relays to the destinations, the received signal for user-i in group-k is expressed as

$$\begin{aligned} y_{k,i}(t) &= \boldsymbol{g}_{k,i}^H \mathbf{x}(t) + v_{k,i}(t) \quad k = 1, ..., G, i = 1, ..., m_k, \\ &= \sum_{j=1}^{G} \sum_{\ell=1}^{L} (g_{k,i}^\ell)^* w_\ell f_j^\ell s_j(t) + \sum_{\ell=1}^{L} (g_{k,i}^\ell)^* w_\ell n^\ell(t) + v_{k,i}(t), \end{aligned} \tag{3}$$

where $\boldsymbol{g}_{k,i} = [g_{k,i}^1, ..., g_{k,i}^\ell, ..., g_{k,i}^L]^T$ is the channel from the relays to user-(k,i) with $g_{k,i}^\ell$ being the channel from relay-ℓ to user-(k,i); $v_{k,i}(t)$ is the Gaussian noise at user-(k,i) with mean zero and variance $\sigma_{k,i}^2$. We assume that channels are perfectly known in the relay network. Then, the signal-to-interference-plus-noise ratio (SINR) for user-(k,i) can be written as

$$\frac{P_k |\sum_{\ell=1}^{L} f_k^\ell (g_{k,i}^\ell)^* w_\ell|^2}{\sum_{m \neq k} P_m |\sum_{\ell=1}^{L} f_m^\ell (g_{k,i}^\ell)^* w_\ell|^2 + \sum_{\ell=1}^{L} |\sigma_\ell^2 (g_{k,i}^\ell)^* w_\ell|^2 + \sigma_{k,i}^2},$$

and the relay signal power is given by $\mathbb{E}[\|\mathbf{x}(t)\|^2] = \mathbf{w}^H \mathbf{D} \mathbf{w}$, where

$$\mathbf{D} = \sum_{j=1}^{G} P_j \mathrm{Diag}(\boldsymbol{f}_j \boldsymbol{f}_j^H) + \mathrm{Diag}(\sigma_1^2, ..., \sigma_L^2). \tag{4}$$

In this way, the design problem for the distributed relay network can be formulated as

$$\begin{aligned} (\mathsf{BF}) \quad \mathbf{w}^\star = \; & \arg \max_{\mathbf{w} \in \mathbb{C}^L} \min_{\substack{i=1,...,m_k \\ k=1,...,G}} \frac{\mathbf{w}^H \mathbf{A}_{k,i} \mathbf{w}}{\mathbf{w}^H \mathbf{C}_{k,i} \mathbf{w} + 1} \\ & \text{subject to} \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P, \end{aligned}$$

where P is the total power limit of the AF signal and

$$\mathbf{A}_{k,i} = P_k (\boldsymbol{f}_k \odot (\boldsymbol{g}_{k,i})^*)(\boldsymbol{f}_k \odot (\boldsymbol{g}_{k,i})^*)^H / \sigma_{k,i}^2, \tag{5}$$

$$\begin{aligned} \mathbf{C}_{k,i} = & \sum_{m \neq k} P_m (\boldsymbol{f}_m \odot (\boldsymbol{g}_{k,i})^*)(\boldsymbol{f}_m \odot (\boldsymbol{g}_{k,i})^*)^H / \sigma_{k,i}^2 \\ & + \mathrm{Diag}(|g_{k,i}^1|^2 \sigma_1^2, |g_{k,i}^2|^2 \sigma_2^2, ..., |g_{k,i}^L|^2 \sigma_L^2) / \sigma_{k,i}^2. \end{aligned} \tag{6}$$

Problem (BF) is a fractional QCQP, which is NP-hard in general [2], [8]–[10]. A popular way to tackle (BF) is to apply the SDR technique [9]. That is, by letting $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and then dropping the non-convex rank constraint, we obtain a fractional SDR form of (BF) as follows

$$\begin{aligned} (\mathrm{SDR}) \quad \mathbf{W}^\star = & \arg \max_{\mathbf{W} \in \mathbb{H}_+^L} \gamma(\mathbf{W}) \\ & \text{subject to} \quad \mathbf{D} \cdot \mathbf{W} \leq P, \end{aligned} \tag{7}$$

