PROBABILISTIC SINR CONSTRAINED ROBUST TRANSMIT BEAMFORMING: A BERNSTEIN-TYPE INEQUALITY BASED CONSERVATIVE APPROACH

Kun-Yu Wang[⋆], *Tsung-Hui Chang*[⋆], *Wing-Kin Ma*[†], *Anthony Man-Cho So*[‡], *and Chong-Yung Chi*[⋆]

[⋆] Institute of Commun. Eng.
National Tsing Hua University,
Hsinchu, Taiwan 30013
E-mail: {kunyuwang7,tsunghui.chang}@gmail.com,
cychi@ee.nthu.edu.tw

[†] Dept. of Electronic Eng.
Chinese Univ. of Hong Kong,
Shatin, Hong Kong
E-mail: wkma@ieee.org

[‡] Dept. of Sys. Eng. & Eng. Mgmt.
Chinese Univ. of Hong Kong,
Shatin, Hong Kong
E-mail: manchoso@se.cuhk.edu.hk

ABSTRACT

Recently, robust transmit beamforming has drawn considerable attention because it can provide guaranteed receiver performance in the presence of channel state information (CSI) errors. Assuming complex Gaussian distributed CSI errors, this paper investigates the robust beamforming design problem that minimizes the transmission power subject to probabilistic signal-to-interference-plus-noise ratio (SINR) constraints. The probabilistic SINR constraints in general have no closed-form expression and are difficult to handle. Based on a Bernstein-type inequality for quadratic forms of complex Gaussian random variables, we propose a conservative formulation to the robust single-cell beamforming design problem. The semidefinite relaxation technique can be applied to efficiently handle the proposed conservative formulation. Simulation results show that, in comparison with existing methods, the proposed method is more power efficient and is able to support higher target SINR values for receivers.

Index Terms— Robust transmit beamforming, semidefinite relaxation, convex optimization.

1. INTRODUCTION

Linear transmit beamforming has been recognized as a powerful technique since it can achieve a large fraction of capacity with low implementation complexity. Conventionally, it is assumed that the transmitter has perfect channel state information (CSI) of the receivers, and the beamforming vectors are optimized such that the signal-to-interference-plus-noise ratio (SINR) requirements of the receivers can be satisfied. In practical situations, however, the CSI at the transmitter is inevitably subject to errors due to finite-energy training and limited feedback. The presence of CSI errors will result in receivers' performance outage. Therefore, robust transmit beamforming designs that take the CSI errors into consideration [1, 2, 3, 4] are of great importance.

In this paper, we assume that the CSI errors are complex Gaussian distributed and study the stochastic robust beamforming design problem under a single-cell system with multiple single-antenna receivers. Specifically, we study the robust design formulation that minimizes the transmission power subject to probabilistic SINR constraints on the receivers [2]. The probabilistic SINR constraints guarantee the receivers' SINR requirements to be satisfied with a predetermined, high probability (i.e., a low outage probability). The robust design problem is difficult to solve because the probabilistic SINR constraints in general do not have closed-form expression and are not convex. To efficiently handle this problem, so-called *conservative* approaches that aim to obtain efficient approximate solutions guaranteeing the probabilistic SINR requirements are proposed [2, 4]. In particular, the authors of [2] used the results in [5] to show that an approximate solution of the robust design problem can be efficiently obtained by solving a semidefinite program (SDP). In [4], a more efficient conservative formulation is presented which also involves solving an SDP.

In this paper, we present a new conservative approach to the probabilistic SINR constrained robust beamforming problem. The idea of the proposed method is based on the observation that each of the probabilistic SINR constraints involves a quadratic form of complex Gaussian random variables, which allows us to use a Bernstein-type inequality[1] [6] to bound the outage probability. By recognizing that the Bernstein-type inequality can be reformulated as a system of conic inequalities, we obtain a new conservative formulation of the robust beamforming problem. Although the proposed formulation is still a nonconvex optimization problem, it can be handled very effectively by a convex relaxation method, called semidefinite relaxation (SDR). The presented simulation results show that the proposed method is more power efficient and less conservative compared to existing methods.

