Modelling, Simulation and Optimisation of Train Traffic with Passenger Movement

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***Abstract* — Purpose of the paper is to present a model for the railway traffic that consists of train arrivals and departures at different stations and also the passenger movement in stations, boarding trains and alighting at different destinations. This is a classic example of hybrid system for which initially the existing models are evaluated and the needs of model for this problem scenario are identified. The model proposed is utilized to simulate an experimental setup with the traffic based on the Train schedule and passenger flow in the stations to analyse whether it can work without any conflicts, impacts on passenger comforts, all in a single model. With this, we can determine the schedule of the train for an optimised headway time based on constrains for passenger waiting times and train running times. The simulation results are presented with assumed figures of passenger flow and train traffic. At the end, the optimization formulation is shown to arrive at an optimum schedule.**

Keywords - Petri nets, Hybrid systems, Railway traffic, Train movement, Passenger flow

I. INTRODUCTION

In the research community, it is common that the study of discrete and continuous systems have been going on from decades. However the study of the hybrid system that comprises of both discrete and continuous parts is always not straightforward from the perspective of finding the right model for a quick adaptation. Each of the models would have certain strengths and draw backs considering different classes of problems.

Railway traffic consists of train arrivals and departures at different stations and also in addition the passenger walking in stations to board the train or after alighting. This is a good example of hybrid system that has been considered in the work of this paper. The train traffic is considered a discrete part while the passenger flow is considered as continuous but the other school of thought is that it can be considered as 'discrete continuous'. An attempt is made to model based on the Place timed Petri nets and also considering aspects from hybrid Petri and coloured Petri nets with additional proposed additions to enable the modelling and simulation of the problem. The model describes the theory and mathematical parts that were developed to build up the analysis. Different classical topologies of the system are also explained. The simulation is performed using MATLAB for a reference scenario and the simulation results and the interpretations are described. The paper is unique in the sense that we attempt to address both the train traffic and passenger traffic using Origin-Destination (O-D) matrix that considers all the platforms of the system. Many other papers consider the system in parts.

II. SYSTEM AND PROBLEM DESCRIPTION

The system that we consider is a single or multiple railway lines with platforms and stations that may be any one of the topologies. Below are the definitions that are referred in the paper:

1) Platform place: Place where the passenger wait to board the train and alight from train. This is referred as paid area from the passengers view and they need to enter via gates.
2) Station places: Place where the passengers walk and via the paid gates to enter the platform. Also, the place to exit via the gates after alighting in the platform. This is referred as unpaid area from the passengers view.
3) Track/Line: These words 'track' and 'line' are interchangeably used to represent the path of train for its movement.
4) Transit Section: We use this term for the transit places which is used for train travel between the platforms.

A. *Platform Places*

A railway line is considered to have multiple running trains which have the origin or the starting point P-S and the last destination P-D as represented in Figure 1. The trains leave the origin P-S and reach destination P-D in accordance with the schedule. The train would go through the stations P-2, P-3… P-n-2, P-D and the time taken to move from one station to another is known but the wait time in each station is pre-determined by the train authority and could vary based on the different dynamic conditions. On moving via these stations, it will go through the sections S-1 followed by the Place P-2, S-2 followed by P-3, S-3 followed by P-4…...S-n-2f followed by P-n-1 to finally reach the last destination P-D. This makes the train line in one direction.

