MULTI-GROUP MULTICAST BEAMFORMING IN COGNITIVE RADIO NETWORKS VIA RANK-TWO TRANSMIT BEAMFORMED ALAMOUTI SPACE-TIME CODING

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ABSTRACT

In this paper, we consider transmit design in multiple-input single-output (MISO) multi-group multicast (MM) cognitive radio (CR) systems. Previously, semidefinite relaxation (SDR)-based transmit beamforming has been very successful in transmit design. However, recent research shows that further performance gain is possible by suitably modifying the transmit structure. Here, we propose a transmit beamformed Alamouti space-time code scheme for MM-CR systems, whose corresponding transmit design problem can be reformulated as a rank-2 constrained fractional semidefinite program. We then develop an SDR framework for this scheme and study its signal-to-interference-and-noise ratio (SINR) performance via both theoretical analysis and simulations. Specifically, we show that the worst-case approximation accuracy of the proposed scheme scales on the order of $\sqrt{M_S}\log M_P$, where M_P (resp. M_S) is the number of primary (resp. secondary) users in the CR network. This unifies and generalizes a number of results in the literature and is, to the best of our knowledge, the first provable bound on the performance of a beamforming scheme in a general MM-CR system. Finally, simulation results show that our proposed scheme indeed has a better performance in both MM and MM-CR scenarios than the traditional beamforming scheme.

***Index Terms*—** multi-group multicast, cognitive radio, transmit beamforming, Alamouti space-time code, semidefinite program

1. INTRODUCTION

In recent years, cognitive radio (CR) has emerged as a promising technology to improve spectrum utilization and bandwidth efficiency [1]. In a CR system, both the primary (licensed) and secondary (unlicensed) users operate at the same frequency bands, and a key design challenge is to provide the latter with certain level of Quality–of–Service (QoS) without causing excessive interference to the former. In the meanwhile, with the rapid growth in the demand for massive content delivery (such as multimedia services), a scenario that has received much attention in the design of modern multiple-antenna communication systems is multi-group multicasting (MM) [2], where the transmitter broadcasts separate information streams to different groups of users. With the advent of 4G systems, like LTE-advanced [3], both CR and MM will play a significant role in supporting resource- and spectral-efficient data services. This motivates us to study multi-group multicast transmission in a CR network (*MM-CR system* for short). In this paper, we will focus on the scenario in which a multi-antenna secondary base station (BS) broadcasts independent data streams to groups of single-antenna secondary users, where users in the same group are interested in a common data stream, and the transmitter has full channel state information. In particular, we are interested in transmit schemes that can provide good QoS for all the secondary users while at the same time control the interference level to the primary users. In this regard, a natural candidate is transmit beamforming [2, 4], where the transmit strategy is single-stream beamforming. Such a strategy has been adopted in various CR systems (see, e.g., [5–8]), and efficient solution methods have been developed to approximately solve the corresponding intractable beamformer optimization problems. One powerful and popular solution method is semidefinite relaxation (SDR) [9], which is a polynomial-time method and whose viability has been demonstrated in many numerical studies. Furthermore, in the multi-group multicast scenario, the approximation accuracy of a certain SDR-based beamforming scheme—i.e., the gap between the worst-user signal-to-interference-and-noise ratio (SINR) achieved by the scheme and the best acheivable worst-user SINR—is provably on the order of M, where M is the number of users [10] (see also [11, 12] for related results). However, to the best of our knowledge, the performance of SDR-based beamforming in MM-CR systems has not been analyzed before. On another front, recent research shows that it is possible, both theoretically and practically, to improve upon the performance of transmit beamforming by modifying the transmit structure [13–16]. In particular, by combining transmit beamforming with the Alamouti space-time code, one obtains a so-called rank-2 transmit beamformed Alamouti scheme, which in the single-group multicast scenario can achieve a worst-user SINR that is only at most a factor of $O(\sqrt{M})$ away from the best achievable worst-user SINR [14]. However, there is no prior work that incorporates the rank-2 transmit beamformed Alamouti scheme in MM-CR systems, let alone its performance analysis.

