Semidefinite Relaxation and Approximation Analysis of a Beamformed Alamouti Scheme for Relay Beamforming Networks

Sissi Xiaoxiao Wu, Anthony Man-Cho So, Jiaxian Pan and Wing-Kin Ma

***Abstract*—In this paper, we study amplify-and-forward (AF) schemes in two-hop one-way relay networks. In particular, we consider multigroup multicast transmission between long-distance users. Assuming that perfect channel state information is perceived, our goal is to design the AF process so that the max-min-fair signal-to-interference-plus-noise ratio (SINR) is optimized while generalized power constraints are satisfied. We propose a beamformed Alamouti (BFA) AF scheme and formulate the corresponding AF design problem as a *two-block* fractional quadratically-constrained quadratic program (QCQP). We then tackle the two-block fractional QCQP using the semidefinite relaxation (SDR) technique and analyze the approximation accuracy of the proposed SDR. From a theoretical perspective, our results are fundamentally new and reveal that the proposed BFA AF scheme can outperform the traditional BF AF scheme, especially when there are many users in the system or many generalized power constraints in the problem formulation. From a practical perspective, our proposed BFA AF scheme improves the receivers' SINR by offering two degrees of freedom (DoFs) in beamformer design, as opposed to only one DoF offered by the BF AF scheme. In the latter part of this paper, we demonstrate how this extra DoF leads to provable performance gain by considering two special relay scenarios, in which the AF process is shown to possess a special structure. Numerical simulations further confirm that the proposed BFA AF scheme outperforms the BF AF scheme and works well for large-scale relay systems.**

Index terms– **MIMO relay network, distributed relay network, cognitive radio, energy harvesting, amplify-and-forward (AF), multigroup multicast, SDR, approximation bounds.**

I. Introduction

The information delivery between multiple wireless devices has shown an increasing importance in up-to-date military networks, relay networks, and 5G networks [1]–[3]. The state-of-the-art technique in this context is to use small smart access points (APs), such as mobile phones, wireless relays, and Wi-Fi APs, to assist information delivery between far-apart transceiver pairs. Nowadays, a new trend is to connect the smart APs by fibers, microwave, or millimeter wave to build up a cloud processing center for facilitating reliable communications. A typical example is the cloud radio access network (C-RAN) [4]–[6], which is recently proposed as a promising network architecture to offer a 1000x increase in capacity to support broadband applications. The key enabling technologies in C-RANs are the cloud processors pool and fronthaul-backhaul links. They coordinate all the base-stations in all cells to form a cloud base-station and serve the users in a jointly optimized manner. Naturally, one can extend the above setting to "cloud relays" by viewing communications between devices as information delivery in a relay network. The intra-network interference will be treated as noise and the inter-network interference will be managed by designing the amplify-and-forward (AF) process at the cloud center. This gives rise to the so-called *cloud relay network* (C-RN) [2], [7]–[9]; see a system model example in Figure 1.

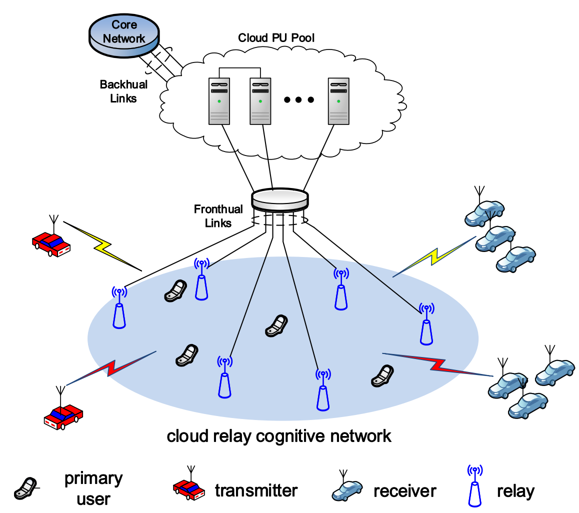

Fig. 1. An example of the cloud relay network.

In this work, we focus on a typical two-hop one-way relay network. In particular, we consider the case where all nodes—i.e., transmitters, receivers, and relays—are equipped with a single antenna. This assumption is reasonable, as nodes in a D2D communication network are usually limited by power and apparatus. In our setting, the transmitters and receivers are far-apart and the direct links between them are negligible.[1] Thus,

[1] We treat signals from direct links as noise terms at the destination.

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the transmitters rely on the relays to AF the information.[2] We assume that the relays are distributively located, and more importantly, they coordinate to form a C-RN. In practice, the capacity of the fronthaul-backhaul links in C-RNs is an important issue. If the link capacity is unlimited, then both the channel state information (CSI) and received signals can be shared within the cloud, and the system becomes an MIMO relay network. On the other hand, if the link capacity is limited in such a way that only CSIs are shared and the received signals are isolated among different relays, then the system is reduced to a distributed relay network. We remark that there are also other types of relay networks corresponding to different link capacities, but in this paper we focus on the two above.

Our goal is to design the relay AF process for multigroup multicast transmission. In the literature, there are different problem formulations; see, e.g., [10]–[18]. Herein, we focus on the max-min-fair (MMF) formulation, in which the worst user's signal-to-interference-plus-noise ratio (SINR) is to be maximized while generalized power constraints, such as total power constraint, per-relay power constraints, interference temperature constraints, or energy harvesting constraints, are to be satisfied. This makes our design approach applicable to many scenarios. A classic design approach for AF relays is to adopt the beamformed (BF) AF scheme [10], [11], which gives rise to an NP-hard *single-block* fractional quadratically-constrained quadratic program (QCQP). An effective way to tackle such fractional QCQPs is to apply the semidefinite relaxation (SDR) technique [19], which involves reformulating the fractional QCQP as a rank-one constrained fractional semidefinite program (SDP) and relaxing the rank constraint to obtain a polynomial-time solvable fractional SDP. It is well known that the SDR is tight if the corresponding fractional SDP has rank-one solutions. Otherwise, a Gaussian randomization algorithm is applied to convert the optimal solution to the fractional SDP into a feasible solution to the fractional QCQP [20], [21]. We shall call this solution the *SDR solution* in the sequel. A fundamental issue here is to quantify the quality of the SDR solution. In our previous work [8], [9], we show that the fractional SDP always has a rank-one solution when $M + J \leq 4$, and that the SINR associated with the SDR solution is at least $\Omega(\frac{1}{M \log J})$ times that associated with the optimal solution to the fractional QCQP when $M + J > 4$. Here, M is the number of users (receivers) in the network and J is the number of generalized power constraints. Although this result provides an SDR approximation bound for the single-block fractional QCQP when multiple constraints are present, from a practical perspective, it actually implies that the SINR associated with the SDR-based BF AF scheme may experience a performance loss on the order of $\frac{1}{M \log J}$ in large-scale systems.

In order to improve the relay beamforming performance, we propose to adopt the Alamouti space-time code in the AF structure. This leads to the *BF Alamouti (BFA) AF* scheme, in which two beamformers are used to process two data symbols jointly. Compared to the BF AF scheme, which uses only one beamformer to process a single data symbol, the BFA AF scheme has one extra degree of freedom (DoF) in the beamformer design. As such, it is expected to yield better system performance. In fact, the extra DoF available to the BFA AF scheme is also manifested in its corresponding design problem, in that it can be shown to admit a *two-block* fractional QCQP formulation. Our analytic results show that the optimal value of the corresponding two-block fractional SDP is always no worse than that of the single-block fractional SDP. Moreover, in a variety of relay scenarios, the SDR is tight when $M + J \leq 5$, and the approximation accuracy of the SDR solution is on the order of $\frac{1}{\sqrt{M} \log J}$ when $M+J > 5$. Clearly, both the tightness and approximation accuracy results are better than their BF AF counterparts.

The idea of using the Alamouti code in single-group multicast beamforming is introduced independently in [22] (see also [23]) and [24] and leads to the first BFA schemes. The subsequent conference paper [21] proposes a BFA scheme for multigroup multicasting without relays. This paper unifies and significantly extends the aforementioned works by developing a BFA AF scheme for multigroup multicast relay networks with generalized power constraints and providing an analysis on the performance of the proposed scheme. Although BFA AF schemes for relay networks have previously been studied in [25], [26], there are fundamental differences between those works and ours. Indeed, the work [25] focuses on a distributed relay network with one power constraint, while our work considers both distributed relay and MIMO relay networks with generalized power constraints. The work [26] allows a direct link to exist between the devices, while our work is targeted at the setting where devices are far-apart and direct links between them are negligible. In addition, both of the works [25], [26] focus on the single-group multicast scenario, while ours considers the multigroup multicast scenario. We remark that generalizing the BFA schemes from single-group multicasting to multigroup multicasting, and from distributed relays to MIMO relays, is non-trivial. More importantly, we establish for the first time the SDR approximation accuracy of a fairly general class of two-block fractional QCQPs, thereby allowing us to obtain a provable guarantee on the performance of our proposed scheme. By contrast, no such guarantee is available for the schemes proposed in [25], [26].

It should also be noted that the problem considered in this paper, namely beamformer design for multi-user to multi-user multigroup multicasting in relay networks, has not been well addressed in the literature. Indeed, existing works on relay transceiver design mainly focus on the point-to-point [13], [14], single-user to multi-user [15], multi-user to single-user [16], [17], and multi-user to multi-user unicast [10] and multicast scenarios [18]. Although the work [27] studies beamformer design in a multigroup multicast relay network, it considers BF AF schemes for single-antenna relays, whereas our focus is on designing a BFA AF scheme in a cloud relay setting. From a computational point of view, some efficient heuristics have recently been proposed to find high-quality solutions to single-block fractional QCQPs; see, e.g., [28]–[31]. We numerically compare those heuristics with the proposed

[2] The relays can also decode-and-forward (DF) the received signals, but this is beyond the scope of this paper.

BFA AF scheme and observe that even though those heuristics can help us find a better QCQP solution, the proposed BFA AF scheme still owns a significantly better performance; see Section IV-F. Lastly, we remark that although it seems one can further improve the system performance by considering higher-dimensional orthogonal space-time block codes (OSTBCs), the overall rate loss associated with the use of such codes (recall that there is no full-rate OSTBC of dimension $n > 2$ [32]) may neutralize any performance gain; see also the discussion in [23, Remark 3]. We should also point out that the usage of the Alamouti code in this work differs from that of the distributed space-time codes in [33], [34], since the former uses the Alamouti code structure at each relay while the latter judiciously designs the code structure at each of the relays so that they form an OSTBC as a whole.

The organization of this paper is as follows. In Section II, we introduce the system model and the SDR-based BF AF scheme for both the MIMO relay and distributed relay networks in the presence of primary users. We show how the design problem for the BF AF scheme can be tackled by the SDR technique and review existing bounds on the approximation accuracy of the SDR solution. In Section III, we introduce the BFA AF scheme and show how the corresponding design problem gives rise to a new two-block fractional QCQP formulation. As one of our main results, we establish a bound on the approximation accuracy of the SDR solution to the two-block fractional QCQP, thereby providing a performance guarantee for the proposed BFA AF scheme. We then apply our results to two types of multicasting relay network and demonstrate how the two DoFs offered by the BFA AF scheme can improve system performance. Lastly, we present simulation results in Section IV and conclude the paper in Section V.

Our notation is standard: $\mathbb{C}^N$ is the set of all complex N-dimensional vectors; $\mathbb{H}_+^N$ is the set of all $N \times N$ positive semidefinite matrices; z^* denotes the complex conjugate of the complex number z; $\|\cdot\|$ is the vector Euclidean norm; $\boldsymbol{A} \bullet \boldsymbol{B}$ stands for the inner product between matrix $\boldsymbol{A}$ and $\boldsymbol{B}$; $\boldsymbol{A} \otimes \boldsymbol{B}$ stands for the Kronecker product between matrix $\boldsymbol{A}$ and $\boldsymbol{B}$; $\boldsymbol{A} \odot \boldsymbol{B}$ stands for the element-wise product between matrix $\boldsymbol{A}$ and $\boldsymbol{B}$; $\mathrm{vec}(\boldsymbol{A})$ is the vectorization operator for matrix $\boldsymbol{A}$; $\mathrm{Diag}(\boldsymbol{x})$ is a diagonal matrix parametrized by the elements of $\boldsymbol{x}$; $\mathrm{rank}(\boldsymbol{X})$ and $\lambda_{\min}(\boldsymbol{X})$ stand for the rank and the smallest eigenvalue, respectively; $\boldsymbol{e}_i$ is a unit vector with the non-zero element in the ith entry; $\boldsymbol{e}$ is the vector of all ones; $\boldsymbol{I}_r$ denotes the r-by-r identity matrix; $\mathbb{E}[\cdot]$ denotes statistical expectation; $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{W})$ denotes the circularly symmetric complex Gaussian distribution with mean vector $\boldsymbol{0}$ and covariance matrix $\boldsymbol{W}$.

II. System Model and the Beamformed Amplify-and-Forward Scheme

In this section, we describe the system model for two-hop one-way relay networks. We consider *multigroup multicast* transmission by a network of single-antenna AF relays. We assume that there are L relays in the network; G single-antenna transmitters (sources) send G independent common information to G groups of single-antenna users (receivers, destinations). Users in the same group require the same information, while users in different groups require different information. In total, there are $\sum_{j=1}^{G} m_j = M$ users in the network, where m_j is the number of users in group j for $j = 1, \ldots, G$. In our target setting, the transmitters and receivers are far-apart so that direct links between them can be ignored. As such, relays play an important role in information delivery by AF-ing received signals from sources to destinations. Under the C-RN setting [7], we assume that the relays are distributively located but connected by a cloud processing unit (PU) pool (i.e., the computation center) via fronthaul and backhaul links, which are typically fibers or microwave connected to fibers. We further assume that all nodes in the network are well synchronized, channels from transmitters to relays and relays to users are frequency flat and quasi-static, and the channels are perfectly perceived at the transmitters and receivers (e.g., by using reference signals) and are fully shared within the cloud PU pool. The cloud PU pool can then coordinate the design of the AF process in the network.