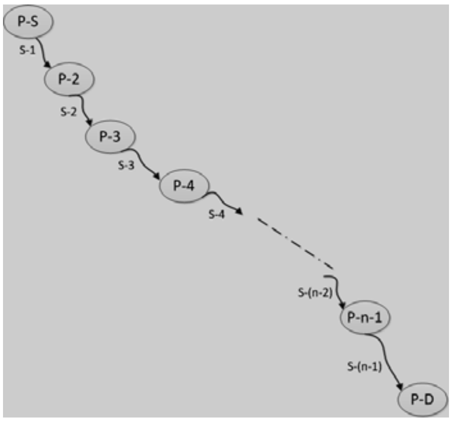

Figure 1 Representation of railway Line

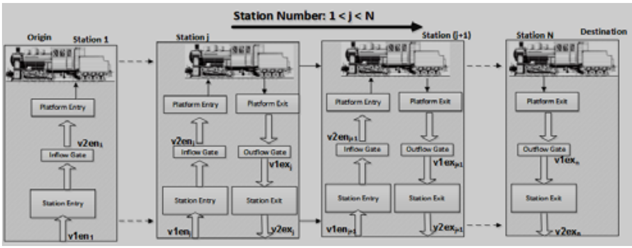

Figure 2 Representation of station

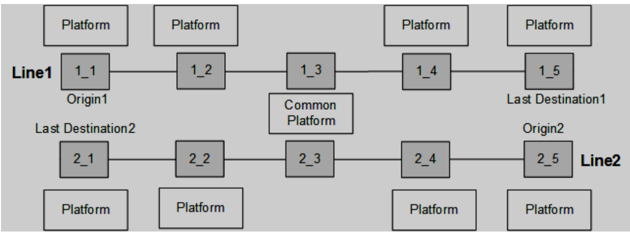

Figure 3 Case of independent lines

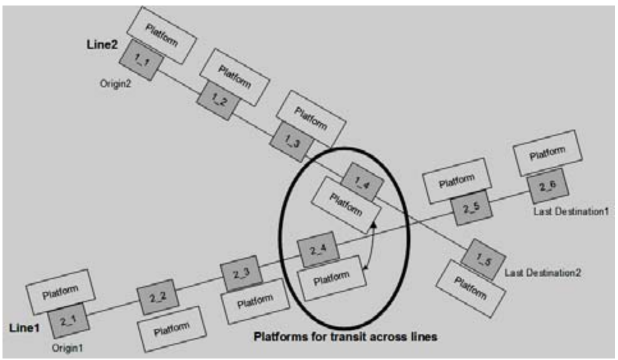

Figure 4 Case of railway lines that share platforms for transit across lines

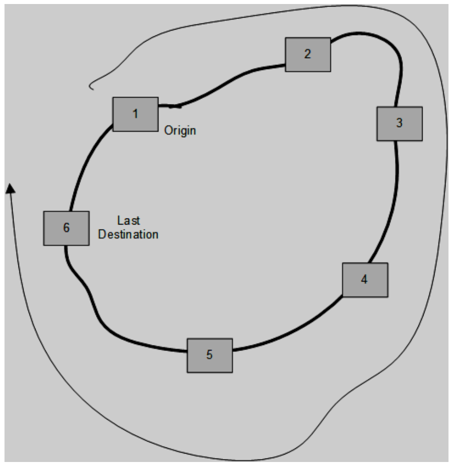

Figure 5: Case of circular line

B. *Station places*

Every station is comprised of places to enter and exit which are referred as station entry and station exit. Platform places to enter and exit the train are referred as Platform entry and Platform exit respectively. Passenger come in freely to the station place and then enters the platform entry place via the gates to wait to board the train. Passengers who alight to platform exit place will go out to station exit place via the gates to finally exit the station. Figure 2 shows the simplified macroscopic diagram for trains travelling in one direction. In the figure, the platforms are also shown where the passenger boards the train and alight from trains. When there is an intersection of the two railway lines they may have a common platform so that the transit is easier and hence alighting from a train and boarding to the other train is the same platform place. When the platforms are not common, we consider different platform places, which mean there is some time spent by passengers for the transits.

C. *Topologies of railway network for local transport*

There are different configurations possible for railway network but the most representative ones are:

1) There could be independent railway lines with trains running and some stations sharing same platforms for passengers to board and alight. This is represented in Figure 3.
2) Circular lines in which the trains run in a circular fashion with one of them considered as origin and other as destination. This is represented in Figure 4.
3) Multiple railway lines intersecting at certain stations with common or different platforms. If it is not common platform, then such case will need longer time for transit. This is represented in Figure 5.

D. *Problem definition*

The model for simulation will need to have the following inputs structurally:

1) Schedule of the trains leaving the origin including the frequency of train and type of train. 2) Distance between the

stations 3) Dwell time in each station 4) Passenger arrivals in each station using origin-destination matrices and this is for every station in the system.

The problem is to analyse the following:
1) Number of passenger in different places based on different passenger flow model 2) Waiting time of passenger in the platform 3) Arrival and departure of the times in different platforms.

The expected output from this modelling and simulation are the following:
1) Simulation of scenarios: a) Single or independent multiple railway lines without any intersection b) Multiple lines having an intersection in common transits that could be common or different platforms c) Circular lines in which the trains repeat the same route.
2) Analysis of the properties of the model: a) Understand the behaviour of the possible discomforts and crowd at different stations b) Schedule impact on system behaviour.
This should lead to optimization of the train schedule considering cost of running the train against the waiting time and hence discomforts of the passenger which is also dealt briefly in the paper.

Assumptions and scope considered for the system for modelling and simulation is as follows:
1) The railway lines are unidirectional and considered for a mass transit railway for local transport.
2) Station places are considered unpaid areas while Platform places as paid areas that have restricted entries which is applicable for many countries and may not hold good everywhere.
3) The physical areas and walk times of passengers are not separately considered. Even though it may not be perfect but these times will get implicitly included as passenger flow rates are considered between stations to platforms for a macroscopic level.