In view of the above discussion, our goal in this paper is to develop a rank-2 transmit beamformed Alamouti scheme for MM-CR systems. Specifically, we will develop an SDR framework for this scheme and study its SINR performance via both simulations and theoretical analysis. Our main contribution is to show that in an MM-CR system with M_P primary users and M_S secondary users, the rank-2 scheme can achieve a worst-user SINR that is at most a factor of $O(\sqrt{M_S}\log M_P)$ away from the best achievable worst-user SINR. In the multi-group multicast scenario, the factor can be improved to $O(\sqrt{M_S})$. Our results not only unify and generalize those for single-group multicasting [11, 12, 14] and multi-group multicasting [10], but also yield the first provable bound on the performance of a beamforming scheme in a general MM-CR system. Finally, we will demonstrate by simulations that the proposed rank-2 beamformed Alamouti scheme can indeed enhance the performance of an

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MM-CR system.

2. PRELIMINARIES

We consider an MM-CR system where a secondary BS is equipped with N transmit antennae, and there are M_P single-antenna primary users (PUs) and M_S single-antenna secondary users (SUs). The SUs are divided into G groups, so that the k-th group consists of m_k users, i.e., $M_S = \sum_{k=1}^{G} m_k$. For simplicity, let us assume that the secondary BS knows all the channel state information for the PUs and SUs. The secondary BS wishes to broadcast a common information to the users in group k. The received signal of user i in group k, where $i = 1, \ldots, m_k$ and $k = 1, \ldots, G$, is modeled as

$$y_{k,i}(t) = \sum_{k=1}^{G} \mathbf{h}_{k,i}^H \mathbf{x}_k(t) + n_{k,i}(t), \quad t = 1, 2, \ldots, T, \tag{1}$$

where $\mathbf{h}_{k,i} \in \mathbb{C}^N$ is the channel vector of user i in group k, $\mathbf{x}_k(t) \in \mathbb{C}^N$ is the signal transmitted by the secondary BS to users in group k, T is the data frame length, and $n_{k,i}(t) \sim \mathcal{CN}(0, \sigma_{k,i}^2)$ is the complex Gaussian noise. Furthermore, let $\bar{\mathbf{h}}_l \in \mathbb{C}^N$ be the channel vector between the secondary BS and the l-th primary receiver.

To handle the secondary BS's transmissions, a traditional approach is SDR-based transmit beamforming (BF), which has been extensively studied in the literature; see, e.g., [2, 4, 5, 8]. In this approach, $\mathbf{x}_k(t)$ takes the form

$$\mathbf{x}_k(t) = \mathbf{w}_k s_k(t), \quad t = 1, 2, \ldots, T,$$

where $\mathbf{w}_k \in \mathbb{C}^N$ is the beamforming vector, and $s_k(t) \in \mathbb{C}$ is a stream of unit-power data symbols. Then, the interference caused to primary user l by the secondary BS is $\sum_{k=1}^{G} |\mathbf{w}_k^H \bar{\mathbf{h}}_l|^2$, and the SINR of the i-th receiver in the k-th group is given by $|\mathbf{w}_k^H \mathbf{h}_{k,i}|^2 / \left(\sum_{j \neq k} |\mathbf{w}_j^H \mathbf{h}_{k,i}|^2 + \sigma_{k,i}^2 \right)$. To optimize the secondary users' SINRs while controlling their interference to the primary users, one can consider the following max-min-fair (MMF) formulation:

$$\begin{aligned} v_{\mathsf{BF}} \quad = \quad & \max_{\{\mathbf{w}_k\}_{k=1}^G} \min_{\substack{i=1,\ldots,m_k \\ k=1,\ldots,G}} \frac{|\mathbf{w}_k^H \mathbf{h}_{k,i}|^2}{\sum_{j \neq k} |\mathbf{w}_j^H \mathbf{h}_{k,i}|^2 + \sigma_{k,i}^2} \\ & \text{subject to} \quad \textstyle\sum_{k=1}^{G} \mathrm{Tr}(\mathbf{w}_k \mathbf{w}_k^H) \le P, \\ & \qquad \textstyle\sum_{k=1}^{G} |\mathbf{w}_k^H \bar{\mathbf{h}}_l|^2 \le \beta_l \quad \text{for } l = 1, \ldots, M_P. \end{aligned} \tag{2}$$

Here, β_l is the interference threshold of primary user l, and P is the maximum allowable transmit power.