A. The BF AF Scheme

The information delivery process in a one-way relay network proceeds in two hops; see Figure 2:

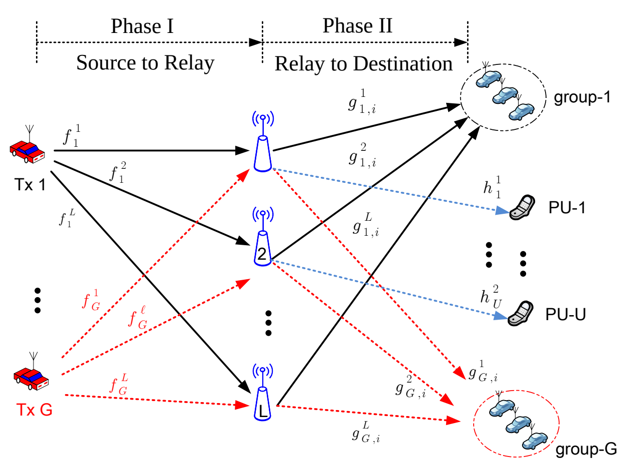

Fig. 2. The two-hop one-way relay network with primary users.

1) *Source-to-Relay Hop: The transmitters send information to relays.* The receive model at the relay is given by

$$\boldsymbol{r}(t) = \sum_{j=1}^{G} \boldsymbol{f}_j s_j(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{r}(t) = [r^1(t), \ldots, r^\ell(t), \ldots, r^L(t)]^T$ with

$$r^\ell(t) = \sum_{j=1}^{G} f_j^\ell s_j(t) + n^\ell(t), \quad \ell = 1, \ldots, L \tag{2}$$

being the received signal at relay ℓ; $s_j(t)$ is the common information specific for group j with $\mathbb{E}[|s_j(t)|^2] = P_j$, where P_j is the transmit power at transmitter j; $\boldsymbol{f}_j = [f_j^1, \ldots, f_j^L]^T$ is the channel from transmitter j to all the relays; $\boldsymbol{n}(t) =$

$[n^1(t),\ldots,n^L(t)]^T$ with $n^\ell(t)$ being the Gaussian noise at relay ℓ, which has mean zero and variance σ_ℓ^2. Throughout this paper, we assume that $\sigma_\ell^2 > 0$ for $\ell = 1,\ldots,L$.

2) *Relay-to-Destination Hop: Relays process the received signals and then forward them to receivers.* A classic AF scheme is the BF AF scheme, which has been widely used in the literature; see, e.g., [10], [11]. The transmit structure at the relay side is given by

$$\boldsymbol{x}(t) = \boldsymbol{V}\boldsymbol{r}(t). \tag{3}$$

The weighting matrix $\boldsymbol{V}$ represents how the relays interact with each other. Specifically, consider the setting where all the single-antenna relays are connected by a cloud, within which the CSIs are fully shared. If the received signals are also shared within the cloud, then there is no constraint on the matrix $\boldsymbol{V}$, and we are dealing with an MIMO relay network [11]. On the other hand, if the received signals are not shared, then $\boldsymbol{V}$ is constrained to be a diagonal matrix, and we are dealing with a distributed relay network [10].

In the case of an MIMO relay network, the received signal for user-(k,i) (i.e., user i in group k) is given by

$$\begin{aligned} y_{k,i}(t) &= \boldsymbol{g}_{k,i}^H\boldsymbol{x}(t) + v_{k,i}(t) \qquad (4)\\ &= \underbrace{\boldsymbol{g}_{k,i}^H\boldsymbol{V}\boldsymbol{f}_k s_k(t)}_{\text{desired signal}} + \underbrace{\boldsymbol{g}_{k,i}^H\boldsymbol{V}\left(\sum_{m\neq k}\boldsymbol{f}_m s_m(t) + \boldsymbol{n}(t)\right) + \mu_{k,i}(t)}_{\text{interference and noise}}, \end{aligned}$$

where $\boldsymbol{g}_{k,i} = [g_{k,i}^1,\ldots,g_{k,i}^\ell,\ldots,g_{k,i}^L]^T$ is the vector of channels from the relays to user-(k,i) and $\mu_{k,i}(t)$ is the Gaussian noise at user-(k,i) with zero mean and variance $\sigma_{k,i}^2$. In this paper, we assume that $\sigma_{k,i}^2 > 0$ for $i = 1,\ldots,m_k$ and $k = 1,\ldots,G$. Accordingly, by letting $\boldsymbol{\Sigma} = \mathrm{Diag}(\sigma_1^2,\ldots,\sigma_\ell^2,\ldots,\sigma_L^2) \succ \boldsymbol{0}$, the receive SINR at user-(k,i) can be expressed as

$$\frac{P_k\left|\boldsymbol{g}_{k,i}^H\boldsymbol{V}\boldsymbol{f}_k\right|^2}{\sum_{m\neq k} P_m\left|\boldsymbol{g}_{k,i}^H\boldsymbol{V}\boldsymbol{f}_m\right|^2 + \boldsymbol{g}_{k,i}^H\boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^H\boldsymbol{g}_{k,i} + \sigma_{k,i}^2}. \tag{5}$$

B. Generalized Power Constraints

We are interested in optimizing the AF weighting matrix $\boldsymbol{V}$ for relay networks under three types of design constraints:

1) *Total Power Constraints.* A natural consideration is that the total transmit power at the relays is below a given threshold. This leads to the total power constraint

$$\begin{aligned} \mathbb{E}[\|\boldsymbol{x}(t)\|^2] =& \mathrm{Tr}\left(\boldsymbol{V}\left(\sum_{j=1}^G P_j\boldsymbol{f}_j\boldsymbol{f}_j^H + \boldsymbol{\Sigma}\right)\boldsymbol{V}^H\right)\\ =& \boldsymbol{w}^H\boldsymbol{D}_0\boldsymbol{w} \le \bar{P}_0, \end{aligned} \tag{6}$$

where (6) is obtained by setting $\boldsymbol{w} = \mathrm{vec}(\boldsymbol{V})$ and applying the identity

$$\mathrm{Tr}\left(\boldsymbol{A}^H\boldsymbol{B}\boldsymbol{C}\boldsymbol{D}\right) = \mathrm{vec}(\boldsymbol{A})^H\left(\boldsymbol{D}^T\otimes\boldsymbol{B}\right)\mathrm{vec}(\boldsymbol{C}),$$

which is valid for arbitrary complex matrices $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D}$ of appropriate dimensions. Also, $\bar{P}_0$ is the total transmit power budget for all the relays and $\boldsymbol{D}_0$ is defined in (10) in Table I.

2) *Per-Relay Power Constraints.* Another common design constraint is to place a power budget at each relay. Such constraint can be formulated as

$$\boldsymbol{e}_\ell^H\boldsymbol{V}\left(\sum_{j=1}^G P_j\boldsymbol{f}_j\boldsymbol{f}_j^H + \boldsymbol{\Sigma}\right)\boldsymbol{V}^H\boldsymbol{e}_\ell \le \bar{P}_\ell,\ \ \ell = 1,\ldots,L,$$

where $\bar{P}_\ell$ is the maximum transmit power allowed at relay ℓ. Similar to (6), we can rewrite the above constraints as

$$\boldsymbol{w}^H\boldsymbol{D}_\ell\boldsymbol{w} \le \bar{P}_\ell, \quad \ell = 1,\ldots,L, \tag{7}$$

where $\boldsymbol{D}_\ell$ is defined in (11) in Table I.

3) *Interference Temperature Constraints.* We may also consider a popular design constraint in the cognitive radio (CR) network setting, which is to control the interference temperature at the primary users [35]–[37]. Suppose that we have a CR relay network with U primary users, at which the interference is simply caused by the AF relays and not by the transmitters. Let $\boldsymbol{h}_u = [h_u^1,\ldots,h_u^\ell,\ldots,h_u^L]^T$ be the vector of channels from the relays to primary user u, for $u = 1,\ldots,U$. Then, the interference temperature constraints can be formulated as

$$\boldsymbol{w}^H\boldsymbol{G}_u\boldsymbol{w} \le \eta_u, \quad u = 1,\ldots,U,$$

where η_u is the maximum interference allowed at primary user u, and $\boldsymbol{G}_u$ is defined in (12) in Table I with $\sigma_u^2 > 0$ being the noise power at primary user u.

C. A Unified AF Design Problem

In this sub-section, we show how the beamformer design problems associated with different types of relay networks admit a unified formulation.

1) Problem Formulation for the MIMO CR Relay Network: In light of the SINR expression in (5), we consider the MMF design approach, which maximizes the worst user's SINR subject to the generalized power constraints. This gives rise to the single-block fractional QCQP problem (R1BF) in Table I.

2) Problem Formulation for the Distributed CR Relay Network: Since the AF weighting matrix is diagonal in the case of a distributed relay network, by writing $\boldsymbol{w} = \mathrm{Diag}(\boldsymbol{V})$, it is not hard to see that the design problem for the distributed CR relay network has exactly the same form as that for the MIMO relay network; i.e., Problem (R1BF) with $\mathcal{L}$, $\boldsymbol{A}_{k,i}$, $\boldsymbol{C}_{k,i}$, $\boldsymbol{D}_\ell$, and $\boldsymbol{G}_u$ given by (13)–(18) in Table I.

3) Problem Formulation for Other Two-Hop Relay Networks: Given different types of interactions among relays, we may formulate different AF design problems by prescribing a set $\mathcal{V}$ of admissible AF matrices and imposing the constraint $\boldsymbol{w} = \mathrm{vec}(\boldsymbol{V}) \in \mathcal{V}$; cf. Sections II-C1 and II-C2. One scenario of potential interest is when the relays are partitioned into several groups; those within the same group can fully communicate with each other, while those belonging to different groups have limited message-passing capacity. This is typical when the backhaul is not powerful enough to support large-scale information sharing. In this paper, we shall

TABLE I
SUMMARY OF THE DESIGN PROBLEMS UNDER THE BF AF SCHEME

(R1BF)
$$\boldsymbol{w}^\star = \arg\max_{\boldsymbol{w}\in\mathbb{C}^{\mathcal{L}}} \min_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}} \frac{\boldsymbol{w}^H\boldsymbol{A}_{k,i}\boldsymbol{w}}{\boldsymbol{w}^H\boldsymbol{C}_{k,i}\boldsymbol{w}+1}$$
$$\text{subject to} \quad \boldsymbol{w}^H\boldsymbol{D}_\ell\boldsymbol{w} \le \bar{P}_\ell,\ \ \ell=0,1,\ldots,L,$$
$$\boldsymbol{w}^H\boldsymbol{G}_u\boldsymbol{w} \le \eta_u,\ \ u=1,\ldots,U.$$

(R1SDR)
$$\boldsymbol{W}^\star = \arg\max_{\boldsymbol{W}\in\mathbb{H}_+^{\mathcal{L}}} \min_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}} \frac{\boldsymbol{A}_{k,i}\bullet\boldsymbol{W}}{\boldsymbol{C}_{k,i}\bullet\boldsymbol{W}+1}$$
$$\text{subject to} \quad \boldsymbol{D}_\ell\bullet\boldsymbol{W} \le \bar{P}_\ell,\ \ \ell=0,1,\ldots,L,$$
$$\boldsymbol{G}_u\bullet\boldsymbol{W} \le \eta_u,\ \ u=1,\ldots,U.$$

MIMO Relay:

$$\mathcal{L} = L^2,$$
$$\boldsymbol{A}_{k,i} = P_k(\boldsymbol{f}_k^*\otimes\boldsymbol{g}_{k,i})(\boldsymbol{f}_k^*\otimes\boldsymbol{g}_{k,i})^H/\sigma_{k,i}^2, \tag{8}$$
$$\boldsymbol{C}_{k,i} = \sum_{m\neq k} P_m(\boldsymbol{f}_m^*\otimes\boldsymbol{g}_{k,i})(\boldsymbol{f}_m^*\otimes\boldsymbol{g}_{k,i})^H/\sigma_{k,i}^2 + \boldsymbol{\Sigma}\otimes(\boldsymbol{g}_{k,i}\boldsymbol{g}_{k,i}^H)/\sigma_{k,i}^2, \tag{9}$$
$$\boldsymbol{D}_0 = \left(\sum_{j=1}^G P_j(\boldsymbol{f}_j^*)(\boldsymbol{f}_j^*)^H + \boldsymbol{\Sigma}\right)\otimes\boldsymbol{I}, \tag{10}$$
$$\boldsymbol{D}_\ell = \left(\sum_{j=1}^G P_j(\boldsymbol{f}_j^*)(\boldsymbol{f}_j^*)^H + \boldsymbol{\Sigma}\right)\otimes(\boldsymbol{e}_\ell\boldsymbol{e}_\ell^H), \tag{11}$$
$$\boldsymbol{G}_u = \sum_{j=1}^G P_j(\boldsymbol{f}_j^*\otimes\boldsymbol{h}_u)(\boldsymbol{f}_j^*\otimes\boldsymbol{h}_u)^H/\sigma_u^2. \tag{12}$$

Distributed Relay:

$$\mathcal{L} = L, \tag{13}$$
$$\boldsymbol{A}_{k,i} = P_k(\boldsymbol{f}_k^*\odot\boldsymbol{g}_{k,i})(\boldsymbol{f}_k^*\odot\boldsymbol{g}_{k,i})^H/\sigma_{k,i}^2, \tag{14}$$
$$\boldsymbol{C}_{k,i} = \sum_{m\neq k} P_m(\boldsymbol{f}_m^*\odot\boldsymbol{g}_{k,i})(\boldsymbol{f}_m^*\odot\boldsymbol{g}_{k,i})^H/\sigma_{k,i}^2 + \mathrm{Diag}(|g_{k,i}^1|^2\sigma_1^2, |g_{k,i}^2|^2\sigma_2^2,\ldots,|g_{k,i}^L|^2\sigma_L^2)/\sigma_{k,i}^2, \tag{15}$$
$$\boldsymbol{D}_0 = \sum_{j=1}^G P_j\mathrm{Diag}((\boldsymbol{f}_j^*)(\boldsymbol{f}_j^*)^H) + \boldsymbol{\Sigma}, \tag{16}$$
$$\boldsymbol{D}_\ell = \left(\sum_{j=1}^G P_j\mathrm{Diag}((\boldsymbol{f}_j^*)(\boldsymbol{f}_j^*)^H) + \boldsymbol{\Sigma}\right)\odot(\boldsymbol{e}_\ell\boldsymbol{e}_\ell^H), \tag{17}$$
$$\boldsymbol{G}_u = \sum_{j=1}^G P_j(\boldsymbol{f}_j^*\odot\boldsymbol{h}_u)(\boldsymbol{f}_j^*\odot\boldsymbol{h}_u)^H/\sigma_u^2. \tag{18}$$

restrict our attention to the MIMO relay and distributed relay networks discussed in Sections II-C1 and II-C2 and provide a comprehensive performance analysis for them.