Focus is to consider train and passenger traffic with trains running on single unidirectional line from origin to destination. A good classification of train traffic problem into strategic, tactic and operational is done in [1]. This paper attempts to address the tactical problem but enables to take inputs from Operational level offline and provide the needed adjustments. We do not consider parts related to the track allocations, train routing, rolling stock and crew scheduling.

Simulation output provides all the output matrices needed using which any kind of Graphic output can be presented but we provide the most essential graphical plots that are needed for analysis.

III. LITERATURE REVIEW

As part of literature, we could classify papers and books to two different categories: 1) The ones that deal with theory and analysis that are generic in nature and can be applied for different practices in varied fields 2) The ones that deal with the actual application taking references of such models. References [2 - 12] fall in category 1 while references [13 - 17] belong to category 2.

A model is proposed in [13] for the railway station using the hybrid Petri nets and good analysis of the qualitative properties. It does consider the arrival and departure of the trains in a single station alone. The train traffic is simulated based on Coloured Petri nets in [14] leading to optimisation of the train schedule. An analysis of railway station using timed Colour Petri nets is made in [15]. It shows an approach to evaluate the operating schedule and also the infrastructure of the station. Pedestrian movement is considered for modelling in [17]. In this paper, our interest is to consider both the train and passenger traffic as a system including the railway lines. It is very much necessary this way, as each of these sub-systems are interdependent and would be meaningful to arrive to the needed analysis and optimisation of train schedules. Passenger traffic is considered in the form of O-D matrix and with this it is important to check the discomfort of people waiting for trains to reach a destination. In this paper, we have considered such a class of system that attempts to emphasize on the enhancements possible over the existing approaches.

It is important to get a brief of the existing models that deals with theory and analysis without detailing out all the basics that are already dealt in the references. The hybrid automata defined a model that describes the evolution in time of the values of a set of discrete and continuous state variables. This is only for autonomous systems that have no inputs and outputs. The railway system problem has inputs and hence this is not a suitable model to consider. Petri nets are usually used to model discrete systems with the marking of a place that may correspond to either Boolean state of device or to an integer that is been used by high level Petri nets. When the Petri net contains a large number of tokens which would be the case for modelling this railway traffic, the number of reachable state explodes and is a limitation. Hybrid Petri net is the extension of the Petri net that is successfully applied for modelling, performance evaluation and manufacturing systems and recently the transportation systems as well. As proposed by Rene David and Hassane Alla in [3], this model will be suitable as worked out by [13] but it would be difficult to apply when we have to view the complete Railway line as a model, carrying passenger and train tokens. There is no concept of different data set to deal with train movement, passenger movement with different stations and also having concurrent transitions which is very much needed. CPN (Colour Petri net) modelling and tools enables a good representation of passenger and trains as different colour sets but this problem will need multiple colours based on the destinations. We would need transitions to be triggered with all colour tokens moving and there is very much a need to have simultaneous multiple transitions getting triggered at the same time. CPN tools do not address the Petri net places with time very effectively. Switching systems model are mostly for hybrid time invariant systems that are viewed as autonomous and is a model where a differential equation needs to be selected depending on the switching signal. In our problem case, the switching happens on different discrete system triggers but the continuous systems are not classical cases of differential equations.

In essence, there are specific needs for the model to address the problem that we defined while some of the existing models examined need some adaptations:

1) Occurrence of simultaneous triggers of two or more transitions will need an order. Hence the priority concept is added.

2) Need different data sets – tokens for train, tokens for passenger movement with different destinations as they all remain at the same time in places. CPN does address this in a programmatic way and not as a pure mathematical operation. We address it considering different variables for the data set so that matrix operations can be performed.

3) The movement of train itself and the passenger in train move together between stations that are discrete while the flow of passenger towards arriving train and leaving trains towards exits is continuous; hence we need discrete places as well as continuous places to be considered. Of course, it can be argued that it is discrete continuous.

4) Continue using the incidence matrix from Petri Nets and enable the mathematical operations to happen the same way with different data sets. Overall, the idea is not to completely ignore the amount of work gone in towards Petri net modelling but to propose the needed adaptations that are needed considering these specific needs. And the model also works closely with the realistic situation so that the visualization of the places will be the same.

IV. SYSTEMIC DESCRIPTION

The paper considers the mass transit railway lines in urban places which consist of multiple one direction lines. In each line, the trains run from station 1 to N. Each station will have a platform pertained to a line and in cases where more than one line pass through the station, there could be more than one platforms.

1) Number of lines l: $\{1, 2 \ldots L\}$
2) Stations in line n: $\{1^l, 2^l \ldots N^l\}$ with the number of stations N^l on the line l.
3) Sections in line n: $\{1^l, 2^l \ldots (N\text{-}1)^l\}$ with the number of sections $(N\text{-}1)^l$ on line l. Section 1 is in between Station 1 and Station 2, Section 2 is between Station 2 and 3…. Section $(N\text{-}1)^l$ is between Station $N\text{-}1$ and N. Since Station N is the destination, there are no more sections following this.
4) Trains running in each line k: $\{1^l, 2^l \ldots K^l\}$ with K^l trains that run on the line l.