In this paper, we shall depart from the above BF scheme and consider the beamformed (BF) Alamouti scheme, which was introduced independently by Wu *et al.* [14] and Wen *et al.* [15] and can be viewed as a generalization the BF scheme. In the BF Alamouti scheme, the unit-power data symbol stream $s_k(t)$ is parsed into blocks via $\mathbf{s}_k(n) = [\, s_k(2n) \;\; s_k(2n+1) \,]^T$. In block n, we transmit $\mathbf{s}_k(n)$ by a transmit beamformed Alamouti space-time code:

$$\mathbf{X}_k(n) = [\, \mathbf{x}_k(2n) \;\; \mathbf{x}_k(2n+1) \,] = \mathbf{B}_k \mathbf{C}(\mathbf{s}_k(n)).$$

Here, $\mathbf{B}_k \in \mathbb{C}^{N \times 2}$ is a transmit beamforming matrix for group k, $\mathbf{C} : \mathbb{C}^2 \to \mathbb{C}^{2 \times 2}$ is the Alamouti space-time block code. From the basic model (1), we have

$$\mathbf{y}_{k,i}(n) = \sum_{k=1}^{G} \mathbf{h}_{k,i}^H \mathbf{B}_k \mathbf{C}(\mathbf{s}_k(n)) + \mathbf{n}_{k,i}(n),$$

where $\mathbf{y}_{k,i}(n) = [\; y_{k,i}(2n) \;\; y_{k,i}(2n+1) \;]$ and $\mathbf{n}_{k,i}(n) = [\, n_{k,i}(2n) \;\; n_{k,i}(2n+1) \,]$. Now, the interference caused to primary user l by the secondary BS can be expressed as $\sum_{k=1}^{G} |\mathbf{B}_k^H \bar{\mathbf{h}}_l|^2$, and the SINR of the i-th receiver in the k-th group is given by $|\mathbf{B}_k^H \mathbf{h}_{k,i}|^2 / \left(\sum_{j \neq k} |\mathbf{B}_j^H \mathbf{h}_{k,i}|^2 + \sigma_{k,i}^2 \right)$. Hence, we can formulate the corresponding MMF problem as follows:

$$\begin{aligned} v_{\mathsf{BF-ALAM}} \quad = \quad & \max_{\{\mathbf{B}_k\}_{k=1}^G} \min_{\substack{i=1,\ldots,m_k \\ k=1,\ldots,G}} \frac{|\mathbf{B}_k^H \mathbf{h}_{k,i}|^2}{\sum_{j \neq k} |\mathbf{B}_j^H \mathbf{h}_{k,i}|^2 + \sigma_{k,i}^2} \\ & \text{subject to} \quad \textstyle\sum_{k=1}^{G} \mathrm{Tr}(\mathbf{B}_k \mathbf{B}_k^H) \le P, \\ & \qquad \textstyle\sum_{k=1}^{G} |\mathbf{B}_k^H \bar{\mathbf{h}}_l|^2 \le \beta_l \quad \forall l. \end{aligned} \tag{3}$$

3. RANK-2 BEAMFORMED ALAMOUTI SCHEME FOR MM-CR SYSTEMS

Although Problems (2) and (3) are NP-hard in general [2,4], by using the relations

$$\begin{aligned} \mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H \quad &\Longleftrightarrow \quad \mathbf{W}_k \succeq \mathbf{0}, \quad \mathrm{rank}(\mathbf{W}_k) \le 1, \\ \mathbf{W}_k = \mathbf{B}_k \mathbf{B}_k^H \quad &\Longleftrightarrow \quad \mathbf{W}_k \succeq \mathbf{0}, \quad \mathrm{rank}(\mathbf{W}_k) \le 2, \end{aligned}$$

they can both be relaxed to the following semidefinite program (SDP) (cf. [14]):

$$\begin{aligned} v_{\mathsf{sdr}} \quad = \quad & \max_{\{\mathbf{W}_k\}_{k=1}^G} \theta\left(\{\mathbf{W}_k\}_{k=1}^G\right) \\ & \text{subject to} \quad \textstyle\sum_{k=1}^{G} \mathrm{Tr}(\mathbf{W}_k) \le P, \\ & \qquad \textstyle\sum_{k=1}^{G} \mathrm{Tr}(\mathbf{G}_l \mathbf{W}_k) \le \beta_l \quad \forall l, \\ & \qquad \mathbf{W}_k \succeq \mathbf{0} \quad \forall k. \end{aligned} \tag{4}$$