D. The SDR Technique and Approximation Bound for Single-Block Fractional QCQPs

It is well known that the single-block fractional QCQP (R1BF) formulated in the previous sub-section is NP-hard in general [10], [19]–[21]. To obtain a tractable solution method, a classic approach is to apply the SDR technique [19]. Specifically, by applying the equivalence $\boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H \iff \boldsymbol{W}\succeq\boldsymbol{0}$, $\mathrm{rank}(\boldsymbol{W}) \le 1$ and dropping the non-convex constraint $\mathrm{rank}(\boldsymbol{W}) \le 1$, we can relax (R1BF) to (R1SDR) shown in Table I. As described in [20], [21], [38], Problem (R1SDR) can be approximated to arbitrary accuracy in polynomial time. Now, define

$$\gamma(\boldsymbol{W}) = \min_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}} \frac{\boldsymbol{A}_{k,i}\bullet\boldsymbol{W}}{\boldsymbol{C}_{k,i}\bullet\boldsymbol{W}+1}$$

and let $\boldsymbol{w}^\star$ and $\boldsymbol{W}^\star$ denote the optimal solutions to (R1BF) and (R1SDR), respectively. It follows that $\gamma(\boldsymbol{W}^\star) \ge \gamma(\boldsymbol{w}^\star{\boldsymbol{w}^\star}^H)$, since (R1SDR) is a relaxation of (R1BF). Moreover, equality holds when (R1SDR) has a rank-one optimal solution. If $\mathrm{rank}(\boldsymbol{W}^\star) > 1$, then a feasible but generally sub-optimal solution $\widehat{\boldsymbol{w}}$ can be extracted from $\boldsymbol{W}^\star$ using a Gaussian randomization algorithm, such as Algorithm 1 in [9]. The solution $\widehat{\boldsymbol{w}}$ clearly satisfies $\gamma(\boldsymbol{w}^\star{\boldsymbol{w}^\star}^H) \ge \gamma(\widehat{\boldsymbol{w}}\widehat{\boldsymbol{w}}^H)$. It is thus natural to ask whether a reverse inequality (approximately) holds. Such inequality, if available, will reveal the approximation accuracy of the SDR solution. We summarize the results in [9], [20] as follows:

Proposition 1 *Let $M \ge 1$ be the total number of users in the relay network and $J = L+U+1 \ge 1$ denote the total number of constraints in (R1BF). Then, the following hold:*

a) *When $M + J \le 4$, Problem (R1SDR) has an optimal solution of rank at most one. Moreover, such a solution can be found in polynomial time.*
b) *When $M + J > 4$, let $\widehat{\boldsymbol{w}}$ be the solution returned by the Gaussian randomization algorithm after N trials. Then, with probability at least $1-(5/6)^N$, we have*

$$\gamma\left(\widehat{\boldsymbol{w}}\widehat{\boldsymbol{w}}^H\right) = \Omega\left(\frac{1}{M\log J}\right)\gamma(\boldsymbol{w}^\star{\boldsymbol{w}^\star}^H).$$

Proposition 1 implies that although the SDR-based BF AF scheme is optimal when $M + J \le 4$, its SINR performance could, in the worst case, degrade at a rate of $M\log J$ when M or J is large. As such, the SDR-based BF AF scheme may not work well in large-scale systems.

Remark 1: Recently, there has been flourishing interest in using RF wave to transfer power to those users in the system who aim at receiving energy rather than information. Such users are usually referred to as *energy receivers* (ERs). It is possible to incorporate $R \ge 1$ harvested energy constraints, one for each ER, into the design problem (R1BF); see, e.g., [39]–[41]. In this case, Proposition 1(a) still holds with $J = L+U+R+1$. However, the analogous result to Proposition 1(b) holds only when $R = 1$. This can be proven using Theorem 2 in [42]. Since our focus is on the BFA AF scheme, we shall not indulge in the proof here.

III. The Beamformed Alamouti Amplify-and-Forward Scheme

Since a multigroup multicast relay network typically aims at serving groups of users and has a number of design constraints, our discussion in the previous section suggests that the BF AF scheme could experience serious performance degradation when deployed in such network. This motivates us to search for more sophisticated AF schemes whose performance can better scale with the size of the network. One approach is to observe that the performance degradation of the BF AF scheme is due to the use of a rank-one AF weight $\widehat{\boldsymbol{W}} = \widehat{\boldsymbol{w}}\widehat{\boldsymbol{w}}^H$ to approximate the high-rank SDP solution $\boldsymbol{W}^\star$; see Proposition 1. By incorporating the Alamouti space-time code structure in the AF process, we can effectively introduce an extra DoF in the design of the AF weight, thereby allowing us to use two rank-one AF weights to approximate the SDP solution.

A. The BFA AF Scheme for MIMO Relay

To realize the above idea for an MIMO relay network, we parse the transmit signal in every two time slots as a group—i.e., $\boldsymbol{s}(m) = [s(2m), s(2m+1)]^T$—and transmit $\boldsymbol{s}(m)$ in each source-to-relay hop. Specifically, following (3), we denote the AF weighting matrix at time slot p as $\boldsymbol{V}_p = [\boldsymbol{v}_1^p, \dots, \boldsymbol{v}_\ell^p, \dots \boldsymbol{v}_L^p]$, where $\boldsymbol{v}_\ell^p = [v_{1,\ell}^p, \dots, v_{\ell,\ell}^p, \dots, v_{L,\ell}^p]^T$ and $p = 1, 2$. Then, we modify the transmit signal at relay ℓ (for every two time slots) to

$$\boldsymbol{X}_\ell(m) = \big[X_\ell(2m), X_\ell(2m+1)\big] = \sum_{c=1}^{L} [v_{\ell,c}^1, v_{\ell,c}^2]\, \mathbf{C}(\boldsymbol{r}^c(m)),$$

where $\mathbf{C} : \mathbb{C}^2 \to \mathbb{C}^{2\times 2}$ is the Alamouti space-time code given by $\mathbf{C}(\boldsymbol{x}) = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$ and $\boldsymbol{r}^\ell(m) = [r^\ell(2m), r^\ell(2m+1)]^T$ with $r^\ell(2m)$ defined in (2). To demonstrate the signal structure, we illustrate the transmit signal at each relay in Figure 3. The corresponding receive signal at user-(k,i) is given by (19). Similar to the derivation of the signal-to-noise ratio (SNR) for the Alamouti-coded signal in point-to-point communication [43, Chapter 3.3.2], we can derive the SINR expression from (19) by treating interference as white noise. Specifically, by letting $\boldsymbol{w}_1 = \mathrm{vec}(\boldsymbol{V}_1)$ and $\boldsymbol{w}_2 = \mathrm{vec}(\boldsymbol{V}_2)$, the SINR at user-$(k,i)$ can be written as

$$\frac{\boldsymbol{w}_1^H \boldsymbol{A}_{k,i} \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{A}}_{k,i} \boldsymbol{w}_2}{\boldsymbol{w}_1^H \boldsymbol{C}_{k,i} \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{C}}_{k,i} \boldsymbol{w}_2 + 1}, \tag{20}$$

where $\boldsymbol{A}_{k,i}$, $\bar{\boldsymbol{A}}_{k,i}$, $\boldsymbol{C}_{k,i}$, and $\bar{\boldsymbol{C}}_{k,i}$ are defined in (8), (21), (9), and (22), respectively; see the companion technical report [44] for details. Similarly, we can obtain $\bar{\boldsymbol{D}}_0$, $\bar{\boldsymbol{D}}_\ell$, and $\bar{\boldsymbol{G}}_u$ in (23), (24), and (25), respectively.

B. The BFA AF Scheme for Distributed Relay

The BFA AF scheme for a distributed relay network is similar to that for the MIMO relay network, except that we have $\boldsymbol{w}_1 = \mathrm{Diag}(\boldsymbol{V}_1)$ and $\boldsymbol{w}_2 = \mathrm{Diag}(\boldsymbol{V}_2)$. We relegate the derivation to the companion technical report [44].

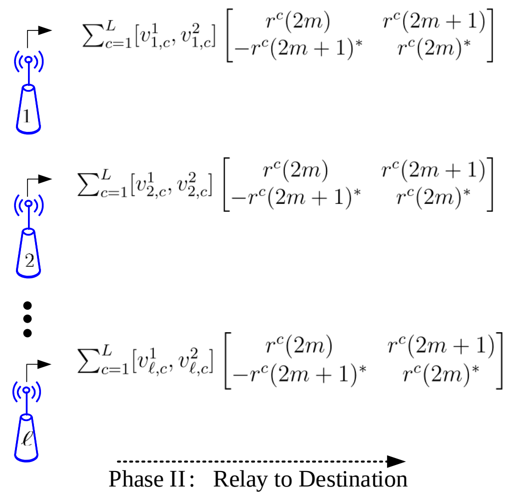

Fig. 3. The BFA AF signal structure for the MIMO relay.

C. The BFA AF Design Problem and Its SDR

Based on the preceding discussion and the ideas in Section II-C, we can formulate the BFA AF design problems associated with MIMO relay and distributed relay networks as the two-block fractional QCQP problem (R2BF) shown in Table II. Note that Problem (R2BF) reduces to Problem (R1BF) when $\bar{\boldsymbol{A}}_{k,i} = \boldsymbol{0}$, $\bar{\boldsymbol{C}}_{k,i} = \boldsymbol{0}$ for $i = 1, \dots, m_k$ and $k = 1, \dots, G$, $\bar{\boldsymbol{D}}_\ell = \boldsymbol{0}$ for $\ell = 0, 1, \dots, L$, and $\bar{\boldsymbol{G}}_u = \boldsymbol{0}$ for $u = 1, \dots, U$. As such, Problem (R2BF) is also NP-hard. Nevertheless, we can apply the SDR technique to it to obtain the relaxation (R2SDR) shown in Table II. Now, define

$$\theta(\boldsymbol{W}_1, \boldsymbol{W}_2) = \min_{\substack{i=1,\dots,m_k \\ k=1,\dots,G}} \frac{\boldsymbol{A}_{k,i} \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{A}}_{k,i} \bullet \boldsymbol{W}_2}{\boldsymbol{C}_{k,i} \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{C}}_{k,i} \bullet \boldsymbol{W}_2 + 1}. \tag{31}$$

It is immediate that if $\boldsymbol{W}$ is feasible for (R1SDR), then $(\boldsymbol{W}, \boldsymbol{0})$ is feasible for (R2SDR) with $\theta(\boldsymbol{W}, \boldsymbol{0}) = \gamma(\boldsymbol{W})$. Thus, the performance of our proposed BFA AF scheme cannot be worse than that of the BF AF scheme. Moreover, if we denote the optimal solutions to (R2BF) and (R2SDR) by $(\boldsymbol{w}_1^\star, \boldsymbol{w}_2^\star)$ and $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$, respectively, then clearly

$$\theta(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star) \geq \theta(\boldsymbol{w}_1^\star \boldsymbol{w}_1^{\star H}, \boldsymbol{w}_2^\star \boldsymbol{w}_2^{\star H}), \tag{32}$$

and equality holds whenever Problem (R2SDR) has an optimal solution of rank at most one.[3] If either $\mathrm{rank}(\boldsymbol{W}_1^\star) > 1$ or $\mathrm{rank}(\boldsymbol{W}_2^\star) > 1$, then a natural generalization of the Gaussian randomization algorithm in [9], which we develop in Algorithm 1, can be used to generate a feasible solution $(\widehat{\boldsymbol{w}}_1, \widehat{\boldsymbol{w}}_2)$ to Problem (R2BF). However, to the best of our knowledge, the approximation accuracy of the SDR solution $(\widehat{\boldsymbol{w}}_1, \widehat{\boldsymbol{w}}_2)$ has not been studied in the literature before. In the next sub-section, we shall tackle this issue and provide a provable performance guarantee for our proposed BFA AF scheme. Curiously, our

[3] By a slight abuse of terminology, we say that a feasible solution $(\boldsymbol{W}_1, \boldsymbol{W}_2)$ to Problem (R2SDR) is of rank (at most) one if both $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are of rank (at most) one.

$$
\begin{aligned}
\boldsymbol{y}_{k,i}(m) &= [y_{k,i}(2m), y_{k,i}(2m+1)] = \sum_{\ell=1}^{L} (g_{k,i}^{\ell})^* \sum_{c=1}^{L} [v_{\ell,c}^1, v_{\ell,c}^2]\, \mathbf{C}(\boldsymbol{r}^c(m)) + [\mu_{k,i}(2m), \mu_{k,i}(2m+1)] \\
&= \sum_{\ell=1}^{L} (g_k^{\ell})^* \sum_{c=1}^{L} [v_{\ell,c}^1, v_{\ell,c}^2] \begin{bmatrix} r^c(2m) & r^c(2m+1) \\ -r^c(2m+1)^* & r^c(2m)^* \end{bmatrix} + [\mu_{k,i}(2m), \mu_{k,i}(2m+1)] \\
&= \underbrace{\sum_{\ell=1}^{L} \sum_{c=1}^{L} [(g_{k,i}^{\ell})^* v_{\ell,c}^1 f_k^c, (g_{k,i}^{\ell})^* v_{\ell,c}^2 (f_k^c)^*] \begin{bmatrix} s_k(2m) & s_k(2m+1) \\ -s_k(2m+1)^* & s_k(2m)^* \end{bmatrix}}_{\text{desired signal}} \\
&\quad + \underbrace{\sum_{\ell=1}^{L} \sum_{c=1}^{L} \sum_{j \neq k} [(g_{k,i}^{\ell})^* v_{\ell,c}^1 f_j^c, (g_{k,i}^{\ell})^* v_{\ell,c}^2 (f_j^c)^*] \begin{bmatrix} s_j(2m) & s_j(2m+1) \\ -s_j(2m+1)^* & s_j(2m)^* \end{bmatrix}}_{\text{interference signal}} \\
&\quad + \underbrace{\sum_{\ell=1}^{L} \sum_{c=1}^{L} [(g_{k,i}^{\ell})^* v_{\ell,c}^1, (g_{k,i}^{\ell})^* v_{\ell,c}^2] \begin{bmatrix} n^{\ell}(2m) & n^{\ell}(2m+1) \\ -n^{\ell}(2m+1)^* & n^{\ell}(2m)^* \end{bmatrix} + [\mu_{k,i}(2m), \mu_{k,i}(2m+1)]}_{\text{noise}}. \qquad (19)
\end{aligned}
$$