where

$$\gamma(\mathbf{W}) = \min_{\substack{k=1,\ldots,G;\\ i=1,\ldots,m_k}} \frac{\mathbf{A}_{k,i}\cdot\mathbf{W}}{\mathbf{C}_{k,i}\cdot\mathbf{W}+1}$$

with $\cdot$ being the matrix inner product operator and $\mathbb{H}_+^L$ being the set of all $L\times L$ positive semidefinite matrices. It is well known that (SDR) can be solved by a bisection method wherein semidefinite programming (SDP) problems are solved in each iteration [8], [10]. In this way, (BF) can be optimally solved when (SDR) has rank-one solutions. If $\mathrm{rank}(\mathbf{W}^\star)>1$, then a Gaussian randomization algorithm [8], [10], [13] is typically used to generate an approximate rank-one solution $\hat{\mathbf{w}}$ from $\mathbf{W}^\star$. From an achievable rate perspective,

$$r_{\mathsf{BF}} = \log(1+\gamma\left(\hat{\mathbf{w}}\hat{\mathbf{w}}^H\right))$$

is the BF achievable rate associated with the SDR approximate solution $\hat{\mathbf{w}}$, and for analysis purpose, we define

$$r_{\mathsf{SDR}} = \log(1+\gamma(\mathbf{W}^\star))$$

as the SDR rate associated with the SDR optimal solution $\mathbf{W}^\star$. It is obvious that we have $r_{\mathsf{SDR}}\geq r_{\mathsf{BF}}$. The other way round relationship between r_{BF} and r_{SDR} is shown in the following proposition.

Proposition 1. *Let M denote the total number of users in the distributed relay network. When $M\leq 3$, we always have $r_{\mathsf{BF}} = r_{\mathsf{SDR}}$. When $M>3$, for non-rank-one cases, let $\hat{\mathbf{w}}$ be the solution returned by the Gaussian randomization Algorithm and N be the number of randomizations. Then, with probability at least $1-(5/6)^N$, we have*

$$r_{\mathsf{SDR}} - r_{\mathsf{BF}} \leq (4.2+\log M)\ \text{nats/s/Hz}. \tag{8}$$

A sketch of the proof is as follows. We first get the SDR approximation bounds in terms of SINRs from the results in [8], [10], and then apply the logarithm relation between SINRs and achievable rates to obtain (8). Proposition 1 implies that the rate gap between the SDR rate and the BF-AF rate will be enlarged as M increases. In other words, the SDR-based BF-AF scheme may not work well when M is large.

Remark 1: In practice, it would be more practical to consider per-relay power constraints, rather than the total relay power constraint, which is adopted in (BF). Due to page limit, we defer this to the full version of this paper. Concisely speaking, with the per-relay power constraints, it can be shown that the rate gap in (8) becomes even worse — increasing at a rate of $\log M + \log\log(L)$, where L is the number of relays.

III. The Stochastic Beamformed AF Schemes in the Distributed One-Way Relay Network

In view of the drawback of the BF-AF scheme, in this section, we develop SBF-AF schemes for improving the achievable rate performance. The essence of the SBF-AF scheme is to adopt time-varying random AF weights, rather than a fixed one in the BF-AF scheme, so that the non-rank-one nature of the SDR optimal solution can be captured by averaging over the random AF weights. To be specific, the source-to-relay link remains the same as the previous BF-AF model, while the relay-to-destination link is modified as

$$\mathbf{x}(t) = \mathrm{Diag}(\boldsymbol{w}(t))\mathbf{r}(t),$$

where $\boldsymbol{w}(t) = [w_1(t),...,w_\ell(t),...w_L(t)]^T$ with $w_\ell(t)$ being the AF weight at relay-ℓ for time t. Note that, unlike the BF-AF scheme, where $\mathbf{w}$ is fixed for a whole data frame transmission, here we let the AF weight $\boldsymbol{w}(t)$ change randomly with time following some prescribed distributions (to be specific shortly). The purpose of adopting the time-varying AF weights is to exploit the temporal degree-of-freedom to mimic a rank-r beamforming. In this way, the receive signal at receiver-(k,i) can be written as

$$y_{k,i}(t) = \sum_{\ell=1}^{L}(g_{k,i}^\ell)^* w_\ell(t) f_k^\ell s_k(t) + \tag{9}$$
$$\sum_{j\neq k}^{G}\sum_{\ell=1}^{L}(g_{k,i}^\ell)^* w_\ell(t) f_j^\ell s_j(t) + \sum_{\ell=1}^{L}(g_{k,i}^\ell)^* w_\ell(t) n^\ell(t) + v_{k,i}(t).$$

