# ROBUST ARTIFICIAL NOISE-AIDED TRANSMIT OPTIMIZATION FOR ACHIEVING SECRECY AND ENERGY HARVESTING

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## ABSTRACT

Consider a wireless scenario in which a multi-antenna transmitter wants to send a confidential message to a single-antenna information receiver (IR) while transferring wireless energy to a number of multi-antenna energy receivers (ERs). In order to keep the ERs from retrieving the confidential message, an artificial noise (AN)-aided physical-layer secrecy approach is employed at the transmitter. The AN has dual purpose: First, it can interfere with the ERs' information receptions and thus help improve security. Secondly, it provides wireless energy for the ERs to harvest. Assuming imperfect channel state information at the transmitter, we jointly optimize the covariances of confidential information and AN such that the secrecy rate at the IR is maximized, while each ER receives a prescribed amount of wireless energy. Although this secrecy-rate maximization problem is non-convex, we show that it can be handled by solving a sequence of convex optimization problems. Numerical results are provided to demonstrate the efficacy of the proposed design.

***Index Terms*—** Physical-layer security, energy harvesting, artificial noise, convex optimization

## 1. INTRODUCTION

Traditionally, electric power transfer and wireless information transmission are independently investigated in the fields of power engineering and communication engineering. Recently, there is growing interest in combining these two topics together to realize simultaneous wireless information and power transfer (SWIPT) [1–10]. The idea is that radio-frequency (RF) signals can not only convey information for the receiver to decode, but also naturally provides electromagnetic energy for receivers to harvest. The latter is particularly important for prolonging receivers' operation time when no sustainable power supply is available at the receiver side — e.g., the receivers are sensors randomly scattered on the battlefield. While the concept of SWIPT is quite simple, it gives rise to a new capacity-energy paradigm for wireless transmission. This has thus triggered lots of recent research endeavors on characterizing capacity-energy tradeoffs for SWIPT under various scenarios, such as point-to-point single-input and single-output (SISO) channels [1, 2], multiuser multiple access channels (MAC) [3], multiuser multi-input and single/multi-output (MISO/MIMO) broadcast channels [4–6], relay channels [9, 10] and wiretap channels [6–8]. Among the various studies, the work in [6] proposed an interesting SWIPT scenario, in which the transmitter or base station (BS) attempts to send confidential information to an information receiver (IR) while transferring wireless energy to a group of energy receivers (ERs). To prevent the ERs from eavesdropping the information, a physical-layer (PHY) secrecy approach is employed at the BS to achieve perfectly secure transmission to IR. PHY secrecy is a means of providing confidentiality at PHY by exploiting the channel capacity difference between the legitimate channel and the eavesdropping channels [11]. The study of PHY secrecy can be traced back to Wyner's seminal work [12] in the 1970s, and recently this kind of approach has gained renewed interest [13–16]. This may partly be attributed to recent advances in multi-antenna techniques, by which one can either employ transmit beamforming or intentionally send spatially selective artificial noise (AN) [13, 15, 16] to degrade eavesdroppers' receptions.

In this work, we consider a similar SWIPT setting as [6]; i.e. one multi-antenna BS sends confidential information to a single-antenna IR while transferring wireless energy to a group of ERs. Different from [6], where ERs are assumed to be single-antenna and the BS has perfect channel state information (CSI) of all the receivers, herein we focus on *multi-antenna* ERs with their CSI *imperfectly* known at the BS. Under the considered setting, our goal is to maximize an achievable secrecy rate for the IR while providing certain amount of energy to the ERs by jointly optimizing the confidential information covariance and the artificial noise (AN) covariance at the BS. This secrecy rate maximization (SRM) problem is non-convex. To handle it, we first reformulate the SRM problem as a two-level optimization problem. Then, we show that the outer problem can be handled by performing an one-dimensional search over a unit interval, while the inner problem admits a tight convex relaxation and hence can be exactly solved in an efficient manner. The crux of our approach is to establish the existence of a rank-one optimal information covariance to the convex relaxation of the inner problem.

We now briefly review some works that are related to our approach. PHY secrecy with energy harvesting has been considered in [6–8]. In particular, the works [7, 8] considered a scenario in which the transmitter/jammer is an energy harvester, but the transmission from the transmitter to the destination does not involve any energy transfer. The work that is most relevant to ours is [6]. As mentioned before, the work [6] focuses on MISO ERs and perfect CSI at the BS, while here we consider a more general setting — MIMO ERs and imperfect CSI at the BS. Finally, we should also mention that the approach developed in this paper is reminiscent of that in our previous work [15], where no energy harvester is present. However, due to the additional energy harvesting constraints, the approach in [15] cannot be directly applied to the problem here.