Algorithm 1 Gaussian Randomization for (R2BF)

Input: optimal solution $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ to (R2SDR), number of trials $N \geq 1$
if $\text{rank}(\boldsymbol{W}_1^\star) \leq 1$ and $\text{rank}(\boldsymbol{W}_2^\star) \leq 1$ **then**
  compute $\widehat{\boldsymbol{w}}_p$ such that $\boldsymbol{W}_p^\star = \widehat{\boldsymbol{w}}_p \widehat{\boldsymbol{w}}_p^H$ for $p = 1, 2$
  return $(\widehat{\boldsymbol{w}}_1, \widehat{\boldsymbol{w}}_2)$
end if
for $n = 1, \ldots, N$ **do**
  generate $\boldsymbol{\xi}_p^n \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{W}_p^\star)$ for $p = 1, 2$
  set $\widehat{\boldsymbol{w}}_p^n = \min\{t_1^n, t_2^n\} \cdot \boldsymbol{\xi}_p^n$ for $p = 1, 2$, where

$$
t_1^n = \min_{\ell} \sqrt{\frac{\bar{P}_\ell}{\boldsymbol{D}_\ell \bullet ((\boldsymbol{\xi}_1^n)(\boldsymbol{\xi}_1^n)^H) + \bar{\boldsymbol{D}}_\ell \bullet ((\boldsymbol{\xi}_2^n)(\boldsymbol{\xi}_2^n)^H)}},
$$

$$
t_2^n = \min_{u} \sqrt{\frac{\eta_u}{\boldsymbol{G}_\ell \bullet ((\boldsymbol{\xi}_1^n)(\boldsymbol{\xi}_1^n)^H) + \bar{\boldsymbol{G}}_\ell \bullet ((\boldsymbol{\xi}_2^n)(\boldsymbol{\xi}_2^n)^H)}}
$$

  set $\theta_n = \theta\left((\widehat{\boldsymbol{w}}_1^n)(\widehat{\boldsymbol{w}}_1^n)^H, (\widehat{\boldsymbol{w}}_2^n)(\widehat{\boldsymbol{w}}_2^n)^H\right)$
end for
set $n^\star = \arg\max_{n=1,\ldots,N} \theta_n$
return $\left(\widehat{\boldsymbol{w}}_1^{n^\star}, \widehat{\boldsymbol{w}}_2^{n^\star}\right)$

---

analysis suggests that in terms of approximability by the SDR technique, the two-block optimization problem (R2BF) can be quite different from its single-block counterpart (R1BF).

D. Approximation Bound for Two-Block Fractional QCQPs

The following result, which constitutes one of the main contributions of this paper, reveals how well (R2BF) can be approximated by its SDR (R2SDR):

Theorem 1 *Let $M \geq 1$ be the total number of users in the relay network and $J = L + U + 1 \geq 1$ denote the total number of constraints in (R2BF). Then, the following hold:*

a) *If every optimal solution $(\widetilde{\boldsymbol{W}}_1, \widetilde{\boldsymbol{W}}_2)$ to (R2SDR) satisfies $\widetilde{\boldsymbol{W}}_1 \neq \mathbf{0}$ and $\widetilde{\boldsymbol{W}}_2 \neq \mathbf{0}$, then one can find in polynomial time an optimal solution to (R2SDR) of rank at most one whenever $M + J \leq 5$. Otherwise, one can find in polynomial time an optimal solution to (R2SDR) of rank at most one only when $M + J \leq 4$.*

b) *When $M + J \geq 5$, let $(\widehat{\boldsymbol{w}}_1, \widehat{\boldsymbol{w}}_2)$ be the solution returned by Algorithm 1 after N trials. Then, with probability at least $1 - (7/8)^N$, we have*

$$
\theta(\widehat{\boldsymbol{w}}_1 \widehat{\boldsymbol{w}}_1^H, \widehat{\boldsymbol{w}}_2 \widehat{\boldsymbol{w}}_2^H) \geq c \cdot \theta(\boldsymbol{w}_1^\star {\boldsymbol{w}_1^\star}^H, \boldsymbol{w}_2^\star {\boldsymbol{w}_2^\star}^H),
$$

where

$$
c = \frac{\max\left\{\frac{\omega}{6\sqrt{M}}, \frac{1}{18M}\right\}}{2\log(16J)} < 1,
$$

$$
\omega = \min_{\substack{i=1,\ldots,m_k \\ k=1,\ldots,G}} \left\{ \frac{\min\{\boldsymbol{A}_{k,i} \bullet \boldsymbol{W}_1^\star,\ \bar{\boldsymbol{A}}_{k,i} \bullet \boldsymbol{W}_2^\star\}}{\boldsymbol{A}_{k,i} \bullet \boldsymbol{W}_1^\star + \bar{\boldsymbol{A}}_{k,i} \bullet \boldsymbol{W}_2^\star} \right\}.
$$

We relegate the proof of Theorem 1 to the Appendix. Theorem 1 is significant, as it not only establishes the first approximation bound for the two-block fractional QCQP (R2BF) but also quantifies the performance gain of our proposed BFA AF scheme over the classic BF AF scheme. In particular, it shows that in the regime where $\frac{\omega}{6\sqrt{M}} \geq \frac{1}{18M}$, the approximation accuracy of the SDR solution under the BFA AF scheme is on the order of $\frac{1}{\sqrt{M}\log J}$, which has a much better scaling with respect to the number of users M than the $\frac{1}{M\log J}$-bound under the BF AF scheme; cf. Proposition 1. Even in the $\frac{\omega}{6\sqrt{M}} < \frac{1}{18M}$ regime, the approximation accuracy of the SDR solution under the BFA AF scheme, which is on the order of $\frac{1}{M\log J}$, matches that under the BF AF scheme. Although the definition of ω involves the optimal solution $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ to (R2SDR) and hence it may be difficult to determine ω *a priori*, in the next sub-section, we will present two relay scenarios in which it can be shown that $\omega = 1/2$. Consequently, Theorem 1 still yields useful information for these scenarios. In particular, we can conclude that the SINR performance of the SDR-based

TABLE II
SUMMARY OF THE DESIGN PROBLEMS UNDER THE BF ALAMOUTI AF SCHEME

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(R2BF) $(\boldsymbol{w}_1^\star, \boldsymbol{w}_2^\star)$

$$= \arg \max_{\boldsymbol{w}_1, \boldsymbol{w}_2 \in \mathbb{C}^{\mathcal{L}}} \min_{\substack{i=1,\dots,m_k \\ k=1,\dots,G}} \frac{\boldsymbol{w}_1^H \boldsymbol{A}_{k,i} \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{A}}_{k,i} \boldsymbol{w}_2}{\boldsymbol{w}_1^H \boldsymbol{C}_{k,i} \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{C}}_{k,i} \boldsymbol{w}_2 + 1}$$

$$\text{subject to} \quad \boldsymbol{w}_1^H \boldsymbol{D}_\ell \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{D}}_\ell \boldsymbol{w}_2 \le \bar{P}_\ell, \ \ell = 0, 1, \dots, L,$$

$$\boldsymbol{w}_1^H \boldsymbol{G}_u \boldsymbol{w}_1 + \boldsymbol{w}_2^H \bar{\boldsymbol{G}}_u \boldsymbol{w}_2 \le \eta_u, \ u = 1, \dots, U.$$

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(R2SDR) $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$

$$= \arg \max_{\boldsymbol{W}_1, \boldsymbol{W}_2 \in \mathbb{H}_+^{\mathcal{L}}} \min_{\substack{i=1,\dots,m_k \\ k=1,\dots,G}} \frac{\boldsymbol{A}_{k,i} \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{A}}_{k,i} \bullet \boldsymbol{W}_2}{\boldsymbol{C}_{k,i} \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{C}}_{k,i} \bullet \boldsymbol{W}_2 + 1}$$

$$\text{subject to} \quad \boldsymbol{D}_\ell \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{D}}_\ell \bullet \boldsymbol{W}_2 \le \bar{P}_\ell, \ \ell = 0, 1, \dots, L,$$

$$\boldsymbol{G}_u \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{G}}_u \bullet \boldsymbol{W}_2 \le \eta_u, \ u = 1, \dots, U.$$

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MIMO Relay:

$$\mathcal{L} = L^2,$$

$$\bar{\boldsymbol{A}}_{k,i} = P_k (\boldsymbol{f}_k \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_k \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2, \tag{21}$$

$$\bar{\boldsymbol{C}}_{k,i} = \sum_{m \neq k} P_m (\boldsymbol{f}_m \otimes \boldsymbol{g}_{k,i})(\boldsymbol{f}_m \otimes \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2 + \boldsymbol{\Sigma} \otimes (\boldsymbol{g}_{k,i} \boldsymbol{g}_{k,i}^H) / \sigma_{v,i}^2, \tag{22}$$

$$\bar{\boldsymbol{D}}_0 = \left(\sum_{j=1}^G P_j \boldsymbol{f}_j \boldsymbol{f}_j^H + \boldsymbol{\Sigma} \right) \otimes \boldsymbol{I}, \tag{23}$$

$$\bar{\boldsymbol{D}}_\ell = \left(\sum_{j=1}^G P_j \boldsymbol{f}_j \boldsymbol{f}_j^H + \boldsymbol{\Sigma} \right) \otimes (\boldsymbol{e}_\ell \boldsymbol{e}_\ell^H), \tag{24}$$

$$\bar{\boldsymbol{G}}_u = \sum_{j=1}^G P_j (\boldsymbol{f}_j \otimes \boldsymbol{h}_u)(\boldsymbol{f}_j \otimes \boldsymbol{h}_u)^H / \sigma_u^2. \tag{25}$$

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Distributed Relay:

$$\mathcal{L} = L,$$

$$\bar{\boldsymbol{A}}_{k,i} = P_k (\boldsymbol{f}_k \odot \boldsymbol{g}_{k,i})(\boldsymbol{f}_k \odot \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2, \tag{26}$$

$$\bar{\boldsymbol{C}}_{k,i} = \sum_{m \neq k} P_m (\boldsymbol{f}_m \odot \boldsymbol{g}_{k,i})(\boldsymbol{f}_m \odot \boldsymbol{g}_{k,i})^H / \sigma_{k,i}^2 + \mathrm{Diag}(|g_{k,i}^1|^2 \sigma_1^2, |g_{k,i}^2|^2 \sigma_2^2, \dots, |g_{k,i}^L|^2 \sigma_L^2) / \sigma_{k,i}^2, \tag{27}$$

$$\bar{\boldsymbol{D}}_0 = \sum_{j=1}^G P_j \mathrm{Diag}(\boldsymbol{f}_j \boldsymbol{f}_j^H) + \boldsymbol{\Sigma}, \tag{28}$$

$$\bar{\boldsymbol{D}}_\ell = \left(\sum_{j=1}^G P_j \mathrm{Diag}(\boldsymbol{f}_j \boldsymbol{f}_j^H) + \boldsymbol{\Sigma} \right) \odot (\boldsymbol{e}_\ell \boldsymbol{e}_\ell^H), \tag{29}$$

$$\bar{\boldsymbol{G}}_u = \sum_{j=1}^G P_j (\boldsymbol{f}_j \odot \boldsymbol{h}_u)(\boldsymbol{f}_j \odot \boldsymbol{h}_u)^H / \sigma_u^2. \tag{30}$$

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BFA AF scheme only degrades at a rate of $\sqrt{M} \log J$ in these scenarios.

Remark 2: The reader may observe that (R2BF) simply reduces to (R1BF) if one stacks the decision vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ into a single decision vector $\boldsymbol{w}$ and uses appropriate block-diagonal matrices to define the objective function and constraints. Thus, Proposition 1 applies to (R2BF) and (R2SDR) as well. However, the conclusions of Proposition 1 are not as sharp as those of Theorem 1. This is because the latter exploits the block structure of (R2BF).

Remark 3: In the case where $\boldsymbol{A}_{k,i} = \bar{\boldsymbol{A}}_{k,i}$ and $\boldsymbol{C}_{k,i} = \bar{\boldsymbol{C}}_{k,i}$ for $i = 1, \dots, m_k$ and $k = 1, \dots, G$; $\boldsymbol{D}_\ell = \bar{\boldsymbol{D}}_\ell$ for $\ell = 0, 1, \dots, L$; $\boldsymbol{G}_u = \bar{\boldsymbol{G}}_u$ for $u = 1, \dots, U$, Problem (R2BF) admits the following equivalent formulation:

$$\begin{aligned} \max_{\boldsymbol{W}_1, \boldsymbol{W}_2 \in \mathbb{H}_+^{\mathcal{L}}} \ & \min_{\substack{i=1,\dots,m_k \\ k=1,\dots,G}} \frac{\boldsymbol{A}_{k,i} \bullet (\boldsymbol{W}_1 + \boldsymbol{W}_2)}{\boldsymbol{C}_{k,i} \bullet (\boldsymbol{W}_1 + \boldsymbol{W}_2) + 1} \\ \text{subject to} \ & \boldsymbol{D}_\ell \bullet (\boldsymbol{W}_1 + \boldsymbol{W}_2) \le \bar{P}_\ell, \ \ell = 0, 1, \dots, L, \\ & \boldsymbol{G}_u \bullet (\boldsymbol{W}_1 + \boldsymbol{W}_2) \le \eta_u, \ u = 1, \dots, U, \\ & \mathrm{rank}(\boldsymbol{W}_1) \le 1, \ \mathrm{rank}(\boldsymbol{W}_2) \le 1. \end{aligned} \tag{33}$$

The above design problem arises in the study of the BFA scheme for multigroup multicast transmission in a network without relays [21]. By relaxing the rank constraints in (33) and noting $\boldsymbol{W} = \boldsymbol{W}_1 + \boldsymbol{W}_2 \in \mathbb{H}_+^{\mathcal{L}}$, we see that (R1SDR) and (R2SDR) are equivalent in this case. In our prior work [21], we developed a Gaussian randomization algorithm for converting an optimal solution to (R1SDR) into a feasible solution to (33) and analyzed its approximation accuracy. Our results in Theorem 1 can be viewed as a generalization of those in [21]. In particular, by noting that $((\boldsymbol{W}_1^\star + \boldsymbol{W}_2^\star)/2, (\boldsymbol{W}_1^\star + \boldsymbol{W}_2^\star)/2)$ is optimal for (R2SDR) whenever $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ is, we can achieve $\omega = 1/2$ in this case. Consequently, the approximation accuracy of the solution $(\widehat{\boldsymbol{w}}_1, \widehat{\boldsymbol{w}}_2)$ returned by Algorithm 1 is on the order of $\frac{1}{\sqrt{M} \log J}$.