When the superscript is not used, it means the reference is to one line that is considered default in most of the descriptions. The trains run in the same order or numbering with train 1 the first; train 2 the next, so on and so forth until K. No train can overtake the other at any station or sections. We do consider different train types with few minimum parameters like acceleration, deceleration and speed to determine the travel time between the stations.

A. Train traffic on lines

The railway transport lines are controlled by timetable and for each station there will be an arrival time, dwell time and departure time as decided by train authority considering the travel times in the transit sections between the stations. Hence there are four important parameters that accounts for the total duration of the travel for passengers. The waiting time of passengers for the train in each of the platforms will also be decided based on this:

1) Headway time ($h_{k,n}$): This is the time between arrivals of successive trains' k and k+1 at the station n. This can also be referred as inter-arrival time. Railways always specify a minimum headway time h_{min} so that the trains arrive and depart safely in stations.

2) Separation time ($s_{k,n}$): This is the time between the departure of a train k to the arrival of the next train at station k+1. This time is also included in the headway time but the separation time will enable the departed train to reach a section with which it is safer for the next train to arrive, dwell and depart without unnecessarily waiting for the just departed train to reach a certain section.

3) Dwell time ($p_{k,n}$): This is the parking time available for the passengers' to board and alight at station n for the train k. The time must be balanced such that the time is enough for the passengers and at the same time, it is not too long for the boarding passengers to wait for a long time to reach the destination and also unnecessarily increasing the passengers in the train and also hindering the next train to arrive.

4) Transit travel times ($r_{k,n}$): This is the running time or the travel time for the train k on the section n between the stations which depend on the distance and the maximum speed of the train.

Considering $a_{n,k}$ and $d_{n,k}$ as the arrival and departure timings respectively of kth train in station n, the following relationships can be directly interpreted:

$$h_{k,n} = a_{k+1,n} - a_{k,n} \quad (1)$$

$$p_{k,n} = d_{k,n} - a_{k,n} \quad (2)$$

$$s_{k,n} = a_{k+1,n} - d_{k,n} \quad (3)$$

From these equations it is clear that

$$h_{k,n} = s_{k,n} - p_{k,n} \quad (4)$$

When the train travels in the sections between the stations, it initially accelerates in phase1, holds on for maximum speed in phase 2 until it starts decelerating in phase 3 till it reaches the next stations. The transit travel time is calculated based on Newton's law considering these acceleration relations with known values of acceleration and deceleration for a train type and distance between stations. Given the distance $dist^l_k$ between stations k and $k+1$ on line

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i, we have the following relations to calculate the transit travel time. Based on train characteristics:

Acceleration: *accn,* Decceleration: *deccn,*
Maximum Speed: *Speed*
Distance covered during acceleration = *distAccn*
$= 0.5 * Speed^2/accn$ (5)
Distance covered during acceleration = *distDecn*
$= 0.5 * Speed^2/deccn$ (6)
Distance covered during the hold speed = *distConstSpeed*
$= (dist^i_k - distAccn - distDecn)$ (7)
Duration of acceleration phase = *Speed/accn* (8)
Duration of acceleration phase = *Speed/deccn* (9)
Duration of hold speed phase = *(distConstSpeed/Speed)* = *(dist/Speed)-(0.5*Speed/accn)-(0.5*Speed/deccn)* (10)
Transit travel time =

$$\left(r^i_{k,n}\right) = \left(\frac{dist^i_k}{Speed}\right) + \left(0.5\,\frac{Speed}{accn}\right) + \left(0.5\,\frac{Speed}{deccn}\right) \quad \text{(11)}$$

B. Passenger traffic in platform/station

a) Station entry and exit Rates

Passengers keep arriving at Station entry places to reach different destination stations and is represented using O-D Matrix:
Simple 1 line O-D matrix form:
StnEntryRate =

$$\begin{bmatrix} 0 & \alpha_{1,2} & \alpha_{1,3} & \dots & \alpha_{1,N-2} & \alpha_{1,N-1} & \alpha_{1,N} \\ 0 & 0 & \alpha_{2,3} & \dots & \alpha_{2,N-2} & \alpha_{2,N-1} & \alpha_{2,N} \\ 0 & 0 & 0 & 0\dots & \alpha_{3,N-2} & \alpha_{3,N-1} & \alpha_{3,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & 0\dots & 0 & \alpha_{N-2,N-1} & \alpha_{N-2,N} \\ 0 & 0 & 0 & 0\dots & 0 & 0 & \alpha_{N-1,N} \\ 0 & 0 & 0 & 0\dots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix} \quad \text{(12)}$$

StnEntryRate is a column matrix with each row representing the entry rate per unit time for each station.