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## 2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a scenario of simultaneous wireless information and power transfer, in which a multi-antenna transmitter sends private information to a single-antenna information receiver (IR) while transferring wireless energy to multiple multi-antenna energy receivers (ER). To prevent the ERs from eavesdropping the private information, an artificial noise (AN)-aided PHY secrecy approach is employed at the transmitter. Specifically, the transmit baseband signal takes the following form:

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{z}(t), \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{C}^{N_t}$ conveys the coded confidential information intended for the IR, which is Gaussian distributed with mean zero and covariance $\mathbf{W} \in \mathbb{H}_+^{N_t}$, i.e., $\mathbf{s}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{W})$ [11]; $N_t$ denotes the number of transmit antennas; $\mathbf{z}(t) \in \mathbb{C}^{N_t}$ is the superimposed artificial noise, which is assumed to be Gaussian distributed and independent of $\mathbf{s}(t)$, i.e., $\mathbf{z}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ with $\mathbf{\Sigma} \in \mathbb{H}_+^{N_t}$ being the covariance matrix of $\mathbf{z}(t)$. Herein, the AN has dual purpose: On the one hand, it acts as interference to cripple the ERs' information reception, just as that in traditional physical-layer security. On the other hand, it provides a source of wireless energy for the ERs.

Assuming frequency-flat and quasi-static fading channels, the received signals at the IR and the $k$th ER are given by

$$y_{\rm I}(t) = \mathbf{h}^H \mathbf{x}(t) + n(t), \tag{2a}$$

$$\mathbf{y}_{{\rm E},k}(t) = \mathbf{G}_k^H \mathbf{x}(t) + \mathbf{v}_k(t), \quad k \in \mathcal{K}, \tag{2b}$$

where $\mathcal{K} \triangleq \{1, \ldots, K\}$; $\mathbf{h} \in \mathbb{C}^{N_t}$ and $\mathbf{G}_k \in \mathbb{C}^{N_t \times N_{e,k}}$ are channel matrices from the transmitter to IR and to the $k$th ER, respectively; $N_{e,k}$ is the number of receive antennas at ER $k$; $n(t) \in \mathbb{C}$ and $\mathbf{v}_k(t) \in \mathbb{C}^{N_{e,k}}$ are complex Gaussian noise, whose distributions follow $n(t) \sim \mathcal{CN}(0, \sigma_{\rm IR}^2)$ and $\mathbf{v}_k(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_{e,k}^2 \mathbf{I})$, respectively.

According to the signal model (2), an achievable secrecy rate at the IR can be calculated as [11]

$$R_s = C_{\rm IR}(\mathbf{W}, \mathbf{\Sigma}) - \max_{k \in \mathcal{K}} C_{e,k}(\mathbf{W}, \mathbf{\Sigma}), \tag{3}$$

where $C_{\rm IR}(\mathbf{W}, \mathbf{\Sigma}) = \log\left(1 + \frac{\mathbf{h}^H \mathbf{W} \mathbf{h}}{\sigma_{\rm IR}^2 + \mathbf{h}^H \mathbf{\Sigma} \mathbf{h}}\right)$ and $C_{e,k}(\mathbf{W}, \mathbf{\Sigma}) = \log\det(\mathbf{I} + (\sigma_{e,k}^2 \mathbf{I} + \mathbf{G}_k^H \mathbf{\Sigma} \mathbf{G}_k)^{-1} \mathbf{G}_k^H \mathbf{W} \mathbf{G}_k)$. The harvested energy (normalized by the baseband symbol duration) at the $k$th ER is [4]

$$Q_k(\mathbf{W}, \mathbf{\Sigma}) = \zeta_k {\rm Tr}(\mathbf{G}_k^H (\mathbf{W} + \mathbf{\Sigma}) \mathbf{G}_k), \ \forall\, k \in \mathcal{K}, \tag{4}$$

where $0 < \zeta_k < 1$ denotes the energy harvesting efficiency at ER $k$.

In this work, we assume that $\mathbf{h}$ is perfectly known at the transmitter, while $\mathbf{G}_k, \forall\, k \in \mathcal{K}$ is imperfectly known. The imperfect channel $\mathbf{G}_k$ is modeled by a deterministically norm-bounded spherical model [15, 16]; i.e.,

$$\mathbf{G}_k \in \mathcal{B}_k \triangleq \{\bar{\mathbf{G}}_k + \Delta\mathbf{G}_k \mid \|\Delta\mathbf{G}_k\|_F \le \epsilon_k\} \tag{5}$$

for some known constant $\epsilon_k \ge 0$, $\forall\, k \in \mathcal{K}$, where $\bar{\mathbf{G}}_k$ and $\Delta\mathbf{G}_k$ are the presumed CSI and the associated CSI error of $\mathbf{G}_k$, respectively. We remark that the perfect CSI case can be recovered from (5) by setting $\epsilon_k = 0$, and all the results in the subsequent development still hold.