E. Application to Multicasting in Relay Networks

In this sub-section, we consider single-group multicasting (i.e., $G = 1$ and $m_1 = M$) in both MIMO relay and distributed relay networks and use Theorem 1 to demonstrate how the two DoFs offered by the BFA AF scheme can lead to provable performance improvement over the BF AF scheme in these two scenarios. Before we proceed, let us note that since $G = 1$, we can drop the index k and write $\boldsymbol{A}_i, \bar{\boldsymbol{A}}_i, \boldsymbol{C}_i, \bar{\boldsymbol{C}}_i$ for $\boldsymbol{A}_{k,i}, \bar{\boldsymbol{A}}_{k,i}, \boldsymbol{C}_{k,i}, \bar{\boldsymbol{C}}_{k,i}$, respectively. Let $\phi_\ell = \arg(f_1^\ell)$ and $\boldsymbol{\phi} = [e^{j2\phi_1}, \dots, e^{j2\phi_\ell}, \dots, e^{j2\phi_L}]^T$, where $j = \sqrt{-1}$.

1) MIMO Relay: A simple calculation shows that $\boldsymbol{f} \otimes \boldsymbol{g}_i = (\boldsymbol{f}^* \otimes \boldsymbol{g}_i) \odot (\boldsymbol{\phi} \otimes \boldsymbol{e})$. It follows from (21) that

$$\boldsymbol{w}_2^H \bar{\boldsymbol{A}}_i \boldsymbol{w}_2 = (\boldsymbol{w}_2 \odot (\boldsymbol{\phi} \otimes \boldsymbol{e})^*)^H \boldsymbol{A}_i (\boldsymbol{w}_2 \odot (\boldsymbol{\phi} \otimes \boldsymbol{e})^*).$$

Similarly, we can use (25) to get

$$\boldsymbol{w}_2^H \bar{\boldsymbol{G}}_u \boldsymbol{w}_2 = (\boldsymbol{w}_2 \odot (\boldsymbol{\phi} \otimes \boldsymbol{e})^*)^H \boldsymbol{G}_u (\boldsymbol{w}_2 \odot (\boldsymbol{\phi} \otimes \boldsymbol{e})^*).$$

Next, since $\boldsymbol{\Sigma}$ is diagonal, $\boldsymbol{\Sigma}\otimes(\boldsymbol{g}_i\boldsymbol{g}_i^H)$ is block diagonal. This, together with the structure of $\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*$ and (22), yields

$$\boldsymbol{w}_2^H\bar{\boldsymbol{C}}_i\boldsymbol{w}_2=(\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*)^H\boldsymbol{C}_i(\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*).$$

Lastly, we see from (23) and (24) that $\bar{\boldsymbol{D}}_\ell$ is also block diagonal. Hence, by a similar reasoning, we have

$$\boldsymbol{w}_2^H\bar{\boldsymbol{D}}_\ell\boldsymbol{w}_2=(\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*)^H\boldsymbol{D}_\ell(\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*).$$

Since the map $\boldsymbol{w}_2\mapsto\boldsymbol{w}_2\odot(\boldsymbol{\phi}\otimes\boldsymbol{e})^*$ is invertible, the above derivation shows that Problem (R2BF) in this scenario has exactly the features discussed in Remark 3 and hence can be put into the form (33). In particular, the discussion in Remark 3 immediately leads to the conclusion that the approximation accuracy of the SDR solution under the BFA AF scheme is on the order of $\frac{1}{\sqrt{M}\log J}$.

2) Distributed Relay: Using (26), (30), and the identities $\boldsymbol{f}\odot\boldsymbol{g}_i=(\boldsymbol{f}^*\odot\boldsymbol{\phi})\odot\boldsymbol{g}_i$ and $\boldsymbol{f}\odot\boldsymbol{h}_u=(\boldsymbol{f}^*\odot\boldsymbol{\phi})\odot\boldsymbol{h}_u$, we have

$$\begin{aligned}\boldsymbol{w}_2^H\bar{\boldsymbol{A}}_i\boldsymbol{w}_2&=(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*)^H\boldsymbol{A}_i(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*),\\ \boldsymbol{w}_2^H\bar{\boldsymbol{G}}_u\boldsymbol{w}_2&=(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*)^H\boldsymbol{G}_u(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*).\end{aligned}$$

Moreover, by (27)–(29), both $\bar{\boldsymbol{C}}_i$ and $\bar{\boldsymbol{D}}_\ell$ are diagonal. This, together with the structure of $\boldsymbol{w}_2\odot\boldsymbol{\phi}^*$, implies that

$$\begin{aligned}\boldsymbol{w}_2^H\bar{\boldsymbol{C}}_i\boldsymbol{w}_2&=(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*)^H\boldsymbol{C}_i(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*),\\ \boldsymbol{w}_2^H\bar{\boldsymbol{D}}_\ell\boldsymbol{w}_2&=(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*)^H\boldsymbol{D}_\ell(\boldsymbol{w}_2\odot\boldsymbol{\phi}^*).\end{aligned}$$

Hence, using the reasoning in Section III-E1, we can see that in this scenario, the approximation accuracy of the SDR solution under the BFA AF scheme is again on the order of $\frac{1}{\sqrt{M}\log J}$.

IV. Numerical Simulations

In this section, we provide numerical simulations to compare the performance of different AF schemes and demonstrate the superiority of the proposed BFA AF scheme. Due to page limit, in Sections IV-A to IV-D, we only present the numerical results for the distributed relay network. Numerical results for the MIMO relay network can be found in the companion technical report [44]. We assume without loss of generality that each multicast group has an equal number of users (i.e., $m_k=M/G$ for $k=1,\ldots,G$). The channels $\boldsymbol{f}_k,\boldsymbol{g}_{k,i}$, where $i=1,\ldots,m_k$ and $k=1,\ldots,G$, are identically and independently distributed according to $\mathcal{CN}(\boldsymbol{0},\boldsymbol{I})$. The power of the transmitted signal at each transmitter is 0dB (i.e., $P_j=0$dB for $j=1,\ldots,G$). Each single-antenna relay has the same noise power (i.e., $\sigma_\ell^2=\sigma_{\text{ant}}^2$, where $\ell=1,\ldots,L$), and all users have the same noise power (i.e., $\sigma_{k,i}^2=\sigma_{\text{user}}^2$ for $i=1,\ldots,m_k$ and $k=1,\ldots,G$). We assume that $\sigma_{\text{ant}}^2>0$ and $\sigma_{\text{user}}^2>0$. The total power threshold for all the relays is $\bar{P}_0$; the power threshold at the ℓth relay is $\bar{P}_\ell$, where $\ell=1,\ldots,L$. For each AF scheme, 100 channel realizations were averaged to get the plots, and $1,000$ trials were made in the Gaussian randomization algorithm to generate the BF AF and BFA AF weights.

A. Worst User's SINR versus Total Power Threshold

In this simulation, we vary the total power threshold for all the relays to see the worst user's SINR performance in the relay network. For simplicity, we impose only the total power constraint. In Figure 4, we assume that there are $L=8$ single-antenna relays and $G=2$ multicast groups with a total of $M=16$ users. For both cases, we set $\sigma_{\text{ant}}^2=\sigma_{\text{user}}^2=0.25$. The objective values (obj.) of (R1SDR) and (R2SDR) serve as upper bounds for the SDR-based BF AF scheme and the SDR-based BFA AF scheme, respectively. From the figure, we see that the BFA AF scheme has significantly better SINR performance than the BF AF scheme in all power regimes.

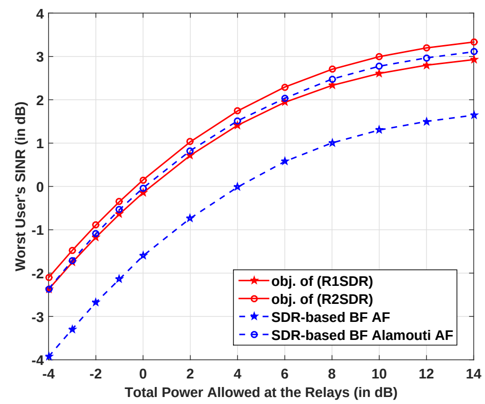

Fig. 4. Worst user's SINR versus total power threshold in the distributed relay network: $L=8$, $G=2$, $M=12$, $\sigma_{\text{ant}}^2=\sigma_{\text{user}}^2=0.25$.

B. Worst User's SINR versus Number of Users

In this simulation, we show how the worst user's SINR scales with the number of users served in the relay network. Again, we impose only the total power constraint. In Figure 5, we have $L=8$, $G=2$, $\sigma_{\text{ant}}^2=\sigma_{\text{user}}^2=0.25$, and the total power threshold is $\bar{P}_0=10$dB. From the figure, we see that the objective value of (R2SDR) is larger than that of (R1SDR), which is consistent with our earlier discussion. Moreover, it shows that the BFA AF scheme has a better SINR performance than the BF AF scheme for all values of M. This corroborates the results in Proposition 1 and Theorem 1.

C. Worst User's SINR versus Number of Per-Relay Power Constraints

In this simulation, we impose both the total power constraint and per-antenna power constraints and investigate how the worst user's SINR scales with the number of per-relay power constraints. Specifically, in Figure 6, we assume that $L=8$, $G=2$, $M=16$, the total power threshold is $\bar{P}_0=7$dB, and the per-relay power threshold is $\bar{P}_\ell=-5$dB for $\ell=1,\ldots,L$. We set $\sigma_{\text{ant}}^2=\sigma_{\text{user}}^2=0.25$ and vary the number of per-relay power constraints from 0 to L to compare the SINR performance of different AF schemes. Our results show that the BFA AF scheme outperforms the BF AF scheme. Moreover, as the number of per-relay power constraints increases, the SINRs of both schemes diverge from their SDR upper bounds and

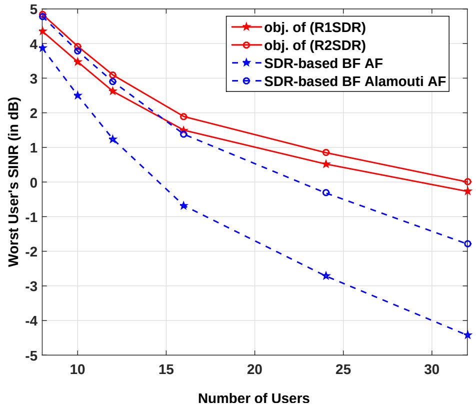

Fig. 5. Worst user's SINR versus number of users in the distributed relay network: $L = 8$, $G = 2$, $\bar{P}_0 = 10$dB, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$.

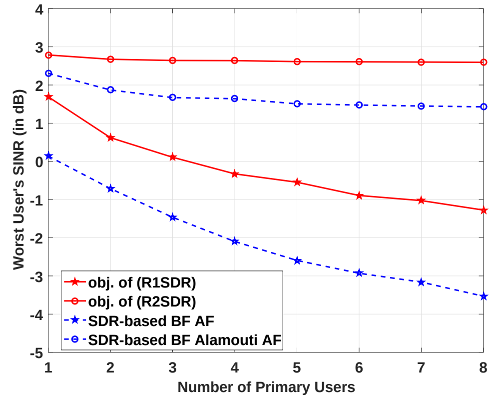

Fig. 7. Worst user's SINR versus number of primary users in the distributed CR relay network: $L = 8$, $G = 2$, $M = 12$, $\bar{P}_0 = 10$dB, $\eta_u = 3$dB for $u = 1, \ldots, U$, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$, $\sigma^2_u = 0.25$ for $u = 1, \ldots, U$.

exhibit the same scaling with respect to L. This is consistent with the results in Proposition 1 and Theorem 1.

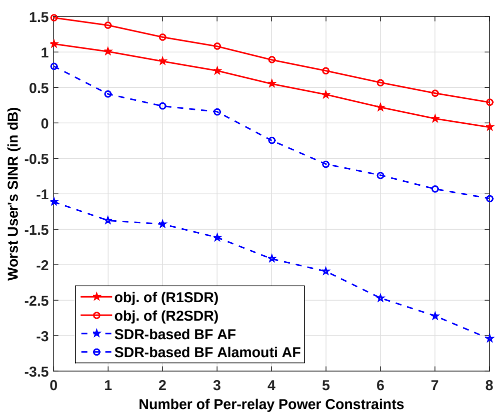

Fig. 6. Worst user's rate achieved by different AF schemes versus number of per-antenna power constraints in the distributed relay network: $L = 8$, $G = 2$, $M = 16$, $\bar{P}_0 = 7$dB, $\bar{P}_\ell = -5$dB for $\ell = 1, \ldots, L$, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$.

D. Worst User's SINR versus Number of Primary Users

In this simulation, we study how the worst user's SINR scales with the number of primary users in a distributed CR relay network. Specifically, we consider the scenario in which both the total power constraint and the primary users' interference temperature constraints are present. We assume that $L = 8$, $G = 2$, $M = 12$, the total power threshold is $\bar{P}_0 = 10$dB, and the interference power threshold is $\eta_u = 3$dB for $u = 1, \ldots, U$. We set $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$ as before and set the noise power at the primary users to be $\sigma^2_u = 0.25$ for $u = 1, \ldots, U$. Figure 7 shows the worst user's SINR as the number of primary users in the network increses. From the figure, we see that as the number of primary users increases, the SINRs of both the BF AF and BFA AF schemes diverge from their SDR upper bounds. Moreover, the BFA AF scheme shows a significantly better performance than the BF AF scheme. These results are consistent with those in Proposition 1 and Theorem 1.