It is not difficult to see that we are actually dealing with multi-user interfering fast fading channels where the fading coefficients comes from the SBF fluctuations. A popular processing is to treat interference as noise [14]–[18]. Thus, in (9), we may as such treat all the interference as noise and define the SBF-AF rate for user-(k,i) as

$$r_{\mathsf{SBF}} = \min_{\substack{k=1,\ldots,G;\\ i=1,\ldots,m_k}} \mathbb{E}\left[\log\left(1+\frac{\boldsymbol{w}^H(t)\mathbf{A}_{k,i}\boldsymbol{w}(t)}{\mathbb{E}[\mathrm{I}_{k,i}(t)]+1}\right)\right]. \tag{10}$$

where $\mathrm{I}_{k,i}(t)$ is the interference term given by $\mathrm{I}_{k,i}(t) = \boldsymbol{w}^H(t)\boldsymbol{C}_{k,i}\boldsymbol{w}(t)$. Note that different from the fixed AF weight in BF-AF, $\boldsymbol{w}(t)$ in (10) is i.i.d. in time, and herein we take the expectation of $\mathrm{I}_{k,i}(t)$ as the noise variance. In this way, r_{SBF} could be analyzed and later on we will show by simulations that the proposed SBF-AF schemes can indeed achieve a relatively good rate by applying random channel codes, e.g., turbo code or LDPC code.

Now, the remaining issue is how to determine the distributions for generating $\boldsymbol{w}(t)$. In general, it is hard to find the optimal distribution of the random AF weights for maximizing r_{SBF}. Thus, we consider two heuristic ways to generate the random AF weights, which are easy to implement and allow for tractable analysis. First of all, we introduce the Gaussian SBF-AF scheme by generating the AF weights following a complex Gaussian distribution; i.e.,

$$\boldsymbol{w}(t)\sim\mathcal{CN}(\mathbf{0},\mathbf{W}^\star),$$

where $\mathbf{W}^\star$ is the optimal solution to (SDR). We call this SBF-AF scheme as *the Gaussian SBF-AF*. Since herein we have $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}(t)^H] = \mathbf{W}^\star$, the power constraint is satisfied on average. In Proposition 2, we show a relationship between the Gaussian SBF-AF rate and the SDR rate.

Proposition 2. *For the Gaussian SBF-AF scheme, we have*

$$r_{\mathsf{SDR}} - r_{\mathsf{SBF}} \leq 0.5772.$$

We provide the proof in the Appendix. Proposition 2 quantifies an upper bound for the rate gap between the SDR rate and the Gaussian SBF rate. It says that, the Gaussian SBF-AF scheme is at most 0.83 bits/s/Hz ($0.5772 \text{ nats}/\log 2 = 0.8317 \text{ bits}$) worse than the SDR rate associated with the SDR optimal solution, irrespective of any other factors, such as the number of users and the transmit power. Thus, in a sharp contrast to Proposition 1, the SBF-AF rate is insensitive to M and exhibits the same scaling as the SDR rate.

We see that the Gaussian SBF-AF rate is within less than 1 bit/s/Hz of the SDR rate. However, from a practical viewpoint, it has a high peak-to-average power ratio (PAPR) since the Gaussian distribution usually has a large spread. Therefore, we are motivated to develop better SBF-AF schemes with smaller power spread. Towards this end, we introduce the elliptic SBF-AF scheme. For the elliptic SBF-AF scheme, we generate the AF weights by

$$\boldsymbol{w}(t) = \frac{\mathbf{L}^H \boldsymbol{\alpha}(t)}{\|\boldsymbol{\alpha}(t)\|/\sqrt{r}}, \quad \boldsymbol{\alpha}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_r), \qquad (11)$$

where $r = \text{rank}(\mathbf{W}^\star)$ and $\mathbf{L} \in \mathbb{C}^{r \times L}$ is a square root decomposition of $\mathbf{W}^\star$, i.e., $\mathbf{L}^H \mathbf{L} = \mathbf{W}^\star$. According to [19], such AF weights are limited by the instantaneous power and follow a complex elliptic distribution with covariance matrix $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}(t)^H] = \mathbf{W}^\star$. It is easy to check that the power constraint is also satisfied. Moreover, in Proposition 3, we prove that the rate gap between the elliptic SBF-AF rate and the SDR rate is always bounded by a constant.