2. SIGNAL MODEL AND PROBLEM STATEMENT

We consider a multiuser wireless system consisting of a multiple-antenna transmitter and K single-antenna receivers. The transmitter, equipped with N_t antennas, wants to communicate with the receivers using transmit beamforming. Let $s_i(t)$ denote the information signal for receiver i. The transmit signal is given by $\sum_{i=1}^{K} \boldsymbol{w}_i s_i(t)$ where $\boldsymbol{w}_i \in \mathbb{C}^{N_t}$ is the beamforming vector for $s_i(t)$. The received signal at receiver i is given by

$$y_i(t) = \mathbf{h}_i^H \left(\sum_{k=1}^{K} \boldsymbol{w}_k s_k(t) \right) + n_i(t), \qquad (1)$$

where $\mathbf{h}_i \in \mathbb{C}^{N_t}$ denotes the channel vector of receiver i, and $n_i(t)$ is the additive noise at receiver i. It is assumed that $n_i(t)$ has zero mean and variance equal to $\sigma_i^2 > 0$ for all $i = 1, \ldots, K$. Assume

This work is supported in part by National Science Council, R.O.C., under Grants NSC 98-2219-E-007-005 and NSC 99-2221-E-007-052-MY3, and in part by a General Research Fund of Hong Kong Research Grant Council (CUHK 415908) and by the Grant Project #MMT-p2-09 of the Shun Hing Institute of Advanced Engineering, The Chinese University of Hong Kong.

[1] Roughly speaking, a Bernstein-type inequality is one which bounds the probability that a sum of random variables deviates from its mean.

that $s_i(t)$ has zero mean and $\mathbb{E}\{|s_i(t)|^2\} = 1$ for all i. The SINR of receiver i can be obtained from (1) as

$$\mathrm{SINR}_i = \frac{|\mathbf{h}_i^H \boldsymbol{w}_i|^2}{\sum_{k\neq i}^K |\mathbf{h}_i^H \boldsymbol{w}_k|^2 + \sigma_i^2}. \tag{2}$$

The goal of transmit beamforming is to design the beamforming vectors $\{\boldsymbol{w}_i\}_{i=1}^K$ such that each of the receivers can achieve a desired SINR performance. To this end, the following design formulation has been frequently employed:

$$\min_{\substack{\boldsymbol{w}_i\in\mathbb{C}^{N_t},\\ i=1,\ldots,K}} \sum_{i=1}^K \|\boldsymbol{w}_i\|^2 \tag{3a}$$

$$\text{s.t.} \ \frac{|\mathbf{h}_i^H \boldsymbol{w}_i|^2}{\sum_{k\neq i}^K |\mathbf{h}_i^H \boldsymbol{w}_k|^2 + \sigma_i^2} \geq \gamma_i, \ \ i = 1,\ldots,K, \tag{3b}$$

where $\|\cdot\|$ denotes the vector Euclidean norm, and $\gamma_i > 0$ stands for the preset target SINR value of receiver i. The design problem (3) aims to find a most power efficient beamforming solution such that the target SINR requirements γ_i are satisfied. There is more than one way to solve problem (3); e.g., by using uplink-downlink duality, by using SDR, or by using a second-order cone program (SOCP) reformulation [7, 8]. Readers are referred to the literature, such as [9], for coverage of this aspect.

2.1. Probabilistic SINR Constrained Robust Beamforming

The conventional design formulation in (3) assumes that the transmitter has perfect knowledge of the channels $\{\mathbf{h}_i\}_{i=1}^K$. In practical wireless environments, however, the transmitter may only have inaccurate CSI due to imperfect channel estimation and limited feedback. Let $\bar{\mathbf{h}}_1,\ldots,\bar{\mathbf{h}}_K \in \mathbb{C}^{N_t}$ denote the channel estimates at the transmitter. The true channels can be expressed as

$$\mathbf{h}_i = \bar{\mathbf{h}}_i + \mathbf{e}_i, \ i = 1,\ldots,K, \tag{4}$$

where $\mathbf{e}_i \in \mathbb{C}^{N_t}$ represents the CSI error vector. In the presence of CSI errors, the beamforming solution of problem (3), denoted by $\{\boldsymbol{w}_i^\star\}_{i=1}^K$, may no longer guarantee the SINR requirements in (3b), that is, for some $\mathbf{e}_i$, it is possible to have

$$\frac{|(\bar{\mathbf{h}}_i + \mathbf{e}_i)^H \boldsymbol{w}_i^\star|^2}{\sum_{k\neq i}^K |(\bar{\mathbf{h}}_i + \mathbf{e}_i)^H \boldsymbol{w}_k^\star|^2 + \sigma_i^2} < \gamma_i. \tag{5}$$

It is desirable to design the beamforming vectors $\{\boldsymbol{w}_i\}_{i=1}^K$ such that (5) occurs only with a small probability.