Based on the above setting, our problem of interest is to maximize the IR's secrecy rate while transferring certain amount of energy to the ERs by jointly designing the information covariance $\mathbf{W}$ and the AN covariance $\mathbf{\Sigma}$; i.e.,

$$R_s^\star = \max_{\mathbf{W}, \mathbf{\Sigma}} \ \left\{C_{\rm IR}(\mathbf{W}, \mathbf{\Sigma}) - \max_{k \in \mathcal{K}} \max_{\mathbf{G}_k \in \mathcal{B}_k} C_{e,k}(\mathbf{W}, \mathbf{\Sigma})\right\} \tag{6a}$$

$$\text{s.t.} \ \min_{\mathbf{G}_k \in \mathcal{B}_k} \ \zeta_k {\rm Tr}(\mathbf{G}_k^H (\mathbf{W} + \mathbf{\Sigma}) \mathbf{G}_k) \ge \eta_k, \ \forall k \in \mathcal{K}, \tag{6b}$$

$${\rm Tr}(\mathbf{W} + \mathbf{\Sigma}) \le P, \ \mathbf{W} \succeq \mathbf{0}, \ \mathbf{\Sigma} \succeq \mathbf{0}, \tag{6c}$$

$$\max_{\mathbf{\Phi}_l \in \mathbf{\Xi}_l} \ {\rm Tr}(\mathbf{\Phi}_l (\mathbf{W} + \mathbf{\Sigma})) \le \rho_l, \ l = 1, \ldots, L, \tag{6d}$$

where $\mathcal{B}_k$ is defined in (5); $P > 0$, $\eta_k > 0$, and $\rho_l \ge 0$, $\forall k, l$ are given constants and $\mathbf{\Xi}_l$ denotes a norm-bounded spherical uncertainty set for the Hermitian positive semidefinite (PSD) matrix $\mathbf{\Phi}_l \in \mathbb{H}_+^{N_t}$ (to be specified shortly). In particular, (6a) corresponds to the worst secrecy rate when taking into account of all possible $\mathbf{G}_k \in \mathcal{B}_k$ for all $k$; (6b) is the energy harvesting constraint, which ensures that the provision of energy for ER $k$ is at least $\eta_k$; (6c) is the standard total power constraint; (6d) is a robust covariance constraint with

$$\mathbf{\Xi}_l \triangleq \{\bar{\mathbf{\Phi}}_l + \Delta\mathbf{\Phi}_l \in \mathbb{H}_+^{N_t} \mid \|\Delta\mathbf{\Phi}_l\|_F \le \delta_l\}, \ \forall l \tag{7}$$

for some known PSD matrix $\bar{\mathbf{\Phi}}_l \in \mathbb{H}_+^{N_t}$ and constant $\delta_l \ge 0$, $\forall\, l$. Generally speaking, the inclusion of (6d) is optional; herein we incorporate (6d) into the design to accommodate some additional design requirements arising from certain specific applications. For example, (6d) may represent per-antenna power constraints by setting $\delta_l = 0$ and $\bar{\mathbf{\Phi}}_l = \boldsymbol{e}_l \boldsymbol{e}_l^H$ for $l = 1, \ldots, N_t$, where $\boldsymbol{e}_l \in \mathbb{R}^{N_t}$ is the $l$th column vector of $\mathbf{I}_{N_t}$. Also, (6d) may represent a robust interference temperature constraint in cognitive radio systems [17]. Readers are referred to [15] for more details on the physical meaning of (6d).

It can be verified that problem (6) is a non-convex semi-infinite optimization problem. In the next section, we will develop a tractable solution to (6) through convex relaxation.

*Remark* 1. As an alternative formulation to (6), one can maximize the weighted sum of harvested energy subject to a minimum secrecy rate constraint for the IR and a total power constraint at the BS. This energy maximization problem is closely related to (6) and can be solved using the same approach described in Sec. 3. Due to page limit, we will only focus on problem (6).

## 3. A TRACTABLE APPROACH TO PROBLEM (6)

In this section, we develop a tractable approach to solving problem (6). To this end, we will first reformulate problem (6) into a two-level optimization problem, and then show that it can be handled by solving a sequence of convex optimization problems.

### 3.1. A Two-Level Reformulation of (6)

Let us introduce a slack variable $\tau$ to rewrite problem (6) as

$$R_s^\star = \max_{\mathbf{W}, \mathbf{\Sigma}, \tau} \ \{C_{\rm IR}(\mathbf{W}, \mathbf{\Sigma}) + \log(\tau)\} \tag{8a}$$

$$\text{s.t.} \ \max_{\mathbf{G}_k \in \mathcal{B}_k} \ C_{e,k}(\mathbf{W}, \mathbf{\Sigma}) \le \log(1/\tau), \ \forall\, k \in \mathcal{K}, \tag{8b}$$

$$\text{(6b)} - \text{(6d) satisfied.} \tag{8c}$$

This can be further rewritten as a two-level optimization problem; i.e., the outer problem with respect to (w.r.t.) the variable $\tau$

$$R_s^\star = \max_{\tau} \ \log(1 + \Gamma(\tau)) + \log(\tau) \tag{9a}$$

$$\text{s.t.} \ \tau_{\min} \le \tau \le 1, \tag{9b}$$

where $\tau_{\min} = (1 + P\|\mathbf{h}\|^2/\sigma_{\rm IR}^2)^{-1}$, and the inner problem that calculates $\Gamma(\tau)$ for a fixed $\tau$:

$$\Gamma(\tau) = \max_{\mathbf{W},\mathbf{\Sigma}} \frac{\mathbf{h}^H\mathbf{W}\mathbf{h}}{\sigma_{\rm IR}^2 + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h}} \tag{10a}$$