E. Simulation Results for Multicasting Relay Networks

In this sub-section, we present numerical results for the multicasting scenarios (i.e., $G = 1$) discussed in Section III-E. We consider the problem formulation with a total power constraint and a single interference temperature constraint (i.e., $U = 1$). Figures 8 and 9 show how the worst user's SINR scales with the number of users. Specifically, in Figure 8, we consider an MIMO relay network with $L = 4$, $M = 16$, $\bar{P}_0 = 10$dB, $\eta_u = 3$dB, and $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = \sigma^2_u = 0.25$. The simulation results show that the objective values of (R1SDR) and (R2SDR) coincide, and that the BFA AF scheme has a better SINR performance than the BF AF scheme. This is consistent with the findings in Section III-E1. In Figure 9, we consider a distributed relay network with $L = 8$, $M = 16$, $\bar{P}_0 = 10$dB, $\eta_u = 3$dB, and $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = \sigma^2_u = 0.25$. Similar to the case of an MIMO relay network, the objective values of (R1SDR) and (R2SDR) are equal, and the SINR performance of the BFA AF scheme is better than that of the BF AF scheme. This is consistent with the findings in Section III-E2.

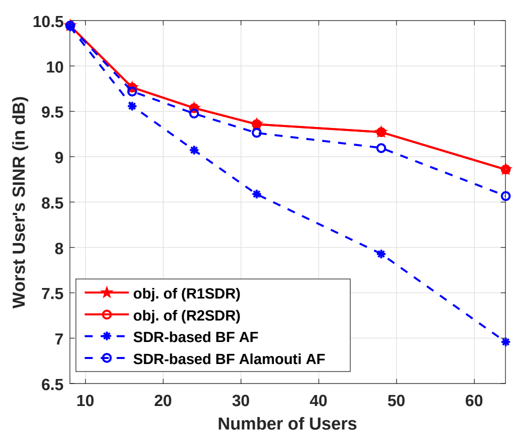

Fig. 8. Worst user's SINR versus number of users in a multicast MIMO relay network: $L = 4$, $G = 1$, $M = 16$, $\bar{P}_0 = 10$dB, $\eta_u = 3$dB, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = \sigma^2_u = 0.25$.

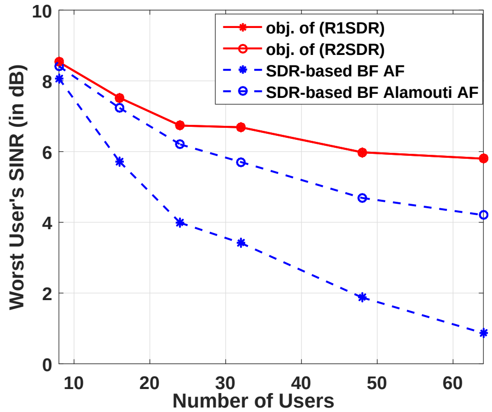

Fig. 9. Worst user's SINR versus number of users in a multicast distributed relay network: $L = 8$, $G = 1$, $M = 16$, $\bar{P}_0 = 10$dB, $b_u = 3$dB, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = \sigma^2_u = 0.25$.

F. Comparison with the Feasible Point Pursuit Algorithm

In this sub-section, we compare the proposed BFA AF scheme with the state-of-the-art algorithm for solving single-block QCQPs, namely, the Feasible Point Pursuit (FPP) algorithm. Specifically, we compare our BFA AF scheme with the FPP scheme [30] in a distributed relay network and with the FPP-SCA scheme [29] in an MIMO relay network. In the left sub-figure of Figure 10, we consider a single total power constraint in a distributed relay network with the system setting $L = 8$, $G = 1$, $M = 16$, and $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$. In the right sub-figure of Figure 10, we consider both the total power constraint and per-relay power constraints in an MIMO relay network with the system setting $L = 4$, $G = 1$, $M = 16$, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$, and $\bar{P}_0 = 3$dB. We set the per-relay power thresholds to be the same for all relays (i.e., $\bar{P}_1 = \cdots = \bar{P}_L$). The results show that the BFA AF scheme exhibits a substantial performance gain over the FPP schemes. It is worth noting that the FPP schemes have been numerically proven to outperform most of the existing algorithms for solving single-block QCQPs.

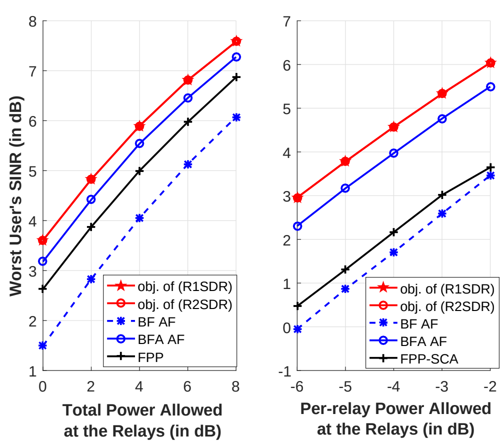

Fig. 10. Comparison with the Feasible Point Pursuit method.

G. Actual Bit Error Rate (BER) Performance

To further demonstrate the efficacy of the proposed BFA AF scheme, we study the actual coded bit error rate (BER) performance of the scenario considered in Section IV-A. Specifically, for each time slot, we simulate the actual AF process by generating $s_j(t), n^\ell(t)$ according to the SISO model in (4) and (19) and detecting and decoding $s_j(t)$ at each receivers. The resulting BERs are shown in Figure 11. To simulate the SDR bound in the BER plots, we assume that there exists an SISO channel whose SINR is equal to $\gamma(\boldsymbol{W}^\star)$ or $\theta(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$. In our simulations, we adopt a gray-coded QPSK modulation scheme and a rate-$1/3$ turbo code in [45] with a codelength of 2,880 bits. We simulate 100 code blocks for each channel realization and thus the BER reliability level is $10e{-4}$. We see that the actual BER performance of the proposed BFA AF scheme indeed outperforms the BF AF scheme at almost all power thresholds. The results are consistent with those SINR results in Figure 4 and show that the BFA AF scheme can achieve a good performance in real applications.

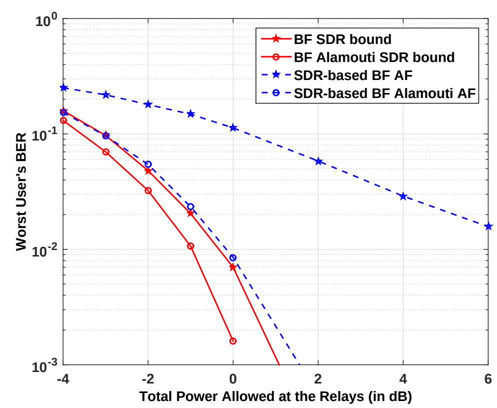

Fig. 11. Worst user's BER achieved by different AF schemes versus total power threshold in the distributed relay network: $L = 8$, $G = 2$, $M = 12$, $\sigma^2_{\text{ant}} = \sigma^2_{\text{user}} = 0.25$. A rate-$\frac{1}{3}$ turbo code with codelength 2,880 is used.

V. Conclusions

In this work, we studied the AF design problem for multigroup multicast transmission in both MIMO relay and distributed relay networks, where the goal is to maximize the worst user's SINR while satisfying generalized power constraints. We proposed a novel BFA AF scheme, which uses two beamformers to process two data symbols jointly and thus has one extra DoF over the classic BF AF scheme. We showed that the design problem corresponding to the BFA AF scheme can be formulated as a two-block fractional QCQP. We then tackled the two-block fractional QCQP using the SDR technique and established a new bound on the approximation accuracy of the SDR solution. Such a bound allows us to demonstrate in a rigorous manner how the two DoFs in the BFA AF scheme can enhance system performance and improve upon the BF AF scheme, especially in large-scale systems. In particular, we used the aforementioned bound to obtain provable guarantees on the performance of our proposed scheme for the special case of multicasting in both the MIMO relay and distributed relay networks. An interesting future direction would be to incorporate CSI uncertainty or consider other types of C-RNs with limited link capacity.

Appendix

A. Proof of Theorem 1(a)

Let $(\bar{\boldsymbol{W}}_1^\star, \bar{\boldsymbol{W}}_2^\star)$ be an optimal solution to (R2SDR). Define $\theta_{k,i}^\star = \frac{\boldsymbol{A}_{k,i}\bullet\bar{\boldsymbol{W}}_1^\star+\bar{\boldsymbol{A}}_{k,i}\bullet\bar{\boldsymbol{W}}_2^\star}{\boldsymbol{C}_{k,i}\bullet\bar{\boldsymbol{W}}_1^\star+\bar{\boldsymbol{C}}_{k,i}\bullet\bar{\boldsymbol{W}}_2^\star+1}$ for $i = 1,\dots,m_k$ and $k = 1,\dots,G$. Observe that $(\bar{\boldsymbol{W}}_1^\star, \bar{\boldsymbol{W}}_2^\star)$ is feasible for the following SDP:

$$
\begin{aligned}
\max\ & \left(\boldsymbol{A}_{1,1} - \theta_{1,1}^\star \boldsymbol{C}_{1,1}\right)\bullet \boldsymbol{W}_1 + \left(\bar{\boldsymbol{A}}_{1,1} - \theta_{1,1}^\star \bar{\boldsymbol{C}}_{1,1}\right)\bullet \boldsymbol{W}_2 \\
\text{s.t.}\ & \left(\boldsymbol{A}_{k,i} - \theta_{k,i}^\star \boldsymbol{C}_{k,i}\right)\bullet \boldsymbol{W}_1 + \left(\bar{\boldsymbol{A}}_{k,i} - \theta_{k,i}^\star \bar{\boldsymbol{C}}_{k,i}\right)\bullet \boldsymbol{W}_2 \\
& \quad = \theta_{k,i}^\star,\ \ (k,i)\neq(1,1), \\
& \boldsymbol{D}_\ell \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{D}}_\ell \bullet \boldsymbol{W}_2 \le \bar{P}_\ell,\ \ \ell = 0,1,\dots,L, \\
& \boldsymbol{G}_u \bullet \boldsymbol{W}_1 + \bar{\boldsymbol{G}}_u \bullet \boldsymbol{W}_2 \le \eta_u,\ \ u = 1,\dots,U, \\
& \boldsymbol{W}_1, \boldsymbol{W}_2 \succeq \boldsymbol{0}.
\end{aligned} \tag{34}
$$

Since $\boldsymbol{D}_0$ and $\bar{\boldsymbol{D}}_0$ are positive definite, the feasible set of Problem (34) is compact. This implies that Problem (34) has an optimal solution. Hence, by [46, Theorem 3.2], we can find in polynomial time an optimal solution $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ to Problem (34) that satisfies $\text{rank}(\boldsymbol{W}_1^\star)^2 + \text{rank}(\boldsymbol{W}_2^\star)^2 \le (M-1) + L + 1 + U = M + L + U$.

Now, note that $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ is also optimal for (R2SDR). Thus, if every optimal solution $(\widetilde{\boldsymbol{W}}_1, \widetilde{\boldsymbol{W}}_2)$ to (R2SDR) satisfies $\widetilde{\boldsymbol{W}}_1 \neq \boldsymbol{0}$ and $\widetilde{\boldsymbol{W}}_2 \neq \boldsymbol{0}$, then $\text{rank}(\widetilde{\boldsymbol{W}}_1) \ge 1$ and $\text{rank}(\widetilde{\boldsymbol{W}}_2) \ge 1$, which implies that $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ must be a rank-one solution when $M+L+U \le 4$, or equivalently, $M + J \le 5$. Otherwise, one of $\boldsymbol{W}_1^\star$ or $\boldsymbol{W}_2^\star$ could be the zero matrix. In this case, we can guarantee that the rank of $(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$ is at most one only when $M + L + U \le 3$, or equivalently, $M + J \le 4$. This completes the proof.

B. Proof of Theorem 1(b)

Consider a particular trial n in Algorithm 1 and let $\boldsymbol{\Xi}_p = (\boldsymbol{\xi}_p^n)(\boldsymbol{\xi}_p^n)^H$ for $p = 1, 2$. For any $\boldsymbol{W}_1, \boldsymbol{W}_2 \in \mathbb{H}_+^{\mathcal{L}}$, define

$$
\theta_{k,i}(\boldsymbol{W}_1, \boldsymbol{W}_2) = \frac{\boldsymbol{A}_{k,i}\bullet \boldsymbol{W}_1 + \bar{\boldsymbol{A}}_{k,i}\bullet \boldsymbol{W}_2}{\boldsymbol{C}_{k,i}\bullet \boldsymbol{W}_1 + \bar{\boldsymbol{C}}_{k,i}\bullet \boldsymbol{W}_2 + 1}
$$

and set $\theta_{k,i}^\star = \theta_{k,i}(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star)$. Furthermore, for any $\gamma \in (0,1)$ and $\delta > 1$, define the events

$$
\begin{aligned}
\mathcal{E}_{k,i}(\gamma) &= \left\{\theta_{k,i}(\boldsymbol{\Xi}_1, \boldsymbol{\Xi}_2) \le \gamma \cdot \theta_{k,i}^\star\right\}, \\
\mathcal{F}_\ell^0(\delta) &= \Big\{\boldsymbol{D}_\ell \bullet \boldsymbol{\Xi}_1 + \bar{\boldsymbol{D}}_\ell \bullet \boldsymbol{\Xi}_2 \\
&\qquad \ge \delta\left(\boldsymbol{D}_\ell \bullet \boldsymbol{W}_1^\star + \bar{\boldsymbol{D}}_\ell \bullet \boldsymbol{W}_2^\star\right)\Big\}, \\
\mathcal{F}_u^1(\delta) &= \Big\{\boldsymbol{G}_\ell \bullet \boldsymbol{\Xi}_1 + \bar{\boldsymbol{G}}_\ell \bullet \boldsymbol{\Xi}_2 \\
&\qquad \ge \delta\left(\boldsymbol{G}_\ell \bullet \boldsymbol{W}_1^\star + \bar{\boldsymbol{G}}_\ell \bullet \boldsymbol{W}_2^\star\right)\Big\}.
\end{aligned}
$$