$v1en_1 = \alpha_{1,2}\, d_2 + \alpha_{1,2}\, d_2 + \dots + \alpha_{1,N-1}\, d_{N-1} + \alpha_{1,N}\, d_N$ (13)
$v1en_2 = \alpha_{2,3}\, d_3 + \alpha_{2,3}\, d_3 + \dots + \alpha_{2,N-1}\, d_{N-1} + \alpha_{2,N}\, d_N$ (14)
………………………………..
$v1en_{N-2} = \alpha_{N-2,N-1}\, d_{N-1} + \alpha_{N-2,N}\, d_N$ (15)
$v1en_{N-1} = \alpha_{N-1,N}\, d_N$ (16)
$v1en_N = 0$ (17)
$\alpha_{i,j}$ represent the passenger flow at station i to reach the destination station j. $i, j\ \varepsilon\ \{2, N\}$, $i < j$ with N being number of stations.
Note that
$\alpha_{1,1},\ \alpha_{2,1},\ \dots\ \alpha_{N,1},\ \alpha_{2,2},\ \alpha_{3,2},\ \dots\ \alpha_{N,2}\ \dots \alpha_{N-1,N-1},\ \alpha_{N,N-1}\dots\ \alpha_{N-1,N-1},\ \dots\alpha_{1,N}\ ,\ \alpha_{2,N}\ \dots\ \alpha_{N,N}$ will all be 0s, which means that the passengers leaving the current station number i with destinations being same station or previous stations is not possible at all. The first column and last row are all zeroes in equation (13), as the origin cannot be destination for passengers and no passengers entering destination respectively. The passengers entering may be of destinations of other railway line as they will make a transit on the appropriate transit stations.

b) Platform entry and exit rates

After entering the station, when the passengers enter the platform, they need to go via gates that need access as the platform is considered as paid areas. We keep $v2en_N = \phi_{\text{in,n}}$ as the platform entry rate at station *n*.

c) Boarding and alighting rates

On arrival of the train in station, with a disciplined crowd, the passengers start alighting followed by boarding. In reality, boarding and alighting may take place simultaneously as all of the vehicle doors are shared. As mentioned in [18], the alighting flow depends on the passengers in the train, the boarding flow, and the passengers in the platform which is again dependent on the platform entry and exit flow. This cannot be solved using analytical methods but need to depend on numerical methods. In the modelling considered here, the alighting and boarding rates are being based on sharing the doors. If the boarding passengers are lower than the number of passengers to alight, the boarding rate will increase in proportion to the reduction of the alighting rate, thus keeping the sum of alighting and boarding rates to depend on the capacity of doors. Based on the discussions in paper [13] which also refers [19] that has more research details, the following approach is taken that is applicable for HK MTR.

Shared gate capacity:

C_{door} = *Door width/Mean width* (18)

Door width is in 'm' and the width of the actual door of the Vehicles of the train and *Mean width* is m/passenger needed for each passenger for crossing the.

Global capacity of the doors for a train

$C_{Train} = N_{vk} * N_{AB/v} * C_{door}$ (19)

where N_{vk} is number of vehicles in a train and $N_{AB/v}$ is number of doors per each vehicle. With this, the flow rate can be approximated using:

Flow Rate = $\phi = C_{Train}/\tau$ (20)

Where τ is the walking time for each passenger to enter or exit the vehicle for entry and exit flow rate. For the discussions in paper, based on this theory, we represent $alightRate_{n,k}$ *and* $boardRate_{n,k}$ for the alighting and boarding rate respectively. In the proposal, as the O-D matrix is known and hence the number of boarding and alighting passengers are known. In reality, the alighting rate of passengers depends on the occupancy in the train and the platform and the boarding rate will depend on the factors of passengers alighting and also the number of passengers in train. This will need numerical techniques to further resolve.

The alighted passengers move out of the unpaid area at the platform exit rate $v1ex_N = \phi_{out,n}$ and finally exit the station at the rate $v2ex_N$.

V. Basic Rules Towards Proposed Model

The model description is considered in different parts and is shown in Fig 3: 1) Model description of train movement 2) Model description of passenger flow from one station to other 3) Model description of passenger flow within stations which comprises of continuous flow model communicating with the discrete model. It is very important to note that this modelling will comprise of continuous and system which requires only integers and does not need real numbers. Below are the sections that provide a quick recap of basics of Petri and explain the concepts and definitions that are been added over the Petri nets before explaining the details of the model.