$$\text{s.t.} \max_{\mathbf{G}_k \in \mathcal{B}_k} C_{e,k}(\mathbf{W},\mathbf{\Sigma}) \le \log(1/\tau),\ \forall\, k \in \mathcal{K}, \tag{10b}$$

$$\min_{\mathbf{G}_k \in \mathcal{B}_k} \zeta_k \mathrm{Tr}(\mathbf{G}_k^H(\mathbf{W}+\mathbf{\Sigma})\mathbf{G}_k) \ge \eta_k,\ \forall\, k \in \mathcal{K}, \tag{10c}$$

$$\mathrm{Tr}(\mathbf{W}+\mathbf{\Sigma}) \le P,\ \mathbf{W} \succeq \mathbf{0},\ \mathbf{\Sigma} \succeq \mathbf{0}, \tag{10d}$$

$$\max_{\mathbf{\Phi}_l \in \mathbf{\Xi}_l} \mathrm{Tr}(\mathbf{\Phi}_l(\mathbf{W}+\mathbf{\Sigma})) \le \rho_l,\ l = 1,\ldots,L. \tag{10e}$$

In (9b), the upper bound on $\tau$ is due to (8b) and the lower bound $\tau_{\min}$ can be deduced as follows: Since $R_s^\star \ge 0$, it follows from (8a) that $\tau \ge (1 + \frac{\mathbf{h}^H\mathbf{W}\mathbf{h}}{\sigma_{\rm IR}^2 + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h}})^{-1} \ge (1 + \frac{\mathbf{h}^H\mathbf{W}\mathbf{h}}{\sigma_{\rm IR}^2})^{-1} \ge (1 + \frac{\lambda_{\max}(\mathbf{W})\|\mathbf{h}\|^2}{\sigma_{\rm IR}^2})^{-1} \ge (1 + \frac{\mathrm{Tr}(\mathbf{W})\|\mathbf{h}\|^2}{\sigma_{\rm IR}^2})^{-1} \ge (1 + P\|\mathbf{h}\|^2/\sigma_{\rm IR}^2)^{-1} \triangleq \tau_{\min}$, where the last inequality is due to the total power constraint.

The outer problem (9) is a single-variable optimization problem with a bounded interval constraint $[\tau_{\min},\ 1]$, which can be handled by performing an one-dimensional line search, provided that $\Gamma(\tau)$ can be evaluated at any feasible $\tau$. Therefore, in the sequel, we will focus on the inner problem (10).

### 3.2. A Tight Convex Relaxation of Problem (10)

Our approach to solving problem (10) is to first derive a convex relaxation of (10), and then show that the relaxation is tight. This would then imply that $\Gamma(\tau)$ is efficiently computable.

Let us consider (10c) first. Upon noting that $\mathrm{Tr}(\mathbf{G}_k^H(\mathbf{W} + \mathbf{\Sigma})\mathbf{G}_k) = \boldsymbol{g}_k^H(\mathbf{I} \otimes (\mathbf{W}+\mathbf{\Sigma}))\boldsymbol{g}_k$ with $\boldsymbol{g}_k \triangleq \mathrm{vec}(\mathbf{G}_k)$, we have

$$(10c) \Longleftrightarrow \zeta_k \boldsymbol{g}_k^H(\mathbf{I} \otimes (\mathbf{W}+\mathbf{\Sigma}))\boldsymbol{g}_k \ge \eta_k,\ \forall\, \mathbf{G}_k \in \mathcal{B}_k,\ \forall k. \tag{11}$$

The inequality on the right-hand side (RHS) of (11) is quadratic in $\boldsymbol{g}_k$, which lies in a bounded sphere (cf. (5)). Hence, by invoking the $\mathcal{S}$-lemma [18], the RHS of (11) is equivalent to a system of linear matrix inequalities (LMIs); i.e.,

$$(10c) \Longleftrightarrow \mathbf{F}_k(\mathbf{W},\mathbf{\Sigma},\xi_k) \succeq \mathbf{0} \ \text{ for some } \xi_k \ge 0,\ \forall k \in \mathcal{K}, \tag{12}$$

where $\mathbf{F}_k(\mathbf{W},\mathbf{\Sigma},\xi_k) \triangleq \left[\mathbf{I},\ \bar{\boldsymbol{g}}_k\right]^H \left(\mathbf{I} \otimes (\mathbf{W}+\mathbf{\Sigma})\right)\left[\mathbf{I},\ \bar{\boldsymbol{g}}_k\right] + \mathrm{Diag}(\xi_k\mathbf{I},\ \ -\eta_k/\zeta_k - \xi_k\epsilon_k^2)$ and $\bar{\boldsymbol{g}}_k = \mathrm{vec}(\bar{\mathbf{G}}_k)$.