To this end, we assume that the CSI errors are complex Gaussian random vectors with zero mean and covariance matrix $\mathbf{C}_i \succeq \mathbf{0}$ (positive semidefinite, PSD), i.e.,

$$\mathbf{e}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_i), \ i = 1,\ldots,K. \tag{6}$$

This model is particularly suitable for CSI errors caused by imperfect channel estimation. Let $\rho_i \in (0, 1]$ denotes the maximum tolerable SINR outage probability of receiver i. We consider the following robust beamforming design formulation [2]:

$$\min_{\substack{\boldsymbol{w}_i\in\mathbb{C}^{N_t},\\ i=1,\ldots,K}} \sum_{i=1}^K \|\boldsymbol{w}_i\|^2 \tag{7a}$$

$$\text{s.t.} \ \Pr\left\{\frac{|(\bar{\mathbf{h}}_i + \mathbf{e}_i)^H \boldsymbol{w}_i|^2}{\sum_{k\neq i}^K |(\bar{\mathbf{h}}_i + \mathbf{e}_i)^H \boldsymbol{w}_k|^2 + \sigma_i^2} \geq \gamma_i\right\} \geq 1 - \rho_i, \quad i = 1,\ldots,K. \tag{7b}$$

It can be seen that problem (7) finds a most power efficient beamforming solution such that the $1 - \rho_i$ SINR satisfaction probability is achieved. Solving problem (7) is a challenging task because the probabilistic SINR constraints in (7b) have no closed-form expression and are not convex in general. To obtain approximate solutions satisfying (7b), efficient *convex conservative* formulations have been proposed; see [2, 4]. In the next section, we present a new conservative formulation for problem (7) that will be shown to outperform the existing methods.

3. PROPOSED CONSERVATIVE FORMULATION

3.1. Bernstein-type Inequality Based Conservative Approach

To present the proposed method, let us express the CSI errors as

$$\mathbf{e}_i = \mathbf{C}_i^{1/2}\mathbf{v}_i, \ i = 1,\ldots,K, \tag{8}$$

where $\mathbf{C}_i^{1/2} \succeq \mathbf{0}$ is the PSD square root of $\mathbf{C}_i$, and $\mathbf{v}_i \in \mathbb{C}^{N_t}$ is a normalized complex Gaussian random vector with zero mean and covariance matrix $\mathbf{I}$ (the N_t by N_t identity matrix), i.e., $\mathbf{v}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. With this expression, the probabilistic constraints in (7b) can be expressed as

$$\Pr\Big\{\mathbf{v}_i^H \mathbf{Q}_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K)\mathbf{v}_i + 2\mathrm{Re}\{\mathbf{v}_i^H \mathbf{u}_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K)\} \geq c_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K)\Big\} \geq 1 - \rho_i, \ i = 1,\ldots,K, \tag{9}$$

where $\mathrm{Re}\{\cdot\}$ represents the real part of the associated argument, and

$$\mathbf{Q}_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K) \triangleq \mathbf{C}_i^{1/2}\Big(\frac{1}{\gamma_i}\boldsymbol{w}_i\boldsymbol{w}_i^H - \sum_{k\neq i}\boldsymbol{w}_k\boldsymbol{w}_k^H\Big)\mathbf{C}_i^{1/2}, \tag{10a}$$

$$\mathbf{u}_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K) \triangleq \mathbf{C}_i^{1/2}\Big(\frac{1}{\gamma_i}\boldsymbol{w}_i\boldsymbol{w}_i^H - \sum_{k\neq i}\boldsymbol{w}_k\boldsymbol{w}_k^H\Big)\bar{\mathbf{h}}_i, \tag{10b}$$

$$c_i(\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K) \triangleq \sigma_i^2 - \bar{\mathbf{h}}_i^H\Big(\frac{1}{\gamma_i}\boldsymbol{w}_i\boldsymbol{w}_i^H - \sum_{k\neq i}\boldsymbol{w}_k\boldsymbol{w}_k^H\Big)\bar{\mathbf{h}}_i. \tag{10c}$$