Proposition 3. *For the elliptic SBF-AF scheme, we have*

$$r_{\mathsf{SDR}} - r_{\mathsf{SBF}} \le \sum_{k=1}^{r-1} \frac{1}{k} - \log(r) \le 0.5772,$$

where $r = \text{rank}(\mathbf{W}^\star)$.

We skip the proof of Proposition 3 due to page limit. This proposition implies that the elliptic SBF-AF rate is insensitive to the number of users and thus it can work well for large-scale systems. Moreover, an important corollary of Proposition 3 is that the worst-case rate gap of the elliptic SBF-AF scheme is no worse than that of the Gaussian SBF-AF scheme. A similar result has been proven in [11] for single-group multicasting. Now we show that it is also true for distributed relay networks.

Remark 2: To implement the SBF-AF schemes, the decision center (which could be the transmitters or PUs pool in the C-RN) sends a random seed and the covariance $\mathbf{W}^\star$ to all the relays and receivers. Therefore, relays can generate the same AF weight by the guide of $\mathbf{W}^\star$ and pick its own coefficient to AF the received signals. At the receivers' side, by knowing the random seed and $\mathbf{W}^\star$, each receiver can perform coherent detection. The proposed SBF-AF schemes are just as efficient as the BF-AF scheme in terms of implementation complexity.

Remark 3: It is worth mentioning that for both BF-AF and SBF-AF, channel coding is needed to resist the Gaussian noise and SBF fluctuations. Theoretically, we need to apply a random channel code with a relatively long codelength for approaching the achievable rates. Our numerical experience is that, an LDPC code or turbo code with a moderate codelength should be efficient enough, and SBF-AF always outperforms BF-AF. Such numerical results will be shown in the next section.

IV. Simulation Results and Conclusions

In this section, we provide numerical results to demonstrate the effectiveness of our proposed schemes. To set up the simulations, we assume that each group has an equal number of users, i.e., $m_j = \frac{M}{G}$; the channels are independently generated by $\boldsymbol{f}_j, \boldsymbol{g}_{k,i} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$; the noise power at relays and at users are both set to be 0.25; and the signal power at each transmitter is 0dB. We averaged 300 channel realizations to get the plots for each AF scheme. The number of randomizations for generating BF-AF weights is $1,000$.

Figure 3 shows the worst user's BF-AF rate and SBF-AF rates scaling w.r.t. the total number of users served in the distributed relay network when $P = 6$dB. Herein the number of relays is $L = 8$, and the number of group is $G = 2$. From the figure, we see that the rates of BF-AF and SBF-AF are upper bounded by the SDR rate. Moreover, the BF-AF rate diverges from the SDR rate as the number of users M increases. It shows that the Gaussian SBF-AF scheme outperforms the BF-AF scheme when $M > 10$, while the elliptic SBF-AF scheme outperforms the BF-AF scheme for all values of M. The SBF-AF schemes exhibit the same scaling as the SDR rate, which is consistent with Propositions 2 and 3. In Figure 4, we compare the coded bit error rate (BER) performances for the case of $M = 12$ to further demonstrate the efficacy of the SBF-AF schemes. Herein, the SDR bound is obtained by assuming that there exists an SISO channel where the SINR is equal to the optimal SDR objective. For all the other AF schemes, we simulate the actual AF process. That is, for each time slot, we generate $s_j(t), n^\ell(t)$ according to the SISO model (3), then detect and decode $s_j(t)$ at each receiver. In our simulations, to fully demonstrate the effectiveness of SBF-AF, we adopt a gray-coded QPSK modulation scheme and a rate-$1/3$ turbo code in [20] with two different codelengths 576 and $2,880$. For each channel realization, we simulate 100 code blocks and thus the BER reliability level is $10\text{e}{-4}$. From the plot, we see that the actual BER performances of the SBF-AF schemes outperform the SDR-based BF-AF scheme almost in all power region, and the elliptic scheme attains the best BER performance, which is consistent with the results in Figure 3. This verifies that the SBF-AF schemes can indeed achieve a relatively good rate.