Our goal is to show that if we choose $\gamma = \max\left\{\frac{\omega}{6\sqrt{M}}, \frac{1}{18M}\right\}$, where

$$
\omega = \min_{\substack{i=1,\dots,m_k\\ k=1,\dots,G}} \left\{\frac{\min\{\boldsymbol{A}_{k,i}\bullet \boldsymbol{W}_1^\star,\ \bar{\boldsymbol{A}}_{k,i}\bullet \boldsymbol{W}_2^\star\}}{\boldsymbol{A}_{k,i}\bullet \boldsymbol{W}_1^\star + \bar{\boldsymbol{A}}_{k,i}\bullet \boldsymbol{W}_2^\star}\right\},
$$

and $\delta = 2\log(16J)$, then

$$
\Pr\left(\bigcup_{\substack{i=1,\dots,m_k\\ k=1,\dots,G}} \mathcal{E}_{k,i}(\gamma) \cup \bigcup_{\ell=0}^{L} \mathcal{F}_\ell^0(\delta) \cup \bigcup_{u=1}^{U} \mathcal{F}_u^1(\delta)\right) \le \frac{7}{8}. \tag{35}
$$

Note that this would imply Theorem 1(b). Indeed, let $\widehat{\boldsymbol{W}}_p = (\widehat{\boldsymbol{w}}_p^n)(\widehat{\boldsymbol{w}}_p^n)^H$ for $p = 1, 2$, where $\widehat{\boldsymbol{w}}_1^n, \widehat{\boldsymbol{w}}_2^n$ are defined in Algorithm 1. Furthermore, let $\mathcal{D}$ be the event that none of the events $\mathcal{E}_{k,i}(\gamma)$ (for $i = 1,\dots,m_k$ and $k = 1,\dots,G$), $\mathcal{F}_\ell^0(\delta)$ (for $\ell = 0,1,\dots,L$), and $\mathcal{F}_u^1(\delta)$ (for $u = 1,\dots,U$) occur. Clearly, the solution $\left(\widehat{\boldsymbol{W}}_1, \widehat{\boldsymbol{W}}_2\right)$ is feasible for (R2SDR). Moreover, under the event $\mathcal{D}$, we have $t = \min\{t_1^n, t_2^n\} \ge 1/\sqrt{\delta}$, where t_1^n, t_2^n are defined in Algorithm 1. Since $\widehat{\boldsymbol{W}}_p = t^2 \boldsymbol{\Xi}_p$ for $p = 1, 2$ and $\Pr(\mathcal{D}) \ge 1/8$ by (35), we see that with probability at least $1/8$,

$$
\begin{aligned}
\theta\left(\widehat{\boldsymbol{W}}_1, \widehat{\boldsymbol{W}}_2\right) &= \min_{\substack{i=1,\dots,m_k\\ k=1,\dots,G}} \frac{\boldsymbol{A}_{k,i}\bullet (t^2\boldsymbol{\Xi}_1) + \bar{\boldsymbol{A}}_{k,i}\bullet (t^2\boldsymbol{\Xi}_2)}{\boldsymbol{C}_{k,i}\bullet (t^2\boldsymbol{\Xi}_1) + \bar{\boldsymbol{C}}_{k,i}\bullet (t^2\boldsymbol{\Xi}_2) + 1} \\
&\ge \frac{\gamma}{\delta} \cdot \theta(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star) \\
&= c \cdot \theta(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star).
\end{aligned}
$$

It follows that after N independent trials, the probability of finding an index $n \in \{1,\dots,N\}$ such that $(\widehat{\boldsymbol{w}}_1^n, \widehat{\boldsymbol{w}}_2^n)$ is feasible for (R2BF) and

$$
\theta\left(\widehat{\boldsymbol{w}}_1^n(\widehat{\boldsymbol{w}}_1^n)^H, \widehat{\boldsymbol{w}}_2^n(\widehat{\boldsymbol{w}}_2^n)^H\right) \ge c \cdot \theta\left(\boldsymbol{W}_1^\star, \boldsymbol{W}_2^\star\right)
$$

is at least $1 - (7/8)^N$, as desired.

To prove (35), we proceed in two steps. First, let us bound $\Pr\left(\bigcup_{\ell=0}^{L} \mathcal{F}_\ell^0(\delta) \cup \bigcup_{u=1}^{U} \mathcal{F}_u^1(\delta)\right)$. This can be achieved with the following proposition:

Proposition 2 *Let $\boldsymbol{Q}_1, \boldsymbol{Q}_2 \in \mathbb{H}_+^N$ be arbitrary and $\boldsymbol{\xi} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{X}_1^\star)$, $\boldsymbol{\eta} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{X}_2^\star)$ be independent. If $\boldsymbol{Q}_1 \bullet \boldsymbol{X}_1^\star + \boldsymbol{Q}_2 \bullet \boldsymbol{X}_2^\star = 0$, then $\boldsymbol{\xi}^H \boldsymbol{Q}_1 \boldsymbol{\xi} + \boldsymbol{\eta}^H \boldsymbol{Q}_2 \boldsymbol{\eta} = 0$ almost surely. Otherwise, for any $\delta \ge 2$, we have*

$$
\begin{aligned}
&\Pr\left(\boldsymbol{\xi}^H \boldsymbol{Q}_1 \boldsymbol{\xi} + \boldsymbol{\eta}^H \boldsymbol{Q}_2 \boldsymbol{\eta} \ge \delta(\boldsymbol{Q}_1 \bullet \boldsymbol{X}_1^\star + \boldsymbol{Q}_2 \bullet \boldsymbol{X}_2^\star)\right) \\
&\le 2\exp\left(-\frac{\delta}{2}\right).
\end{aligned}
$$

Before we prove Proposition 2, observe that it immediately implies that for any $\delta \ge 2$, we have $\Pr(\mathcal{F}_\ell^0(\delta)) \le 2\exp(-\delta/2)$ for $\ell = 0,1,\dots,L$ and $\Pr(\mathcal{F}_u^1(\delta)) \le 2\exp(-\delta/2)$ for $u = 1,\dots,U$. Hence, by setting $\delta = 2\log(16J)$ and applying the union bound, we obtain

$$
\Pr\left(\bigcup_{\ell=0}^{L} \mathcal{F}_\ell^0(\delta) \cup \bigcup_{u=1}^{U} \mathcal{F}_u^1(\delta)\right) \le \frac{2(L+U+1)}{16J} = \frac{1}{8}. \tag{36}
$$

Proof of Proposition 2: If $\boldsymbol{Q}_1 \bullet \boldsymbol{X}_1^\star + \boldsymbol{Q}_2 \bullet \boldsymbol{X}_2^\star = 0$, then $\mathbb{E}\left[\boldsymbol{\xi}^H \boldsymbol{Q}_1 \boldsymbol{\xi} + \boldsymbol{\eta}^H \boldsymbol{Q}_2 \boldsymbol{\eta}\right] = 0$. Since $\boldsymbol{Q}_1, \boldsymbol{Q}_2 \in \mathbb{H}_+^N$, we have $\boldsymbol{\xi}^H \boldsymbol{Q}_1 \boldsymbol{\xi} + \boldsymbol{\eta}^H \boldsymbol{Q}_2 \boldsymbol{\eta} \ge 0$. It follows that $\boldsymbol{\xi}^H \boldsymbol{Q}_1 \boldsymbol{\xi} + \boldsymbol{\eta}^H \boldsymbol{Q}_2 \boldsymbol{\eta} = 0$ almost surely.

Otherwise, let $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ be unitary matrices satisfying $(\boldsymbol{X}_1^\star)^{1/2} \boldsymbol{Q}_1 (\boldsymbol{X}_1^\star)^{1/2} = \boldsymbol{U}_1^H \boldsymbol{\Lambda}_1 \boldsymbol{U}_1$ and $(\boldsymbol{X}_2^\star)^{1/2} \boldsymbol{Q}_2 (\boldsymbol{X}_2^\star)^{1/2} = \boldsymbol{U}_2^H \boldsymbol{\Lambda}_2 \boldsymbol{U}_2$, where $\boldsymbol{\Lambda}_1 = \text{Diag}(\lambda_1,\dots,\lambda_{r_1},0,\dots,0)$ with $\lambda_1 \ge \cdots \ge \lambda_{r_1} > 0$ and $\boldsymbol{\Lambda}_2 = \text{Diag}(\mu_1,\dots,\mu_{r_2},0,\dots,0)$

with $\mu_1 \geq \cdots \geq \mu_{r_2} > 0$. Then, we have $\boldsymbol{\xi} \sim (\boldsymbol{X}_1^\star)^{1/2}\boldsymbol{U}_1^H\boldsymbol{x}$ and $\boldsymbol{\eta} \sim (\boldsymbol{X}_2^\star)^{1/2}\boldsymbol{U}_2^H\boldsymbol{y}$, where $\boldsymbol{x},\boldsymbol{y} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I})$ are independent. It follows that

$$\begin{aligned}&\Pr\left(\boldsymbol{\xi}^H\boldsymbol{Q}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{Q}_2\boldsymbol{\eta} \geq \delta(\boldsymbol{Q}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{Q}_2\bullet\boldsymbol{X}_2^\star)\right)\\ &=\Pr\left[\sum_{i=1}^{r_1}\lambda_i|x_i|^2+\sum_{i=1}^{r_2}\mu_i|y_i|^2 \geq \delta\left(\sum_{i=1}^{r_1}\lambda_i+\sum_{i=1}^{r_2}\mu_i\right)\right].\end{aligned}$$

Since $\boldsymbol{Q}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{Q}_2\bullet\boldsymbol{X}_2^\star > 0$, we have $\sum_{i=1}^{r_1}\lambda_i+\sum_{i=1}^{r_2}\mu_i > 0$. Thus, we may let $\alpha_i = \lambda_i/(\sum_{j=1}^{r_1}\lambda_j+\sum_{j=1}^{r_2}\mu_j)$ for $i=1,\ldots,r_1$ and $\alpha_{r_1+i} = \mu_i/(\sum_{j=1}^{r_1}\lambda_j+\sum_{j=1}^{r_2}\mu_j)$ for $i=1,\ldots,r_2$, so that $\sum_{i=1}^{r_1+r_2}\alpha_i = 1$. Furthermore, let $g_1,\ldots,g_{2(r_1+r_2)}$ be $2(r_1+r_2)$ independent real Gaussian random variables with mean 0 and variance 1/2. Since both $|x_i|^2$ and $|y_i|^2$ have the same distribution as $g_1^2+g_2^2$, we have

$$\begin{aligned}&\Pr\left(\boldsymbol{\xi}^H\boldsymbol{Q}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{Q}_2\boldsymbol{\eta} \geq \delta(\boldsymbol{Q}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{Q}_2\bullet\boldsymbol{X}_2^\star)\right)\\ &=\Pr\left(\sum_{i=1}^{r_1+r_2}\sum_{j=0}^{1}\alpha_i g_{2i-j}^2 \geq \delta\right).\end{aligned}$$

The above probability can be bounded using the arguments in [47, Section 2]. In particular, following the remark after the proof of [47, Proposition 2.2]), we have

$$\Pr\left(\sum_{i=1}^{r_1+r_2}\sum_{j=0}^{1}\alpha_i g_{2i-j}^2 \geq \delta\right) \leq 2\exp\left(-\frac{\delta}{2}\right).$$

This completes the proof. ■

Next, we bound $\Pr\left(\bigcup_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}}\mathcal{E}_{k,i}(\gamma)\right)$ by establishing the following proposition:

Proposition 3 *Let $\boldsymbol{A}_1,\boldsymbol{A}_2,\boldsymbol{C}_1,\boldsymbol{C}_2 \in \mathbb{H}_+^N$ be arbitrary with* $\text{rank}(\boldsymbol{A}_1) = \text{rank}(\boldsymbol{A}_2) = 1$. *Furthermore, let $\boldsymbol{\xi} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{X}_1^\star)$, $\boldsymbol{\eta} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{X}_2^\star)$ be independent. Suppose that $\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star > 0$. Then, the following hold:*

1) *For any $\gamma \in (0,\omega/2)$, where $\omega = \frac{\min\{\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star,\ \boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star\}}{\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star} > 0$,*

$$\begin{aligned}&\Pr\left(\frac{\boldsymbol{\xi}^H\boldsymbol{A}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{A}_2\boldsymbol{\eta}}{\boldsymbol{\xi}^H\boldsymbol{C}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{C}_2\boldsymbol{\eta}+1} \leq \gamma\frac{\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star}{\boldsymbol{C}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{C}_2\bullet\boldsymbol{X}_2^\star+1}\right)\\ &\leq\left(\frac{3\gamma}{\omega-2\gamma}\right)^2.\end{aligned}\tag{37}$$

2) *For any $\gamma \in (0,1/4)$,*

$$\begin{aligned}&\Pr\left(\frac{\boldsymbol{\xi}^H\boldsymbol{A}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{A}_2\boldsymbol{\eta}}{\boldsymbol{\xi}^H\boldsymbol{C}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{C}_2\boldsymbol{\eta}+1} \leq \gamma\frac{\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star}{\boldsymbol{C}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{C}_2\bullet\boldsymbol{X}_2^\star+1}\right)\\ &\leq\frac{10\gamma}{1-4\gamma}.\end{aligned}\tag{38}$$

Let us defer the proof of Proposition 3 to later and first see how it implies a bound on $\Pr\left(\bigcup_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}}\mathcal{E}_{k,i}(\gamma)\right)$. From Tables I and II, we see that $\text{rank}(\boldsymbol{A}_{k,i}) = \text{rank}(\bar{\boldsymbol{A}}_{k,i}) = 1$ for $i=1,\ldots,m_k$ and $k=1,\ldots,G$. Set $\omega_{k,i} = \frac{\min\{\boldsymbol{A}_{k,i}\bullet\boldsymbol{W}_1^\star,\ \bar{\boldsymbol{A}}_{k,i}\bullet\boldsymbol{W}_2^\star\}}{\boldsymbol{A}_{k,i}\bullet\boldsymbol{W}_1^\star+\bar{\boldsymbol{A}}_{k,i}\bullet\boldsymbol{W}_2^\star}$ and $\omega = \min_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}}\{\omega_{k,i}\}$. If $\omega > 0$, then by taking $\gamma = \frac{\omega}{6\sqrt{M}}$ in (37), we obtain

$$\Pr(\mathcal{E}_{k,i}) \leq \left(\frac{3}{6\sqrt{M}-2}\right)^2 < \frac{3}{4M}.\tag{39}$$

On the other hand, regardless of the value of ω, we can take $\gamma = \frac{1}{18M}$ in (38) to get

$$\Pr(\mathcal{E}_{k,i}) \leq \frac{10}{18M-4} < \frac{3}{4M}.\tag{40}$$

Thus, we see from (39), (40), and the union bound that when $\gamma = \max\left\{\frac{\omega}{6\sqrt{M}},\frac{1}{18M}\right\}$,

$$\Pr\left(\bigcup_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}}\mathcal{E}_{k,i}\right) \leq \sum_{\substack{i=1,\ldots,m_k\\k=1,\ldots,G}}\Pr(\mathcal{E}_{k,i}) < \frac{3}{4}.\tag{41}$$

Now, by combining (36), (41) and applying once again the union bound, we obtain (35). Thus, to complete the proof of Theorem 1(b), it remains to prove Proposition 3.