Note that (9) is a probability inequality involving a quadratic form of complex Gaussian random variables. The idea of conservative approaches is to find computationally tractable forms that are sufficient to achieve (9). To implement this idea, we use the following lemma:

Lemma 1 [6] *Let $G = \mathbf{v}^H\mathbf{Q}\mathbf{v} + 2\mathrm{Re}\{\mathbf{v}^H\mathbf{u}\}$ where $\mathbf{Q} \in \mathbb{H}^{N_t}$ is a complex Hermitian matrix*[2], *$\mathbf{u} \in \mathbb{C}^{N_t}$, and $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Then for any $\delta > 0$, we have*

$$\Pr\Big\{G \geq \mathrm{Tr}(\mathbf{Q}) - \sqrt{2\delta}\sqrt{\|\mathbf{Q}\|_F^2 + 2\|\mathbf{u}\|^2} - \delta s^+(\mathbf{Q})\Big\} \geq 1 - e^{-\delta}, \tag{11}$$

where $s^+(\mathbf{Q}) = \max\{\lambda_{\max}(-\mathbf{Q}), 0\}$ in which $\lambda_{\max}(-\mathbf{Q})$ denotes the maximum eigenvalue of matrix $-\mathbf{Q}$, and $\|\cdot\|_F$ denotes the matrix Frobenius norm.

The inequality in (11) is a Bernstein-type inequality, which bounds the probability that the quadratic form G of complex Gaussian random variables deviates from its mean $\mathrm{Tr}(\mathbf{Q})$. Let $\delta \triangleq -\ln(\rho)$ where $\rho \in (0, 1]$. Lemma 1 implies that the inequality

$$\Pr\Big\{\mathbf{v}^H\mathbf{Q}\mathbf{v} + 2\mathrm{Re}\{\mathbf{v}^H\mathbf{u}\} \geq c\Big\} \geq 1 - \rho \tag{12}$$

[2]Here, $\mathbb{H}^N$ is the set of all N-by-N complex Hermitian matrices.

holds true if the following inequality is satisfied

$$\mathrm{Tr}(\mathbf{Q}) - \sqrt{2\delta}\sqrt{\|\mathbf{Q}\|_F^2 + 2\|\mathbf{u}\|^2} - \delta s^+(\mathbf{Q}) \geq c. \tag{13}$$

Equation (13) thus serves as a conservative formulation for (12). Now, a crucial observation is that (13) can be represented by

$$\mathrm{Tr}\,(\mathbf{Q}) - \sqrt{2\delta}x - \delta y \geq c, \tag{14a}$$

$$\sqrt{\|\mathbf{Q}\|_F^2 + 2\|\mathbf{u}\|^2} \leq x, \tag{14b}$$

$$y\mathbf{I} + \mathbf{Q} \succeq \mathbf{0}, \tag{14c}$$

$$y \geq 0, \tag{14d}$$

where $x, y \in \mathbb{R}$ are slack variables. By defining

$$\delta_i \triangleq -\ln(\rho_i),\ i = 1, \ldots, K, \tag{15}$$

and applying (14) to (9), we obtain the following problem

$$\min_{\substack{\boldsymbol{w}_i \in \mathbb{C}^{N_t}, x_i, y_i \in \mathbb{R},\\ i=1,\ldots,K}} \sum_{i=1}^{K} \|\boldsymbol{w}_i\|^2 \tag{16}$$

$$\begin{aligned} \text{s.t. } & \mathrm{Tr}\,(\mathbf{Q}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)) - \sqrt{2\delta_i}x_i - \delta_i y_i \\ & \qquad \geq c_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K),\ i = 1, \ldots, K, \\ & \left\| \begin{bmatrix} \mathrm{vec}\,(\mathbf{Q}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)) \\ \sqrt{2}\mathbf{u}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K) \end{bmatrix} \right\| \leq x_i,\ i = 1, \ldots, K, \\ & y_i\mathbf{I} + \mathbf{Q}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K) \succeq \mathbf{0},\ i = 1, \ldots, K, \\ & y_i \geq 0,\ i = 1, \ldots, K \end{aligned}$$

as a conservative formulation for problem (7), where $\mathrm{vec}(\cdot)$ denotes the column-by-column matrix vectorization.