Here,

$$\theta\left(\{\mathbf{W}_k\}_{k=1}^G\right) = \min_{\substack{i=1,\ldots,m_k \\ k=1,\ldots,G}} \frac{\mathrm{Tr}(\mathbf{A}_{k,i} \mathbf{W}_k)}{\sum_{j \neq k} \mathrm{Tr}(\mathbf{A}_{k,i} \mathbf{W}_j) + \sigma_{k,i}^2}, \tag{5}$$

$\mathbf{A}_{k,i} = \mathbf{h}_{k,i} \mathbf{h}_{k,i}^H$ and $\mathbf{G}_l = \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H$. Note that Problem (2) (resp. (3)) corresponds to finding a collection of rank-1 (resp. rank-2) matrices $\{\mathbf{W}_k\}_{k=1}^G$ that is feasible for (4) and maximizes $\theta\left(\{\mathbf{W}_k\}_{k=1}^G\right)$. As such, it is intuitively clear that the BF Alamouti scheme should perform better than the BF scheme. Now, Problem (4) is a quasi-convex optimization problem that can be solved in polynomial time using bisection [17] in conjunction with an algorithm for solving SDPs (e.g., interior-point algorithm; see [9]). However, the solution $\{\mathbf{W}_k^*\}_{k=1}^G$ obtained by solving (4) may not satisfy $\mathrm{rank}(\mathbf{W}_k^*) \le 2$ for all k. To extract a feasible solution $\{\hat{\mathbf{B}}_k\}_{k=1}^G$ to Problem (3) from $\{\mathbf{W}_k^*\}_{k=1}^G$, we can apply the Gaussian randomization procedure in Algorithm 1.

The description of our proposed BF Alamouti scheme is now complete. Before we proceed to analyze its approximation accuracy, it would be interesting to compare its performance with that of the traditional SDR-based BF scheme in the multi-group multicast scenario (i.e., when $M_P = 0$). Specifically, Figure 1 shows the performance of our proposed scheme and the SDR-based BF scheme in [10], respectively. As can be seen from the figure, the proposed BF Alamouti scheme already has some advantage in terms of SINR performance in the multi-group multicast scenario.

Now, let us state the main result of this paper:

Theorem 1 *Let* $\{\hat{\mathbf{B}}_k\}_{k=1}^G$ *be the solution returned by Algorithm 1. Then,* $\{\hat{\mathbf{B}}_k\}_{k=1}^G$ *is feasible for Problem* (3)*. Moreover, with probability at least* $1 - (7/8)^L$,

$$\theta\left(\{\hat{\mathbf{B}}_k \hat{\mathbf{B}}_k^H\}_{k=1}^G\right) \ge \frac{v_{\mathsf{BF-ALAM}}}{8\sqrt{M_S}(3\log 8(M_P+1)+2)}.$$

Algorithm 1 Gaussian Randomization Procedure for (4)

1: **input**: an optimal solution $\{\mathbf{W}_k^*\}_{k=1}^G$ to (4), number of randomizations $L \geq 1$
2: **for** $j = 1, 2, \ldots, L$; $k = 1, \ldots, G$ **do**
3: generate 2 independent circularly symmetric complex Gaussian random vectors $\boldsymbol{\xi}_k^j, \boldsymbol{\eta}_k^j \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_k^*)$, and define

$$\bar{\mathbf{B}}_{j,k} = \frac{1}{\sqrt{2}} \left[\begin{array}{cc} \boldsymbol{\xi}_k^j & \boldsymbol{\eta}_k^j \end{array} \right],$$

$$\bar{T}_{j,k} = \max \left\{ \max_{l=1,\ldots,M_P} \left\{ \frac{1}{\beta_l} \sum_{k=1}^G \mathrm{Tr}(\mathbf{G}_l \bar{\mathbf{B}}_{j,k} \bar{\mathbf{B}}_{j,k}^H) \right\}, \frac{1}{P} \sum_{k=1}^G \mathrm{Tr}(\bar{\mathbf{B}}_{j,k} \bar{\mathbf{B}}_{j,k}^H) \right\}$$