A. Brief on Petri nets and definitions

We do not want to repeat all the basics of Petri nets here for which readers may refer [4] [7] [9]. For the benefit and quick recap, few basics are briefed here. Petri net (PN), also known as Place/Transition net is a graphical and mathematical modelling tool which was applied for discrete systems when it was introduced. It allows a way of graphically depicting the structure of the system as a directed bipartite graph. The graph consists of places, transitions and arcs. Arcs are either from a place to a transition or from a transition to a place. Places contain a natural number (nonnegative integer) of tokens. A distribution of tokens over the places of a PN is called a marking. In graphical representation, places are drawn as circles, transitions as bars and tokens as black dots. Petri nets are based on theoretical foundations that allow checking that the model is, for example, reversible, live, deadlock-free, bounded, etc. The main methods of analysis of these properties are the cover-ability tree, matrix equations, and reduction techniques in [4] and [8] .

1) Incidence matrix and resultant matrix

The resultant marking can be always found using

$$M = W.S + M0 \tag{21}$$

Where M [Px1] is the new marking after the firing S
$M0$ - Initial marking of dimension [P x 1]
W - Incidence matrix [P x T]
S - Trigger matrix that of dimension [T x 1]
P - Number of places in the model
T - Number of triggers in the model

2) Example of a Petri net representation and matrices

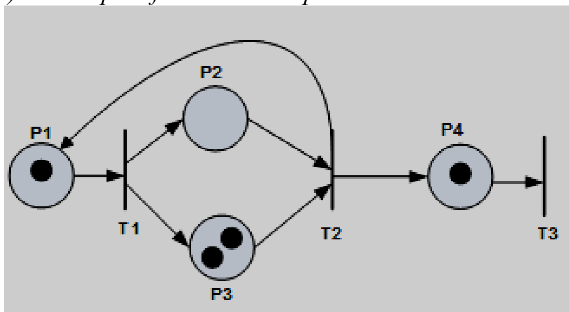

Figure 6: Example of Petri Net with an initial marking

Input Incidence Matrix (4x3) $W^{+} = \begin{array}{cccc} T1 & T2 & T3 & * \\ \left[\begin{array}{c} 0 \\ 1 \\ 1 \\ 0 \end{array}\right. & \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right] & \begin{array}{c} P1 \\ P2 \\ P3 \\ P4 \end{array} \end{array}$

Output Incidence Matrix (4x3) $W^{-} = \begin{array}{cccc} T1 & T2 & T3 & * \\ \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array}\right. & \begin{array}{c} 0 \\ 1 \\ 1 \\ 0 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \end{array}\right] & \begin{array}{c} P1 \\ P2 \\ P3 \\ P4 \end{array} \end{array}$

Incidence Matrix (4x4) $W = W^{+} - W^{-}$

$= \begin{array}{cccc} T1 & T2 & T3 & * \\ \left[\begin{array}{c} -1 \\ 1 \\ 1 \\ 0 \end{array}\right. & \begin{array}{c} 1 \\ -1 \\ -1 \\ 0 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ 0 \\ -1 \end{array}\right] & \begin{array}{c} P1 \\ P2 \\ P3 \\ P4 \end{array} \end{array}$

$M0 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

In the Fig 3 with the initial marking M0, the triggers that are enabled are T1, T3.

3) Hybrid Petri net and Coloured Petri nets

David and Rene introduced Hybrid Petri Nets that comprises of both continuous and discrete parts. Places P have subsets PD and PC that represent discrete and continuous places respectively and similarly triggers T have subsets TD and TC that represent discrete and continuous triggers respectively. Continuous places are represented by double lined circles and Continuous Triggers are represented using unfilled or white filled boxes. All the details of the theory on the rules and properties of Hybrid Petri nets are available in [3].

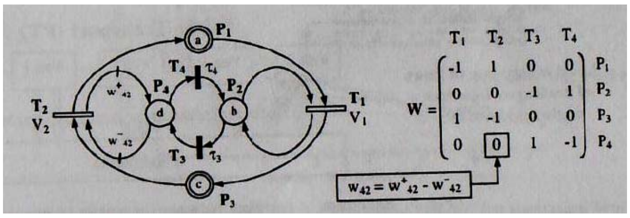

Figure 7 Example of hybrid Petri net taken from [3]

There are other extensions to Petri nets that enable the study of complex systems. To name, the two main ones are coloured Petri nets and hybrid petri Nets. Coloured Petri Nets are defined to carry data values using the colour tokens unlike the basic Petri Nets that use only black tokens. Colour Petri Nets is established to use complex data sets as colour sets and the theory and tools are generalized. A good understanding of the Coloured Petri Nets is available at [9] [20]. Other classes are called extensions as they add properties and/or nodes to basic PNs for modelling specific behaviour or constraints that cannot be modelled with the original paradigm. For example, timed Petri nets (TPNs) are a well-known extension which introduces time delays associated with transitions or places for performance evaluation of different systems. A good list of references on this topic are [8] [21] and [22].