Our derived conservative formulation (16) is desirable since it has closed-form expressions with the constraints. However, problem (16) is nonconvex, owing to the fact that $\mathbf{Q}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)$, $\mathbf{u}_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)$ and $c_i(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)$ are indefinite quadratic in $\{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K\}$ [see (10)]. Next, we will handle this problem by using the SDR technique [10].

3.2. Semidefinite Relaxation

To apply SDR to the conservative formulation (16), we let $\mathbf{W}_i = \boldsymbol{w}_i\boldsymbol{w}_i^H$, for $i = 1, \ldots, K$. By replacing $\boldsymbol{w}_i\boldsymbol{w}_i^H$ with $\mathbf{W}_i$ in (10) and then constraining $\mathbf{W}_i$ to be positive semidefinite only, we obtain the following problem

$$\min_{\substack{\mathbf{W}_i \in \mathbb{H}^{N_t}, x_i, y_i \in \mathbb{R},\\ i=1,\ldots,K}} \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{W}_i) \tag{17a}$$

$$\begin{aligned} \text{s.t. } & \mathrm{Tr}\,(\mathbf{Q}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K)) - \sqrt{2\delta_i}x_i - \delta_i y_i \\ & \qquad \geq c_i(\mathbf{W}_1, \ldots, \mathbf{W}_K), && \text{(17b)} \\ & \left\| \begin{bmatrix} \mathrm{vec}\,(\mathbf{Q}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K)) \\ \sqrt{2}\mathbf{u}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K) \end{bmatrix} \right\| \leq x_i, && \text{(17c)} \\ & y_i\mathbf{I} + \mathbf{Q}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K) \succeq \mathbf{0}, && \text{(17d)} \\ & y_i \geq 0,\ \mathbf{W}_i \succeq \mathbf{0},\ i = 1, \ldots, K, && \text{(17e)} \end{aligned}$$

where, with a slight abuse of notations, we define

$$\mathbf{Q}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K) \triangleq \mathbf{C}_i^{1/2}\Big(\frac{1}{\gamma_i}\mathbf{W}_i - \sum_{k\neq i}\mathbf{W}_k\Big)\mathbf{C}_i^{1/2},$$

$$\mathbf{u}_i(\mathbf{W}_1, \ldots, \mathbf{W}_K) \triangleq \mathbf{C}_i^{1/2}\Big(\frac{1}{\gamma_i}\mathbf{W}_i - \sum_{k\neq i}\mathbf{W}_k\Big)\bar{\mathbf{h}}_i,$$

$$c_i(\mathbf{W}_1, \ldots, \mathbf{W}_K) \triangleq \sigma_i^2 - \bar{\mathbf{h}}_i^H\Big(\frac{1}{\gamma_i}\mathbf{W}_i - \sum_{k\neq i}\mathbf{W}_k\Big)\bar{\mathbf{h}}_i,$$

for $i = 1, \ldots, K$. Note that the constraints in (17b), (17c) and (17d) are respectively a linear constraint, a convex second-order cone (SOC) constraint and a convex PSD constraint. Hence problem (17) is a convex conic problem and can be efficiently solved by standard solvers such as CVX [11].

The SDR problem (17) is in general a relaxation of problem (16) because the associated optimal $\{\mathbf{W}_i\}_{i=1}^K$ may not be of rank one. If the optimal $\{\mathbf{W}_i\}_{i=1}^K$ of (17) is of rank one, i.e., $\mathbf{W}_i = \boldsymbol{w}_i\boldsymbol{w}_i^H$ for all i, then $\{\boldsymbol{w}_i\}_{i=1}^K$ is an optimal solution to problem (16); otherwise additional solution approximation procedures to turn the optimum $\{\mathbf{W}_i\}_{i=1}^K$ into a rank-one approximate solution of problem (16) is needed [10]. Fortunately and rather surprisingly, it is found by simulations that problem (17) always yields rank-one optimal $\{\mathbf{W}_i\}_{i=1}^K$. This implies that the globally optimal solution of the conservation formulation (16) may be attained by SDR, at least for all the problem instances we tested in simulations. Also, under the same argument, the feasibility of (16) may be equivalent to that of (17).

3.3. Reducing the Level of Conservatism by Bisection

Analogous to the methods presented in [2, 4], it is found that the proposed conservative formulation (16) with δ_i set as in (15) may yield beamforming solutions that correspond to an SINR satisfaction probability [in (7b)] much higher than $1 - \rho_i$. According to (11), the SINR satisfaction probability achieved by formulation (16) can be reduced by decreasing the parameter δ_i. Hence the bisection method presented in [2, 4] can be used to reduce the level of conservatism of problem (16).