4: let $\hat{\mathbf{B}}_{j,k} = \bar{\mathbf{B}}_{j,k} / \sqrt{\bar{T}_{j,k}}$
5: **end for**
6: let

$$j^* = \arg \max_{j=1,\ldots,L} \theta \left(\{\hat{\mathbf{B}}_{j,k} \hat{\mathbf{B}}_{j,k}^H\}_{k=1}^G \right),$$

where $\theta(\cdot)$ is defined in (5)
7: **return** $\hat{\mathbf{B}}_k = \hat{\mathbf{B}}_{j^*,k}$ for $k = 1, \ldots, G$

The proof of Theorem 1 can be found in the Appendix. In essence, Theorem 1 states that the gap between the worst-user SINR achieved by our proposed scheme and the best achievable worst-user SINR scales on the order of $\sqrt{M_S} \log M_P$. This result generalizes that in [14], which applies only to the single-group multicast scenario (i.e., $G = 1$ and $M_P = 0$). Moreover, Theorem 1 shows that in the multi-group multicast scenario (i.e., when $M_P = 0$), the gap scales only on the order of $\sqrt{M_S}$. This is substantially better than the traditional BF scheme, where the provable gap scales on the order of M_S [10].

4. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the performance of the proposed scheme. The simulations are based on the model setting in Section II. In particular, we run 300 channel trials with the channel vector $\mathbf{h}_{k,i}, \bar{\mathbf{h}}_l \in \mathbb{C}^N$ i.i.d Rayleigh distributed with unit power. The noise power is set to be 1. Besides, without loss of generality, we assume that each group has the same number of users.

Fig. 2 shows the worst secondary user (SU) SINR scaling with respective to (w.r.t) the number of SUs (M_S) for the SDR upper bound (v_{sdr}), traditional transmit beamforming (BF) scheme and the proposed beamformed (BF) Alamouti scheme. We assume that the secondary BS is equipped with $N = 8$ transmit antennae, and there are $M_P = 2$ PUs. The transmit power is set to be 20dB, and PU interference threshold is set to be 0dB (i.e., $\beta_l = 1$ for all l). The SUs are divided into $G = 3$ groups, i.e., each group has 4 users when $M_S = 12$. Problem (4) is solved using bisection (see [17]), in which the SDPs are solved using SeDuMi, and $L = 1000$ randomizations in Algorithm 1 are used to generate a rank-1 or rank-2 solution. It reflects that the proposed scheme is more capable than the traditional scheme, especially when dealing with more SUs.

Fig. 3 shows the worst SU SINR scaling w.r.t the number of PUs. We assume that there are two groups and each one has 8 users. Also, we set $P = 20$dB, $\beta_l = 0$dB as before. It can be shown that, with the increasing number of PUs, the BF Alamouti scheme shows an improvement in terms of the worst SU SINR over the BF scheme, which demonstrates that performing an SDR-based rank-two approximation can narrow the performance loss of the rank-one approximation in traditional BF. It is worth mentioning that, conceptually, we may apply a rank-r ($r \geq 3$) approximation by an orthogonal space-time coding (OSTBC) scheme. However, as we have mentioned in [14], this will induce a rate loss, as full-rate OSTBC does not exist when dimension larger than two.

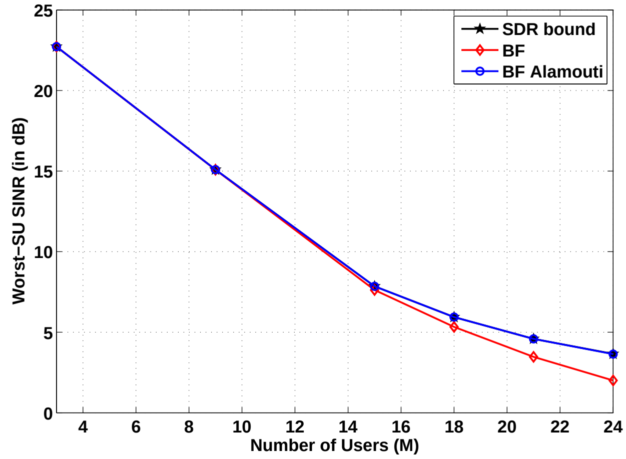

Fig. 1. Users' SINR scaling w.r.t. number of users M. $N = 8$, $G = 3$, $P = 20$dB