Next we proceed with (10e). Substituting (7) into (10e) yields

$$(10e) \Leftrightarrow \mathrm{Tr}(\bar{\mathbf{\Phi}}_l(\mathbf{W}+\mathbf{\Sigma})) + \max_{\|\Delta\mathbf{\Phi}_l\|_F \le \delta_l} \mathrm{Tr}(\Delta\mathbf{\Phi}_l(\mathbf{W}+\mathbf{\Sigma})) \le \rho_l,\ \forall l.$$

By the Cauchy-Schwarz inequality, we have $\mathrm{Tr}(\Delta\mathbf{\Phi}_l(\mathbf{W} + \mathbf{\Sigma})) \le \|\Delta\mathbf{\Phi}_l\|_F\|\mathbf{W}+\mathbf{\Sigma}\|_F$. Hence,

$$(10e) \Longleftrightarrow \mathrm{Tr}(\bar{\mathbf{\Phi}}_l(\mathbf{W}+\mathbf{\Sigma})) + \delta_l\|\mathbf{W}+\mathbf{\Sigma}\|_F \le \rho_l,\ \forall l. \tag{13}$$

The RHS of (13) is a second-order cone constraint, which is convex in $(\mathbf{W},\mathbf{\Sigma})$.

Finally, to handle (10b), we resort to a certain relaxation approach to deriving a convex relaxation of (10b). To this end, we need the following lemma.

**Lemma 1** ( [15, Prop. 1]). *The following implication holds:*

$$\log\det\left(\mathbf{I} + \left(\sigma^2\mathbf{I} + \mathbf{G}^H\mathbf{\Sigma}\mathbf{G}\right)^{-1}\mathbf{G}^H\mathbf{W}\mathbf{G}\right) \le \log\beta \tag{14a}$$

$$\Longrightarrow\ (\beta-1)(\sigma^2\mathbf{I} + \mathbf{G}^H\mathbf{\Sigma}\mathbf{G}) - \mathbf{G}^H\mathbf{W}\mathbf{G} \succeq \mathbf{0} \tag{14b}$$

*for any* $\sigma \neq 0$, $\mathbf{G} \in \mathbb{C}^{N\times M}$, $\mathbf{W} \in \mathbb{H}_+^N$ *and* $\mathbf{\Sigma} \in \mathbb{H}_+^N$. *Moreover,* (14a) *and* (14b) *are equivalent if* $\mathrm{rank}(\mathbf{W}) \le 1$.

By applying Lemma 1 to (10b), we have

$$(10b) \Rightarrow (\tau^{-1}-1)(\sigma_{e,k}^2\mathbf{I} + \mathbf{G}_k^H\mathbf{\Sigma}\mathbf{G}_k) \succeq \mathbf{G}_k^H\mathbf{W}\mathbf{G}_k, \forall \mathbf{G}_k \in \mathcal{B}_k, \forall k. \tag{15}$$

While the inequality on the RHS of (15) is already convex in $(\mathbf{W},\mathbf{\Sigma})$, it is still not convenient to process as it involves an infinite number of LMIs. Nevertheless, by employing a matrix form of $\mathcal{S}$-lemma (cf. Theorem 3.3 and Proposition 3.4 of [19]), the RHS of (15) can be equivalently expressed as a single LMI:

$$\text{RHS of (15)} \Leftrightarrow \mathbf{T}_k(\tau,\mathbf{W},\mathbf{\Sigma},t_k) \succeq \mathbf{0} \text{ for some } t_k \ge 0, \forall k, \tag{16}$$

where $\mathbf{T}_k(\tau,\mathbf{W},\mathbf{\Sigma},t_k) \triangleq [\bar{\mathbf{G}}_k,\ \mathbf{I}]^H((\tau^{-1}-1)\mathbf{\Sigma} - \mathbf{W})[\bar{\mathbf{G}}_k,\ \mathbf{I}] + \mathrm{Diag}((\sigma_{e,k}^2(\tau^{-1}-1) - t_k)\mathbf{I},\ t_k/\epsilon_k^2\mathbf{I})$.

Now by replacing (10b), (10c) and (10e) with (16), (12) and (13), respectively, we obtain a relaxation of problem (10):

$$\hat{\Gamma}(\tau) = \max_{\mathbf{W},\mathbf{\Sigma},\{t_k,\xi_k\}_{k\in\mathcal{K}}} \frac{\mathbf{h}^H\mathbf{W}\mathbf{h}}{\sigma_{\rm IR}^2 + \mathbf{h}^H\mathbf{\Sigma}\mathbf{h}} \tag{17a}$$

$$\text{s.t. } \mathbf{T}_k(\tau,\mathbf{W},\mathbf{\Sigma},t_k) \succeq \mathbf{0},\ t_k \ge 0,\ \forall\, k \in \mathcal{K}, \tag{17b}$$

$$\mathbf{F}_k(\mathbf{W},\mathbf{\Sigma},\xi_k) \succeq \mathbf{0},\ \xi_k \ge 0,\ \forall\, k \in \mathcal{K}, \tag{17c}$$

$$\mathrm{Tr}(\mathbf{W}+\mathbf{\Sigma}) \le P,\ \mathbf{W} \succeq \mathbf{0},\ \mathbf{\Sigma} \succeq \mathbf{0}, \tag{17d}$$

$$\mathrm{Tr}(\bar{\mathbf{\Phi}}_l(\mathbf{W}+\mathbf{\Sigma})) + \delta_l\|\mathbf{W}+\mathbf{\Sigma}\|_F \le \rho_l,\ \forall\, l. \tag{17e}$$

We note that $\hat{\Gamma}(\tau) \ge \Gamma(\tau)$ for any feasible $\tau$, because the implication in Lemma 1 shows that (17b) is a relaxation of (10b). Moreover, every feasible solution to problem (10) is feasible for problem (17). Finally, if an optimal solution $\mathbf{W}^\star$ to problem (17) is of rank one, then it can be checked from Lemma 1 that $\hat{\Gamma}(\tau) = \Gamma(\tau)$ holds; i.e., the relaxation is tight. Interestingly, as we will show shortly, there always exists a rank-one optimal solution $\mathbf{W}^\star$ to problem (17). As a result, it suffices to solve the relaxed problem to obtain $\Gamma(\tau)$. Before delving into the details of the tightness proof, let us explain how to efficiently solve problem (17).