To illustrate how this method works, let us assume that all the receivers have the same SINR outage probability, i.e., $\rho \triangleq \rho_1 = \cdots = \rho_K$. Thus we can let $\delta \triangleq \delta_1 = \cdots = \delta_K$ [see (15)]. For a given δ, one can obtain a beamforming solution by solving (16) and use the validation procedure in [5] to test if the associated probabilistic SINR constraints in (7b) are empirically satisfied or not. If yes, the parameter δ can be reduced; otherwise it should be increased. The procedure is repeated until a predefined stopping criterion is met. Readers are referred to [2, 4] for further details.

4. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present some simulation results to demonstrate the performance of the proposed method. We consider the wireless system as described in Section 2 with three antennas at the transmitter and with three receivers ($N_t = K = 3$). For simplicity, we consider independent and identically distributed (i.i.d.) complex Gaussian CSI errors, i.e., $\mathbf{C}_i = \epsilon\mathbf{I}$ for all i, and set $\epsilon = 0.002$. The noise variances of all receivers are set to 0.01 ($\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0.01$), and the outage probabilities are set to 0.1 ($\rho_1 = \rho_2 = \rho_3 = 0.1$), i.e., 90% satisfaction probability. The target SINR values of all receivers are also set to be the same, i.e., $\gamma \triangleq \gamma_1 = \gamma_2 = \gamma_3$. We compare the proposed conservative formulation in (16) with the method presented in [4] and the Formulation I in [2]. The bisection technique mentioned in Section 3.3 was also implemented for the three methods under test. The parameter setting of this technique follows that in [4]. All three methods were implemented using CVX [11].

Since a less conservative method is more likely to be feasible, in the first example, we examine the feasibility rates of the three methods under test and their bisection counterparts. To this end, 500 sets of channel estimates $\{\bar{\mathbf{h}}_i\}_{i=1}^3$ were generated according to complex Gaussian with zero mean and covariance matrix $\mathbf{I}$. Figure 1 shows the simulation results of feasibility rate ($\%$) versus target SINR γ.

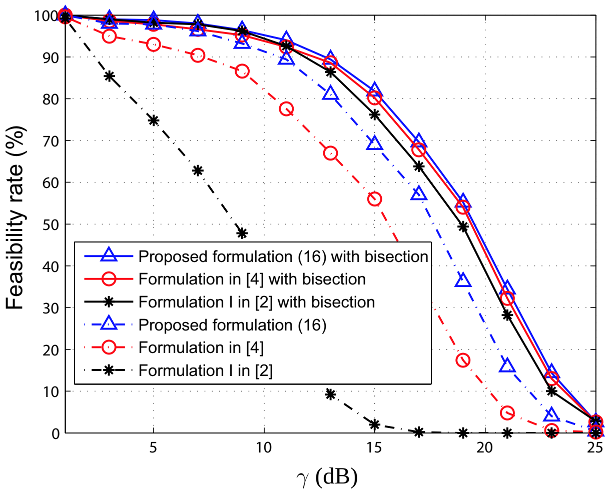

Fig. 1. Feasibility rate (%) versus target SINR γ.

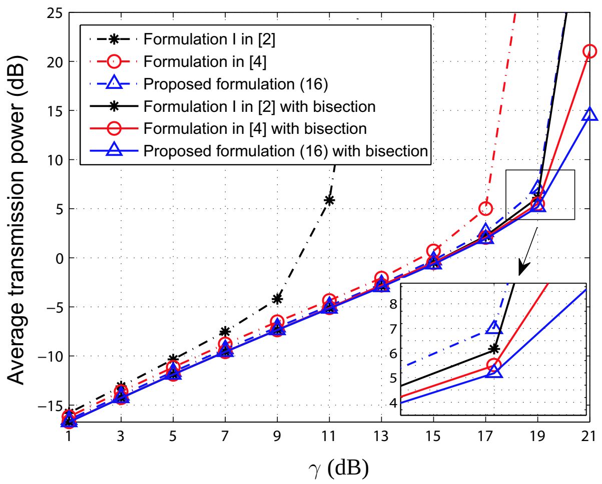

Fig. 2. Average transmission power (dB) versus target SINR γ.