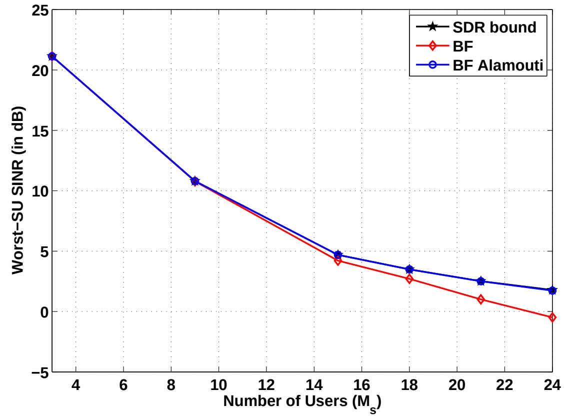

Fig. 2. Worst SU SINR scaling w.r.t. number of SUs M_S. $N = 8$, $G = 3$, $M_P = 2$, $P = 20$dB, $\beta_l = 0$dB.

5. CONCLUSION

In this paper, we proposed a rank-2 transmit beamformed Alamouti scheme for MM-CR systems. To handle the resulting beamformer optimization problem, we developed a polynomial-time SDR-based method. We then analyzed the worst-case approximation accuracy of the proposed scheme. Our analysis unifies and generalizes those in [10, 14]. Moreover, when specialized to the multi-group multicast scenario, our result shows that the proposed scheme has a provably

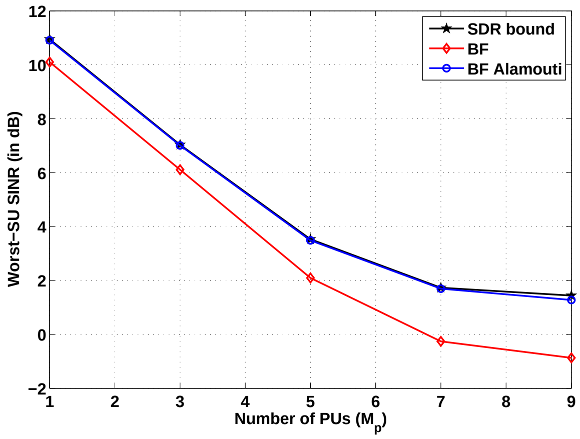

Fig. 3. Worst SU SINR scaling w.r.t. number of PUs M_P. $N = 8$, $M_S = 16$, $G = 2$, $P = 20$dB, $\beta_l = 0$dB.

better performance than the traditional beamforming scheme in [10]. Finally, the aforementioned theoretical findings are supported by our simulation results.

6. APPENDIX: PROOF OF THEOREM 1

Consider a fixed j in Algorithm 1 and let $\hat{\mathbf{W}}_k = \hat{\mathbf{B}}_{j,k}\hat{\mathbf{B}}_{j,k}^H$ for $k = 1,\ldots,G$. For any $\beta,\gamma > 0$, consider the events

$$E_{k,i}^{\beta} = \left\{ \frac{\text{Tr}(\mathbf{A}_{k,i}\hat{\mathbf{W}}_k)}{\sum_{j\neq k}\text{Tr}(\mathbf{A}_{k,i}\hat{\mathbf{W}}_k) + \sigma_{k,i}^2} \le \frac{\beta \text{Tr}(\mathbf{A}_{k,i}\mathbf{W}_k^*)}{\sum_{j\neq k}\text{Tr}(\mathbf{A}_{k,i}\mathbf{W}_j^*) + \sigma_{k,i}^2} \right\},$$

$$F_l^{\gamma} = \left\{ \sum_{k=1}^{G}\text{Tr}(\mathbf{G}_l\hat{\mathbf{W}}_k) \ge \gamma \sum_{k=1}^{G}\text{Tr}(\mathbf{G}_l\mathbf{W}_k^*) \right\},$$

where $k = 1,\ldots,G$; $i = 1,\ldots,m_k$; $l = 0,1,\ldots,M_P$; $\mathbf{G}_0 = \mathbf{I}$ (i.e., $\mathbf{G}_0$ is the identity matrix). We shall use the following lemmas to bound $\Pr(E_{k,i}^{\beta})$ and $\Pr(F_l^{\gamma})$.