To conclude, in this paper we have considered relay AF schemes in a distributed one-way relay network, where the received signals cannot be shared among relays. We proposed two SBF-AF schemes which adopt time-varying AF weights to explore the temporal degree of freedom to perform a rank-r beamforming. The proposed SBF-AF schemes outperform the traditional rank-one BF-AF scheme, especially when the number of users served in the system is large. We prove that, in the worst case, the proposed SBF-AF schemes are only 0.83 bits/s/Hz worse than the SDR rate. The actual BER performance comparisons further validate the superiority of the proposed SBF-AF schemes.

V. Appendix

For Gaussian SBF-AF, we have $\mathbb{E}[\mathrm{I}_{k,i}(t)] = \mathbf{C}_{k,i} \cdot \mathbf{W}^{\star}$. Then, letting $\rho_{\min} = \min_{k,i} \frac{\mathbf{A}_{k,i} \cdot \mathbf{W}^{\star}}{\mathbf{C}_{k,i} \cdot \mathbf{W}^{\star}+1}$, we have

$$
\begin{aligned}
r_{\mathsf{SDR}} - r_{\mathsf{SBF}} &= \log(1 + \min_{k,i} \frac{\mathbf{A}_{k,i} \cdot \mathbf{W}^{\star}}{\mathbf{C}_{k,i} \cdot \mathbf{W}^{\star} + 1}) \\
&- \mathbb{E}_{\boldsymbol{w}(t)}[\log(1 + \min_{k,i} \frac{\boldsymbol{w}^H(t)\mathbf{A}_{k,i}\boldsymbol{w}(t)}{\mathbf{C}_{k,i} \cdot \mathbf{W}^{\star} + 1})] \\
&= \log(1 + \rho_{\min}) - \mathbb{E}_{\xi}[\log(1 + \rho_{\min}\xi)],
\end{aligned}
$$

where ξ follows an exponential distribution with unit mean. Let $g(\rho_{\min}) = r_{\mathsf{SDR}} - r_{\mathsf{SBF}}$, which is a function of $\rho_{\min}$. We have

$$
g'(\rho_{\min}) \geq \left(\frac{1}{1+\rho_{\min}} - \frac{\mathbb{E}_{\xi}[\xi]}{1+\rho_{\min}\mathbb{E}_{\xi}[\xi]} \right) \rho_{\min} = 0,
$$

which means that $g(\rho_{\min})$ is non-decreasing w.r.t. $\rho_{\min}$. Observe that $0 \leq \rho_{\min} < +\infty$. Therefore, we must have

$$
r_{\mathsf{SDR}} - r_{\mathsf{SBF}} \leq g(+\infty) = 0.5772,
$$

where the equality comes from the derivation of Theorem 1 in [11], which completes the whole proof.

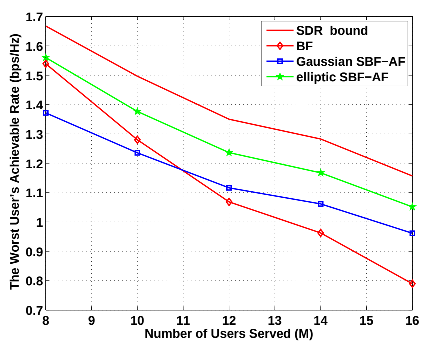

Fig. 3. The worst user's BF-AF rate and SBF-AF rates versus the number of users served in the relay system.

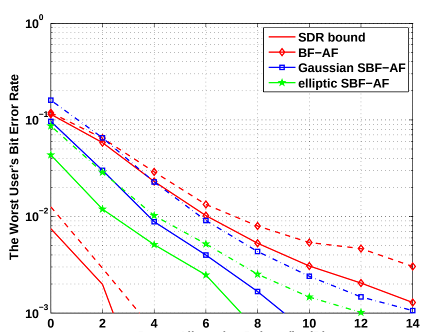

Fig. 4. The worst user's BER curves for different AF schemes. The solid and dashed curves are for the codelength 2880 and 576, respectively.

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