As seen, the proposed formulation in (16) exhibits a higher feasibility rate than the other two methods. When the bisection technique is employed, we can see that all methods have improved performance; whereas the proposed method still has the highest feasibility rate.

In the second example, we compare the average transmission powers of the three formulations. Five hundred sets of $\{\bar{\mathbf{h}}_i\}_{i=1}^3$ were randomly generated, and 135 sets of them were selected for which all the methods under test are feasible for $\gamma = 11$ dB. The 135 sets of channel estimates were used to test the six methods for various values of γ, and the average transmission power of each method was obtained by averaging over these channel realizations. Note that the average power would be set to infinity if there exist at least one channel realizations such that the method under test is infeasible. Figure 2 shows the simulation results. One can observe from this figure that the proposed method yields less average transmission powers than the other two methods, regardless of whether bisection is used or not. In fact, it is observed in simulations that the proposed method always consumes the least power for every feasible channel realization. The results in Fig. 1 and Fig. 2 imply that the proposed method is amenable to support a wider range of target SINRs.

In the last example, we examine the average computation time (in seconds) of CVX for solving the proposed method and the methods in [2] and [4]. The simulation results for $\gamma = 7$ dB over 50 feasible channel realizations are shown in Fig. 3. We can see that the proposed method has a larger computation time than the method in [4]; this shows that there is a tradeoff between complexity and performance for these two methods. On the other hand, we can observe from this figure that both the proposed method and that in [4] are computationally cheaper than the method in [2], especially when $N_t = K \geq 3$.

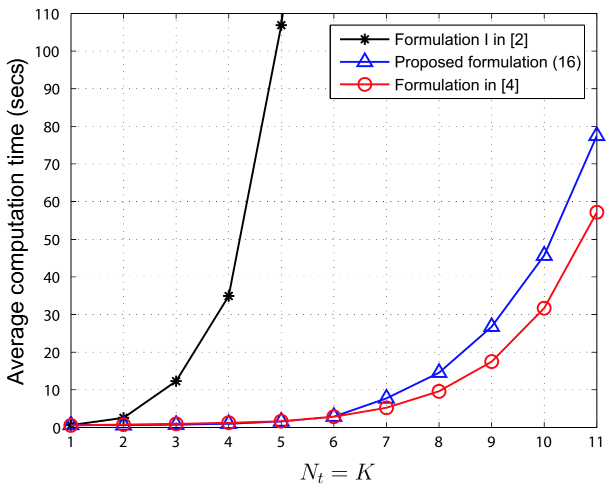

Fig. 3. Average computation time (secs) versus number of transmit antennas N_t ($N_t = K$).

5. REFERENCES

[1] B. K. Chalise, S. Shahbazpanahi, A. Czylwik, and A. B. Gershman, "Robust downlink beamforming based on outage probability specifications," *IEEE Trans. Wireless Commun.*, vol. 6, no. 10, pp. 3498–3503, Oct. 2007.

[2] M. B. Shenouda and T. N. Davidson, "Probabilistically-constrained approaches to the design of the multiple antenna downlink," in *Proc. 42nd Asilomar Conference*, Pacific Grove, Oct. 26-29, 2008, pp. 1120–1124.

[3] ——, "Outage-based designs for multi-user transceivers," in *Proc. IEEE ICASSP*, Taipei, Taiwan, Apr. 19-24, 2009, pp. 2389–2392.

[4] K.-Y. Wang, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "A semidefinite relaxation based conservative approach to robust transmit beamforming with probabilistic SINR constraints," in *Proc. EUSIPCO*, Aalborg, Denmark, Aug. 23-27, 2010, pp. 407–411.

[5] A. Ben-Tal and A. Nemirovski, "On safe tractable approximations of chance-constrained linear matrix inequalities," *Math Oper. Res.*, vol. 34, no. 1, pp. 1–25, Feb. 2009.

[6] I. Bechar, "A Bernstein-type inequality for stochastic processes of quadratic forms of Gaussian variables," avaliable online: http://arxiv.org/abs/0909.3595.

[7] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1999.

[8] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," Chapter 18 in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed., CRC Press, Aug. 2001.

[9] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Process. Mag.*, pp. 62–75, May 2010.

[10] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, pp. 20–34, May 2010.

[11] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," http://stanford.edu/∼boyd/cvx, Jun. 2009.