Lemma 1 *Let $\mathbf{A},\mathbf{B}$ be arbitrary Hermitian positive semidefinite matrices, and let $\boldsymbol{\xi},\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0},\mathbf{W}^*)$ be independent random vectors. Consider the matrix $\mathbf{W} = \frac{1}{2}[\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,][\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,]^H$. Then,*

$$\Pr\left(\frac{\text{Tr}(\mathbf{AW})}{\text{Tr}(\mathbf{BW})+1} \le \beta\frac{\text{Tr}(\mathbf{AW}^*)}{\text{Tr}(\mathbf{BW}^*)+1}\right) \le \left(\frac{5\beta}{\alpha - 2\beta}\right)^2,$$

where $\frac{1}{\alpha} \ge r_A = \text{rank}((\mathbf{W}^*)^{1/2}\mathbf{A}(\mathbf{W}^*)^{1/2})$ and $0 < \beta < \frac{\alpha}{2}$.

Lemma 2 *(cf. [12, 14] and [18, Proposition A5.5]) Let $\mathbf{H}$ be an arbitrary rank-1 Hermitian positive semidefinite matrix, and let $\boldsymbol{\xi},\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0},\mathbf{W}^*)$ be independent random vectors. Consider the matrix $\mathbf{W} = \frac{1}{2}[\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,][\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,]^H$. Then, for any $\gamma \ge 4/3$, we have*

$$\begin{aligned} \Pr(\text{Tr}(\mathbf{HW}) \ge \gamma\text{Tr}(\mathbf{HW}^*)) &\le \exp[2(1-\gamma+\ln\gamma)], \\ \Pr(\text{Tr}(\mathbf{W}) \ge \gamma\text{Tr}(\mathbf{W}^*)) &\le \exp[-(\gamma+4\ln(3/4))/2]. \end{aligned}$$

Armed with Lemmas 1 and 2, we now proceed to prove Theorem 1. Since $\text{rank}(\mathbf{A}_{k,i}) = 1$, according to Lemma 1, we have $\Pr(E_{k,i}^{\beta}) \le (5\beta/(1-2\beta))^2$ for $k = 1,\ldots,G$ and $i = 1,\ldots,m_k$. Now, let $E = \bigcup_{k=1}^{G}\bigcup_{i=1}^{m_k} E_{k,i}^{\beta}$ and $F = \bigcup_{l=0}^{M_P} F_l^{\gamma}$. Upon choosing $\beta = 1/(8\sqrt{M_S})$, $\gamma = 3\log 8(M_P+1)+2$ and applying Lemma 2, we obtain

$$\Pr(E) \le \sum_{k=1}^{G}\sum_{i=1}^{m_k}\Pr(E_{k,i}^{\beta}) \le M_S\left(\frac{5\beta}{1-2\beta}\right)^2 < \frac{3}{4},$$

$$\Pr(F) \le \sum_{l=0}^{M_P}\Pr(F_l^{\gamma}) < (M_P+1)\times\frac{1}{8(M_P+1)} = \frac{1}{8}.$$

Thus, we conclude that $\Pr(E^c \cap F^c) \ge 1/8$, i.e., with probability at least $1/8$, we have the desired result:

$$\gamma\left(\{\hat{\mathbf{W}}_k\}_{k=1}^{G}\right) \ge \frac{\beta}{\gamma}v_{\text{sdr}} \ge \frac{v_{\text{BF-ALAM}}}{8\sqrt{M_S}(3\log 8(M_P+1)+2)}.$$

6.1. Proof of Lemma 1

Let $\mathbf{Q}$ be a unitary matrix satisfying $(\mathbf{W}^*)^{1/2}\mathbf{A}(\mathbf{W}^*)^{1/2} = \mathbf{Q}^H\boldsymbol{\Lambda}_{\mathbf{A}}\mathbf{Q}$, where $\boldsymbol{\Lambda}_{\mathbf{A}} = \text{diag}(\lambda_1,\ldots,\lambda_{r_A},0,\ldots,0)$ and $\lambda_1 \ge \cdots \ge \lambda_{r_A} > 0$. Observe that $\boldsymbol{\xi} \sim (\mathbf{W}^*)^{1/2}\mathbf{Q}^H\mathbf{x}$ and $\boldsymbol{\eta} \sim (\mathbf{W}^*)^{1/2}\mathbf{Q}^H\mathbf{y}$, where $\mathbf{x},\mathbf{y} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$ are independent. Since $\mathbf{W} = \frac{1}{2}[\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,][\,\boldsymbol{\xi}\ \boldsymbol{\eta}\,]^H$, we have