Problem (17) is a quasi-convex problem, which can be turned into a convex problem by the Charnes-Cooper transformation [20]. Specifically, let us introduce the following change of variables:

$$\mathbf{W} = \tilde{\mathbf{W}}/\mu,\ \mathbf{\Sigma} = \tilde{\mathbf{\Sigma}}/\mu,\ \xi_k = \tilde{\xi}_k/\mu,\ t_k = \tilde{t}_k/\mu,\ \forall\, k \in \mathcal{K}, \tag{18}$$

where $\mu > 0$ is a parameter. Then, problem (17) amounts to

$$\hat{\Gamma}(\tau) = \max_{\tilde{\mathbf{W}},\tilde{\mathbf{\Sigma}},\mu,\{\tilde{t}_k,\tilde{\xi}_k\}_{k\in\mathcal{K}}} \mathbf{h}^H\tilde{\mathbf{W}}\mathbf{h} \tag{19a}$$

$$\text{s.t. } \sigma_{\rm IR}^2\mu + \mathbf{h}^H\tilde{\mathbf{\Sigma}}\mathbf{h} = 1, \tag{19b}$$

$$\tilde{\mathbf{T}}_k(\tau,\mu,\tilde{\mathbf{W}},\tilde{\mathbf{\Sigma}},\tilde{t}_k) \succeq \mathbf{0},\ \tilde{t}_k \ge 0,\ \forall\, k \in \mathcal{K}, \tag{19c}$$

$$\tilde{\mathbf{F}}_k(\mu,\tilde{\mathbf{W}},\tilde{\mathbf{\Sigma}},\tilde{\xi}_k) \succeq \mathbf{0},\ \tilde{\xi}_k \ge 0,\ \forall\, k \in \mathcal{K}, \tag{19d}$$

$$\mathrm{Tr}(\tilde{\mathbf{W}}+\tilde{\mathbf{\Sigma}}) \le \mu P,\ \tilde{\mathbf{W}} \succeq \mathbf{0},\ \tilde{\mathbf{\Sigma}} \succeq \mathbf{0}, \tag{19e}$$

$$\mathrm{Tr}(\bar{\mathbf{\Phi}}_l(\tilde{\mathbf{W}}+\tilde{\mathbf{\Sigma}})) + \delta_l\|\tilde{\mathbf{W}}+\tilde{\mathbf{\Sigma}}\|_F \le \mu\rho_l,\ \forall\, l, \tag{19f}$$

where $\tilde{\mathbf{T}}_k(\tau,\mu,\tilde{\mathbf{W}},\tilde{\mathbf{\Sigma}},\tilde{t}_k) \triangleq [\bar{\mathbf{G}}_k,\ \mathbf{I}]^H((\tau^{-1}-1)\tilde{\mathbf{\Sigma}} - \tilde{\mathbf{W}})[\bar{\mathbf{G}}_k,\ \mathbf{I}] + \mathrm{Diag}((\mu\sigma_{e,k}^2(\tau^{-1}-1) - \tilde{t}_k)\mathbf{I},\ \tilde{t}_k/\epsilon_k^2\mathbf{I})$ and $\tilde{\mathbf{F}}_k(\mu,\tilde{\mathbf{W}},\tilde{\mathbf{\Sigma}},\tilde{\xi}_k) \triangleq \left[\mathbf{I},\ \bar{\boldsymbol{g}}_k\right]^H\left(\mathbf{I} \otimes (\tilde{\mathbf{W}}+\tilde{\mathbf{\Sigma}})\right)\left[\mathbf{I},\ \bar{\boldsymbol{g}}_k\right] + \mathrm{Diag}(\tilde{\xi}_k\mathbf{I},\ -\mu\eta_k/\zeta_k - \tilde{\xi}_k\epsilon_k^2)$. For the equivalence between problems (17) and (19), readers are referred to [15] for a detailed proof. Problem (19) is a convex conic optimization problem, which can be efficiently solved with interior-point methods [18]. Once (19) is solved, an optimal solution to (17) can be recovered through (18).