In the proposed model, we consider the hybrid Petri nets as the basis but also take the background of coloured Petri nets in an algebraic form and at the same time adding new concepts of representing transitions and consideration of handling simultaneous occurrence of triggers.

B. Representation of data sets

The data sets needed are for representation of train and passengers with different destinations. This is referred as origin to destination matrix in the railway system domain. In the description of the model, the train is represented as Tr. The number of passengers is represented using k. Dn where k represents the number of passengers that intend to alight at destination n. As an example, 10*D2 will represent passengers intend to alight at destination station 2. All the places will comprise of the algebraic expressions of the data sets. If the tokens in a place is represented using 'Tr + 10 d2 + 8 d3 + 9 d4', this means there is a train in a place that is carrying 10 passengers who intend to alight at destination station 2, 8 passengers at destination station 3 and 9 passengers at destination station 4. The place and trigger will both comprise of expressions using the data set.

C. Addition and removal of triggers

When there are data sets involved, the weight on the arcs indicates the token to be added or removed with the default as one. If the weight is on the arc that is input to place, then this weight is to be added to the input place while if it is on output place then it is to be removed from output place. In the example shown in Figure 8, each of the arcs from places to triggers and triggers to places have a weight indicated

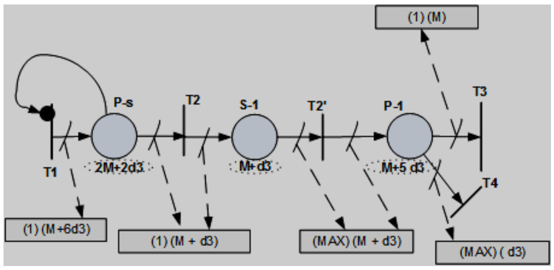

Figure 8: Initial markings for occurrence of triggers

Rule 1: MAX is the weight that signifies that the respective data set associated with MAX is to be removed from the output place and added to the respective input place. This simply represents that the maximum firings occur to empty the input place for the firing of the trigger.

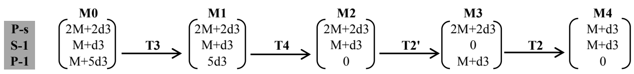

Figure 9: Reachable Marking examples – T3, T4, T2', T2

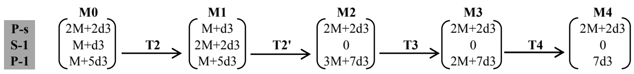

Figure 10: Reachable marking examples – T2, T2', T3, T4

In Figure 9 and 10, reachable markings of system in Figure 8 are shown with M0 as the initial markings with triggers being fired one after the other. There are two data sets M and d3 in the system. With the markings M1 is in Figure 9, on firing trigger T4, all the data set that belongs to d3 is removed as MAX (d3) is the weight on the arc. When the marking is M0 in Figure 10, T2 is fired to reach M1 and T2 is to be fired again in case if S-1 is to become 0. In this case, the arc weight is (1) (M+d3) and hence multiple firing is needed to make Ps as 0 and this is unlike using the MAX as the weight which enables a single firing to remove all the tokens in one shot.

D. Simultaneous occurrence of triggers

When there is simultaneous occurrence of more than one trigger that have mutual relationship, it is extremely important to consider which has higher priority as it has to reflect the real situation and also the system behaviour appropriately. A good overview is given in [23] that include cases of multiple enabling of transitions including some basic theory and semantics. When the simultaneous trigger have no relationship then it is absolutely fine to consider any of them as high and low priority. This is explained with an example.

Rule 2: When there are two triggers enabled, the trigger that does not affect the marking for the trigger of the other must have higher priority to be fired. This means if there are two

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triggers Tx and Ty and if Tx has output places that are input places for Ty, then Ty has higher priority than Tx.

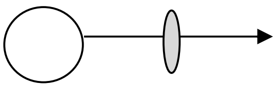
In the example in Fig 8, with P-s and P-1 having markings (2M+2d3) and (M+5d3) respectively, trigger T2 is not enabled because of inhibitor from P-1 which means that T2 needs also a condition with no tokens in P-1 in addition to any token in P-s. Let's say the triggers are checked every 't's with T2' being high priority trigger than T3, then the situation will be:

1) At t=0, it is found that T1 cannot be triggered with tokens in P-s. T2, T2', T3 and T4 are all enabled with tokens available in P-s, S-1 and P-1.
2) Below shows different markings when the order of trigger is T2, T2', T3 and T4 as compared to the order T3, T4, T2' T2. Refer Figures 9 and 10 for the same. In such circumstance, based on the rule, the order of priority must be chosen as T3, T4, T2' and then T2. T3 and T4 priority interchangeable as they do not have mutual dependence on markings and the removal of tokens from common place P-1 is mutually exclusive.