$$\begin{aligned} &\Pr\left(\frac{\text{Tr}(\mathbf{AW})}{\text{Tr}(\mathbf{BW})+1} \le \beta\frac{\text{Tr}(\mathbf{AW}^*)}{\text{Tr}(\mathbf{BW}^*)+1}\right) \\ = &\Pr\left(\frac{\sum_{i=1}^{r_A}\lambda_i(|x_i|^2+|y_i|^2)}{\sum_{i=1}^{r_A}\lambda_i} \le \beta\frac{\mathbf{x}^H\bar{\mathbf{B}}\mathbf{x}+\mathbf{y}^H\bar{\mathbf{B}}\mathbf{y}+2}{\text{Tr}(\bar{\mathbf{B}})+1}\right), \end{aligned}$$

where $\bar{\mathbf{B}} = \mathbf{Q}(\mathbf{W}^*)^{1/2}\mathbf{B}(\mathbf{W}^*)^{1/2}\mathbf{Q}^H$. Since $\bar{\mathbf{B}} \succeq \mathbf{0}$, we may write $\bar{\mathbf{B}} = \mathbf{U}^H\boldsymbol{\Lambda}_{\mathbf{B}}\mathbf{U}$, where $\mathbf{U}$ is unitary and $\boldsymbol{\Lambda}_{\mathbf{B}} = \text{diag}(\mu_1,\ldots,\mu_{r_B},0,\ldots,0)$ with $\mu_1 \ge \cdots \ge \mu_{r_B} > 0$. Now, let $\mathbf{z} = \mathbf{Ux}$, $\mathbf{s} = \mathbf{Uy}$, and define $\bar{\lambda}_i = \lambda_i/\sum_i\lambda_i$, $\bar{\mu}_i = \mu_i/\sum_i\mu_i$. Obviously, we have $\bar{\lambda}_1 \ge 1/r_A \ge \alpha$. Hence,

$$\begin{aligned} &\Pr\left(\frac{\sum_{i=1}^{r_A}\lambda_i(|x_i|^2+|y_i|^2)}{\sum_{i=1}^{r_A}\lambda_i} \le \beta\frac{\mathbf{x}^H\bar{\mathbf{B}}\mathbf{x}+\mathbf{y}^H\bar{\mathbf{B}}\mathbf{y}+2}{\text{Tr}(\bar{\mathbf{B}})+1}\right) \\ \le &\Pr\left(\alpha(|x_1|^2+|y_1|^2) \le \beta\sum_{i=1}^{r_B}\bar{\mu}_i(|z_i|^2+|s_i|^2)+2\right) \\ \le &\Pr\left[\alpha(|x_1|^2+|y_1|^2) \le \beta\sum_{i=1}^{r_B}\bar{\mu}_i\left(2|U_{i1}|^2(|x_1|^2+|y_1|^2)\right.\right. \\ &\left.\left. +2\left|\sum_{j=2}^{n}U_{ij}x_j\right|^2 + 2\left|\sum_{j=2}^{n}U_{ij}y_j\right|^2\right)+2\right] \\ \le &\Pr\left[|x_1|^2+|y_1|^2 \le \frac{2\beta}{\alpha-2\beta}\left(\sum_{i=1}^{r_B}\bar{\mu}_i\left(\left|\sum_{j=2}^{n}U_{ij}x_j\right|^2\right.\right.\right. \\ &\left.\left.\left. +\left|\sum_{j=2}^{n}U_{ij}y_j\right|^2\right)+1\right)\right] := \Gamma \end{aligned}$$

Since $\Pr(|x|^2+|y|^2 \le t) = 1-(t+1)e^{-t} \le t^2/2$ for all $t \ge 0$, we conclude that

$$\begin{aligned} \Gamma &\le \frac{2\beta^2}{(\alpha-2\beta)^2}\mathbb{E}\left[\sum_{i=1}^{r_B}\bar{\mu}_i\left(\left|\sum_{j=2}^{n}U_{ij}x_j\right|^2+\left|\sum_{j=2}^{n}U_{ij}y_j\right|^2+1\right)\right]^2 \\ &\le \frac{2\beta^2}{(\alpha-2\beta)^2}\left(2\mathbb{E}\left[\sum_{i=1}^{r_B}\bar{\mu}_i\left|\sum_{j=2}^{n}U_{ij}x_j\right|^2\right]^2+7\right) < \frac{25\beta^2}{(\alpha-2\beta)^2}. \end{aligned}$$

This completes the proof.

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