*3.2.1. Tightness Proof for the Relaxation* (17)

Suppose that we have solved (19) with the optimal value $\hat{\Gamma}(\tau)$. Then, we consider the following power minimization problem:

$$\min_{\mathbf{W},\mathbf{\Sigma},\{t_k,\xi_k\}_{k\in\mathcal{K}}} \quad \text{Tr}(\mathbf{W}) \tag{20a}$$

$$\text{s.t. } \mathbf{h}^H(\mathbf{W}-\hat{\Gamma}(\tau)\mathbf{\Sigma})\mathbf{h} \geq \hat{\Gamma}(\tau)\sigma_{\text{IR}}^2, \tag{20b}$$

$$\text{(17b)} - \text{(17e) satisfied.} \tag{20c}$$

Here, (20b) is rewritten from $\frac{\mathbf{h}^H\mathbf{W}\mathbf{h}}{\sigma_{\text{IR}}^2+\mathbf{h}^H\mathbf{\Sigma}\mathbf{h}} \geq \hat{\Gamma}(\tau)$. Problem (20) is closely related to (17) and has some interesting properties:

**Property 1.** *Every feasible solution to* (20) *is optimal for* (17).

**Property 2.** *Every optimal* $\mathbf{W}^\star$ *to* (20) *satisfies* $\text{rank}(\mathbf{W}^\star) \leq 1$.

Property 1 can be easily deduced from (20b) and (20c). Property 2 is obtained by checking the Karush-Kuhn-Tucker (KKT) conditions of (20). The detailed proof is given in the Appendix. From Properties 1 and 2, the following theorem is immediate:

**Theorem 1.** *Suppose that problem* (17) *is feasible. Then, there exists an optimal solution* $(\mathbf{W}^\star,\mathbf{\Sigma}^\star)$ *to* (17) *with* $\text{rank}(\mathbf{W}^\star) \leq 1$. *Moreover, such an optimal solution can be obtained by solving* (20).

From Theorem 1, we conclude that (17) is a tight relaxation of (10) (cf. Lemma 1), and that $\hat{\Gamma}(\tau) = \Gamma(\tau)$ holds for all feasible $\tau$.

*Remark* 2. We should point out that the construction of problem (20) is crucial for the tightness proof. Also, it is different from that in [15] (cf. problem (11) in [15]), as Property 2 may not hold for problem (11) in [15] when the EH constraints are present.

## 4. SIMULATION RESULTS AND CONCLUSIONS

Two numerical results are provided to demonstrate the efficacy of the proposed design. For comparison, we also present the result of an isotropic AN-based design [13], which fixes $\mathbf{W} = \alpha P\mathbf{h}\mathbf{h}^H/\|\mathbf{h}\|^2$ and $\mathbf{\Sigma} = (1-\alpha)P/(N_t-1)(\mathbf{I}-\mathbf{h}\mathbf{h}^H/\|\mathbf{h}\|^2)$. Here, $0 \leq \alpha \leq 1$ is an optimal power allocation ratio that can be computed by substituting the above $\mathbf{W}$ and $\mathbf{\Sigma}$ into (6) and solving (6) w.r.t. $\alpha$. The simulation settings are as follows, unless otherwise specified: The number of transmit antenna is $N_t = 5$. There are two ERs, each having two receive antennas, i.e., $K = 2$ and $N_{e,k} = 2, \forall k$. All the receivers have the same noise level $\sigma_{\text{IR}}^2 = \sigma_{e,k}^2 = -40$dBm for all $k$. Each element of $\mathbf{h}$ (resp. $\mathbf{G}_k$) is i.i.d. and generated from a complex Gaussian distribution with mean zero and variance $-30$dB (resp. $-10$dB). The channel uncertainty level for $\mathbf{G}_k$ is $\epsilon_k = 0.1, \forall\, k$. The energy harvesting efficiency $\zeta_k = 50\%, \forall\, k$ and all the ERs have the same energy harvesting threshold, i.e., $\eta_1 = \eta_2$.

Fig. 1 shows the achievable rate-energy regions of the proposed design and the isotropic AN design for one random channel realization. Here, the transmit power $P$ is fixed at 10dBm, and for simplicity, the general covariance constraint (6d) is not considered. As seen, the proposed design can achieve a much larger rate-energy region than isotropic AN.

Fig. 2 plots the worst-case secrecy rate against the transmit power $P$ for the two schemes when fixing $\eta_1 = \eta_2 = 0.5$mW. In this example, besides the total power constraint, we also consider the per-antenna power constraints; i.e., by setting $L = N_t$, $\rho_l = 2P/N_t$, $\delta_l = 0$ , $\forall\, l$ and $\mathbf{\Phi}_l = \boldsymbol{e}_l\boldsymbol{e}_l^H$ in (6d). From Fig. 2, we see that the proposed design outperforms the isotropic AN design over the whole range of powers tested. In particular, with the increase of the transmit power, the two designs grows nearly linearly w.r.t. $P$, and there is a constant rate gap 1.5 bps/Hz between the two schemes.

To conclude, we have considered transmit covariances optimization for simultaneous confidential information transmission and wireless energy transfer. While the transmit covariances optimization problem is non-convex in its original form, we show that it can be recast as a two-level optimization problem, which can be handled by solving multiple convex optimization problems. As a future direction, it would be interesting to consider multiple MIMO IRs case.