Proof of Proposition 3: The proof can be seen as a generalization of that in [21]. Let

$$\begin{aligned}\mathcal{Q} = \Pr\Bigg(&\frac{\boldsymbol{\xi}^H\boldsymbol{A}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{A}_2\boldsymbol{\eta}}{\boldsymbol{\xi}^H\boldsymbol{C}_1\boldsymbol{\xi}+\boldsymbol{\eta}^H\boldsymbol{C}_2\boldsymbol{\eta}+1}\\ &\leq\gamma\frac{\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star}{\boldsymbol{C}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{C}_2\bullet\boldsymbol{X}_2^\star+1}\Bigg)\end{aligned}$$

be the probability that we wish to bound. Let $\boldsymbol{U}_1,\boldsymbol{U}_2$ be unitary matrices satisfying $(\boldsymbol{X}_1^\star)^{1/2}\boldsymbol{A}_1(\boldsymbol{X}_1^\star)^{1/2} = \boldsymbol{U}_1^H\boldsymbol{\Lambda}_1\boldsymbol{U}_1$ and $(\boldsymbol{X}_2^\star)^{1/2}\boldsymbol{A}_2(\boldsymbol{X}_2^\star)^{1/2} = \boldsymbol{U}_2^H\boldsymbol{\Lambda}_2\boldsymbol{U}_2$, where $\boldsymbol{\Lambda}_1 = \text{Diag}(\lambda_1,0,\ldots,0)$ and $\boldsymbol{\Lambda}_2 = \text{Diag}(\mu_1,0,\ldots,0)$ (recall that $\text{rank}(\boldsymbol{A}_1) = 1$ and $\text{rank}(\boldsymbol{A}_2) = 1$). Note that $\lambda_1+\mu_1 > 0$ because $\boldsymbol{A}_1\bullet\boldsymbol{X}_1^\star+\boldsymbol{A}_2\bullet\boldsymbol{X}_2^\star > 0$ by assumption. Moreover, we have $\boldsymbol{\xi} \sim (\boldsymbol{X}_1^\star)^{1/2}\boldsymbol{U}_1^H\boldsymbol{x}^1$ and $\boldsymbol{\eta} \sim (\boldsymbol{X}_2^\star)^{1/2}\boldsymbol{U}_2^H\boldsymbol{x}^2$, where $\boldsymbol{x}^1,\boldsymbol{x}^2 \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I})$ are independent. Hence, we obtain

$$\begin{aligned}\mathcal{Q} = \Pr\Bigg(&\frac{\lambda_1|x_1^1|^2+\mu_1|x_1^2|^2}{\lambda_1+\mu_1}\\ &\leq\gamma\frac{(\boldsymbol{x}^1)^H\boldsymbol{B}_1(\boldsymbol{x}^1)+(\boldsymbol{x}^2)^H\boldsymbol{B}_2(\boldsymbol{x}^2)+1}{\boldsymbol{B}_1\bullet\boldsymbol{I}+\boldsymbol{B}_2\bullet\boldsymbol{I}+1}\Bigg),\end{aligned}\tag{42}$$

where $\boldsymbol{B}_1 = \boldsymbol{U}_1(\boldsymbol{X}_1^\star)^{1/2}\boldsymbol{C}_1(\boldsymbol{X}_1^\star)^{1/2}\boldsymbol{U}_1^H \succeq \boldsymbol{0}$ and $\boldsymbol{B}_2 = \boldsymbol{U}_2(\boldsymbol{X}_2^\star)^{1/2}\boldsymbol{C}_2(\boldsymbol{X}_2^\star)^{1/2}\boldsymbol{U}_2^H \succeq \boldsymbol{0}$. In particular, we may write $\boldsymbol{B}_1 = \boldsymbol{V}_1^H\boldsymbol{\Sigma}_1\boldsymbol{V}_1$ and $\boldsymbol{B}_2 = \boldsymbol{V}_2^H\boldsymbol{\Sigma}_2\boldsymbol{V}_2$, where $\boldsymbol{V}_1$, $\boldsymbol{V}_2$ are unitary matrices satisfying $\boldsymbol{\Sigma}_1 = \text{Diag}(\nu_1,\ldots,\nu_{r_1},0,\ldots,0)$ with $\nu_1 \geq \cdots \geq \nu_{r_1} > 0$ and $\boldsymbol{\Sigma}_2 = \text{Diag}(\zeta_1,\ldots,\zeta_{r_2},0,\ldots,0)$ with $\zeta_1 \geq \cdots \geq \zeta_{r_2} > 0$. In the sequel, we shall assume that $\sum_{j=1}^{r_1}\nu_j+\sum_{j=1}^{r_2}\zeta_j > 0$, though our proof works (and in fact simplifies considerably) even when $\sum_{j=1}^{r_1}\nu_j+\sum_{j=1}^{r_2}\zeta_j = 0$.

Let $\alpha = \lambda_1/(\lambda_1+\mu_1)$, $\beta = \mu_1/(\lambda_1+\mu_1)$, $\phi_i^1 = \nu_i/(\sum_{j=1}^{r_1}\nu_j+\sum_{j=1}^{r_2}\zeta_j)$ for $i=1,\ldots,r_1$, and $\phi_i^2 = \zeta_i/(\sum_{j=1}^{r_1}\nu_j+\sum_{j=1}^{r_2}\zeta_j)$ for $i=1,\ldots,r_2$. It is then clear that $\alpha+\beta = 1$, $\omega = \min\{\alpha,\beta\}$, and $\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i = 1$. Now, let $\boldsymbol{z}^1 = \boldsymbol{V}_1\boldsymbol{x}^1$, $\boldsymbol{z}^2 = \boldsymbol{V}_2\boldsymbol{x}^2$ and consider the following two cases:

Case 1: $\omega > 0$ and $\gamma \in (0,\omega/2)$. Let $(\boldsymbol{V}_i)_{jk}$ denote the (j,k)th element of $\boldsymbol{V}_i$. We then have the chain of inequalities (43)–(47), where (43) follows from (42) and the definitions of ω, $\boldsymbol{z}^1$, and $\boldsymbol{z}^2$; (44) follows from the inequality $(a+b)^2 \leq 2(a^2+b^2)$, which is valid for any $a,b \in \mathbb{R}$; (45) follows from the fact that $\boldsymbol{V}_i$ is unitary and hence $|(\boldsymbol{V}_i)_{j1}| \leq 1$; (46) follows from the fact that $\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i = 1$; (47) follows from the fact that x_1^1, x_1^2

$$\mathcal{Q} \leq \Pr\left[\omega(|x_1^1|^2+|x_1^2|^2) \leq \gamma\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left|(\boldsymbol{V}_i)_{j1}x_1^i+\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2+1\right)\right] \tag{43}$$

$$\leq \Pr\left[\omega(|x_1^1|^2+|x_1^2|^2) \leq 2\gamma\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left(\left|(\boldsymbol{V}_i)_{j1}x_1^i\right|^2+\left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2\right)+\frac{1}{2}\right)\right] \tag{44}$$

$$\leq \Pr\left[\omega(|x_1^1|^2+|x_1^2|^2) \leq 2\gamma\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left(|x_1^i|^2+\left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2\right)+\frac{1}{2}\right)\right] \tag{45}$$

$$\leq \Pr\left[|x_1^1|^2+|x_1^2|^2 \leq \frac{2\gamma}{\omega-2\gamma}\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2+\frac{1}{2}\right)\right] \tag{46}$$

$$\leq \frac{2\gamma^2}{(\omega-2\gamma)^2}\mathbb{E}\left[\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2+\frac{1}{2}\right)^2\right]. \tag{47}$$

are standard complex Gaussian random variables and hence for all $t>0$, $\Pr(|x_1^1|^2+|x_1^2|^2 \leq t) = 1-(t+1)e^{-t} \leq t^2/2$. Note that the expectation in (47) is taken with respect to the random variables x_k^i for $i=1,2$ and $k=2,\ldots,N$. Now, define $W_j^i = \left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2$ for $i=1,2$ and $j=2,\ldots,r_i$. We compute

$$\begin{aligned}
&\mathbb{E}\left[\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^iW_j^i+\frac{1}{2}\right)^2\right] \\
&= \sum_{i=1}^{2}\mathbb{E}\left[\left(\sum_{j=1}^{r_i}\phi_j^iW_j^i\right)^2\right]+\frac{1}{4}+\sum_{i=1}^{2}\mathbb{E}\left[\sum_{j=1}^{r_i}\phi_j^iW_j^i\right] \\
&\quad + 2\mathbb{E}\left[\left(\sum_{j=1}^{r_1}\phi_j^1W_j^1\right)\left(\sum_{j=1}^{r_2}\phi_j^2W_j^2\right)\right].
\end{aligned} \tag{48}$$

Let us consider each term on the right-hand side of the above expression separately. First, we have (49) at the top of the next page. Using the fact that $\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i = 1$ and $\boldsymbol{V}_i$ is unitary, we deduce that

$$\sum_{i=1}^{2}\mathbb{E}\left[\left(\sum_{j=1}^{r_i}\phi_j^iW_j^i\right)^2\right] \leq \sum_{i=1}^{2}\sum_{j,k=1}^{r_i}\phi_j^i\phi_k^i \leq 2. \tag{50}$$

Next, we compute

$$\begin{aligned}
\sum_{i=1}^{2}\mathbb{E}\left[\sum_{j=1}^{r_i}\phi_j^iW_j^i\right] &= \sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\sum_{k,l=2}^{N}(\boldsymbol{V}_i)_{jk}(\boldsymbol{V}_i)_{jl}^*\mathbb{E}[x_k^i(x_l^i)^*] \\
&= \sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\sum_{k=2}^{N}|(\boldsymbol{V}_i)_{jk}|^2 \leq 1.
\end{aligned} \tag{51}$$

Finally, since W_j^1 and W_k^2 are independent for $j=1,\ldots,r_1$ and $k=1,\ldots,r_2$, it follows from (51) that

$$\mathbb{E}\left[\left(\sum_{j=1}^{r_1}\phi_j^1W_j^1\right)\left(\sum_{j=1}^{r_2}\phi_j^2W_j^2\right)\right] \leq \frac{1}{4}. \tag{52}$$

Upon substituting (50)–(52) into (48), we obtain

$$\mathbb{E}\left[\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^iW_j^i+\frac{1}{2}\right)^2\right] \leq \frac{15}{4}.$$

This, together with (47), yields

$$\mathcal{Q} \leq \frac{15\gamma^2}{2(\omega-2\gamma)^2} < \left(\frac{3\gamma}{\omega-2\gamma}\right)^2.$$

Case 2: $\gamma \in (0,1/4)$. Without loss of generality, suppose that $0 \leq \beta \leq \alpha \leq 1$. Then, we have $\alpha \geq 1/2$. It follows from (42) that

$$\mathcal{Q} \leq \Pr\left[|x_1^1|^2 \leq 2\gamma\left(\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i|z_j^i|^2+1\right)\right].$$

Since x_1^1 is a standard complex Gaussian random variable, we have $\Pr(|x_1^1|^2 \leq t) = 1-e^{-t} \leq t$ for all $t>0$. Thus, using similar arguments as in the derivation of the chain of inequalities (43)–(47), we obtain

$$\begin{aligned}
\mathcal{Q} \leq \frac{4\gamma}{1-4\gamma}\mathbb{E}&\left[\sum_{j=1}^{r_2}\phi_j^2\left|(\boldsymbol{V}_2)_{j1}x_1^2\right|^2\right. \\
&\left.+\sum_{i=1}^{2}\sum_{j=1}^{r_i}\phi_j^i\left|\sum_{k=2}^{N}(\boldsymbol{V}_i)_{jk}x_k^i\right|^2+\frac{1}{2}\right],
\end{aligned}$$

where the expectation is taken with respect to the random variables x_1^2 and x_k^i for $i=1,2$ and $k=2,\ldots,N$. Now, a simple calculation gives $\sum_{j=1}^{r_2}\phi_j^2\left|(\boldsymbol{V}_2)_{j1}\right|^2\mathbb{E}[|x_1^2|^2] \leq 1$, which, together with (51), implies that $\mathcal{Q} \leq \frac{10\gamma}{1-4\gamma}$. This completes the proof of Proposition 3. ∎

$$\mathbb{E}\left[\left(\sum_{j=1}^{r_i}\phi_j^i W_j^i\right)^2\right]=\mathbb{E}\left[\sum_{j,k=1}^{r_i}\phi_j^i\phi_k^i W_j^i W_k^i\right] \tag{49}$$

$$=\sum_{j,k=1}^{r_i}\phi_j^i\phi_k^i\sum_{l,m=2}^{N}\sum_{p,q=2}^{N}\Big((\boldsymbol{V}_i)_{jl}(\boldsymbol{V}_i)_{jm}^*(\boldsymbol{V}_i)_{kp}(\boldsymbol{V}_i)_{kq}^*\times\mathbb{E}[x_l^i(x_m^i)^*x_p^i(x_q^i)^*]\Big)$$

$$=\sum_{j,k=1}^{r_i}\phi_j^i\phi_k^i\left(\sum_{l=2}^{N}|(\boldsymbol{V}_i)_{jl}|^2|(\boldsymbol{V}_i)_{kl}|^2\sum_{l,m=2}^{N}|(\boldsymbol{V}_i)_{jl}|^2|(\boldsymbol{V}_i)_{km}|^2\right).$$

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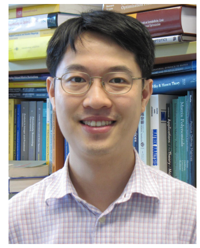

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