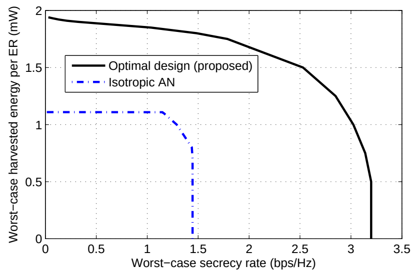


**Fig. 1**: Achievable rate-energy region for $P = 10$ dBm

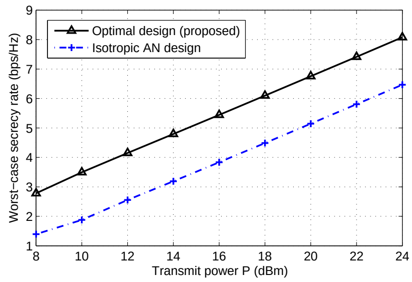


**Fig. 2**: Secrecy rate vs. transmit power $P$ for $\eta_1 = \eta_2 = 0.5$mW

## 5. APPENDIX

We check the KKT conditions of (20). Let $\lambda \in \mathbb{R}_+$, $\mathbf{A}_k \in \mathbb{H}_+^{N_t+N_{e,k}}$, $\mathbf{B}_k \in \mathbb{H}_+^{N_tN_{e,k}+1}$, $\nu \in \mathbb{R}_+$, $\gamma_l \in \mathbb{R}_+$, $\mathbf{Q} \in \mathbb{H}_+^{N_t}$ and $\mathbf{M} \in \mathbb{H}_+^{N_t}$ be the Lagrangian multipliers associated with (20b), $\mathbf{T}_k \succeq \mathbf{0}$, $\mathbf{F}_k \succeq \mathbf{0}$, $\text{Tr}(\mathbf{W}+\mathbf{\Sigma}) \leq P$, (17e), $\mathbf{W} \succeq \mathbf{0}$ and $\mathbf{\Sigma} \succeq \mathbf{0}$, respectively. Assuming that problem (20) satisfies some constraint qualifications [18], we have the following KKT conditions of (20):

$$\begin{aligned}\mathbf{Q} =& \mathbf{I} - \lambda\mathbf{h}\mathbf{h}^H + \textstyle\sum_k \bar{\boldsymbol{\mathcal{G}}}_k\mathbf{A}_k\bar{\boldsymbol{\mathcal{G}}}_k^H - \sum_k \nabla_{\mathbf{W}}\text{Tr}(\mathbf{B}_k\mathbf{F}_k) \\ &+ \textstyle\sum_l \gamma_l\big(\bar{\mathbf{\Phi}}_l + \delta_l\|\mathbf{W}+\mathbf{\Sigma}\|_F^{-1}\mathbf{I}\big) + \nu\mathbf{I}, \end{aligned} \tag{21a}$$

$$\begin{aligned}\mathbf{M} =& \lambda\hat{\Gamma}(\tau)\mathbf{h}\mathbf{h}^H - (\tau^{-1}-1)\textstyle\sum_k \bar{\boldsymbol{\mathcal{G}}}_k\mathbf{A}_k\bar{\boldsymbol{\mathcal{G}}}_k^H - \sum_k \nabla_{\mathbf{\Sigma}}\text{Tr}(\mathbf{B}_k\mathbf{F}_k) \\ &+ \textstyle\sum_l \gamma_l\big(\bar{\mathbf{\Phi}}_l + \delta_l\|\mathbf{W}+\mathbf{\Sigma}\|_F^{-1}\mathbf{I}\big) + \nu\mathbf{I}, \end{aligned} \tag{21b}$$

$$\mathbf{Q}\mathbf{W} = \mathbf{0},\ \mathbf{M} \succeq \mathbf{0},\ \mathbf{A}_k \succeq \mathbf{0},\ \forall\, k \in \mathcal{K}, \tag{21c}$$

where $\bar{\boldsymbol{\mathcal{G}}}_k \triangleq [\bar{\mathbf{G}}_k,\ \mathbf{I}]$. It can be easily shown that $\nabla_{\mathbf{\Sigma}}\text{Tr}(\mathbf{B}_k\mathbf{F}_k) = \nabla_{\mathbf{W}}\text{Tr}(\mathbf{B}_k\mathbf{F}_k)$ by checking the gradients directly. Hence, subtracting (21b) from (21a) yields

$$\mathbf{Q}-\mathbf{M} = \mathbf{I} - \lambda(1+\hat{\Gamma}(\tau))\mathbf{h}\mathbf{h}^H + \tau^{-1}\textstyle\sum_k \bar{\boldsymbol{\mathcal{G}}}_k\mathbf{A}_k\bar{\boldsymbol{\mathcal{G}}}_k^H. \tag{22}$$

By post-multiplying (22) by $\mathbf{W}$ and making use of (21c), we get

$$\mathbf{W} = \lambda(1+\hat{\Gamma}(\tau))(\mathbf{I}+\mathbf{M}+\tau^{-1}\textstyle\sum_k \bar{\boldsymbol{\mathcal{G}}}_k\mathbf{A}_k\bar{\boldsymbol{\mathcal{G}}}_k^H)^{-1}\mathbf{h}\mathbf{h}^H\mathbf{W}. \tag{23}$$

Clearly, the rank of the matrix on the right-hand side of (23) is no greater than 1, which completes the proof.

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