E. Channel arc transition

The need for this is to enable the firing of the transition that moves the passenger from platform to train on arrival of train and the train continues on the track.

Definition 1: Let N=(P, T, B, F, M0) be a Petri net where P and T are two disjoint sets called places and transitions; B and F are two functions from P x T to N called backward and forward incidence functions, M0 is a function from P to N called the initial marking. A new transition set Tc will have the form: (Te) : (Tr).



Tc = (Te):(Tr)

Figure 11: Representation of a channel arc transition

Te – Data set to be available in place for transition to fire. This is the symbolic form of the data set.

Tr – Data set token numbers to be removed from the place when transition is fired.

Definition 2: A channel arc transition Tc is enabled in M(t) with t as the transition and $M: P \rightarrow N$, a marking iff $\forall p \in P: M(p) \geq Te(p, t)$ leading to that it can be fired to a new marking M' with the removal and addition of tokens as defined by Tr.

Channel arcs are the arcs which allow the firing based on a channelling mechanism. Instead of having a weight of 1 or a finite number 'n' for arcs, the idea is to define the weights to control the enabling based on the dataset. The channel arcs are represented with an oval shape on the arcs defined from basic Petri nets as explained with the help of Fig 12 and Fig 13.

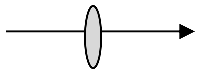
Tc = (Te=N1 d1 or N2 d2 or N3 d3) : (Tr=ds)   Tc = (Te=N1 d1 and N2 d2 and N3 d3) : (Tr=ds)

Figure 12: Channel filter arc representation

This channel filter arcs are the arcs which has a weight of two forms AND and OR. First form is AND: (N1 d1) and (N2 d2) and (N3 d3). This means, the place to which the arc is an output must have N1 tokens of dataset d1, N2 tokens of dataset d2, N3 tokens of dataset d3 etc. to satisfy the firing condition of the transition to which this arc is connected as an input. When the firing takes place, the tokens that will be removed from the place will be ds.

The other form of weight indication is: (N1d1) or (N2 d2) or (N3 d3). In this form, if any of the condition is satisfied then the firing condition is satisfied.

Enable Arcs



Tc = (Te = 1 d) : (Tr = 0 d)

Figure 13 Channel enable arc

Fig 13 shows arcs that act as an enabler when at least one token of the dataset 'd' is available while on firing, no token is removed. The next sections explain how these channel arc transitions are used in the proposed model Fig 14.

F. Reference model description

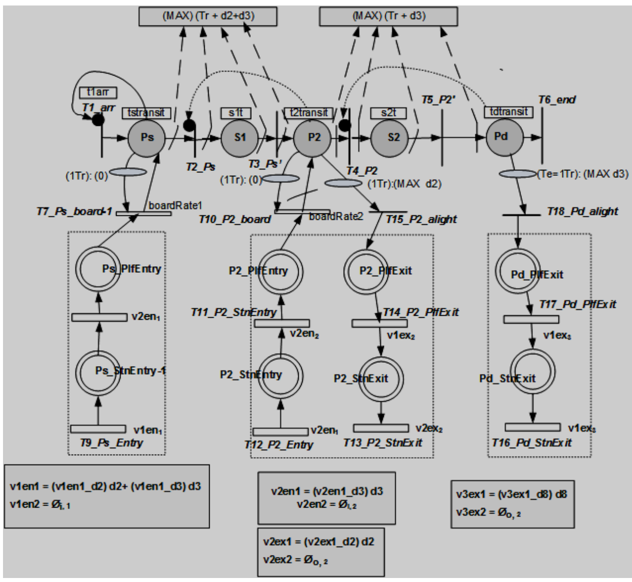


Figure 14: Model for a simple case of a railway line

G. Train movement

The train movement is purely a discrete system based Petri net which starts at origin Ps and reaches the last destination Pd. T1_arr is fired whenever a train is to arrive at the origin Ps and this will be based on the schedule of the train that is modelled as Trigger timer t1arr that represents the inter-arrival of the train to origin. It may be cyclic or acyclic depending on the timetable of the train. Ps, P2 and Pd are platform places while S1 and S2 are the travel places for trains which are between the platform places. Ps and Pd represent the origin and destination places respectively. The train movement can be modelled with the incidence matrix shown in Table 1. The timing behaviour is modelled using the P-timed places. The P-times *tstransit*, *t2transit* and *tdtransit* represent how long the train waits at each Platform. The P-times *s1t* and *s2t* represents the time taken to travel between the platform places and this is referred as section place timings.

Table 1 INCIDENCE MATRIX FOR TRAIN MOVEMENT

T1_arr	T2_Ps	T3_Ps'	T4_P1	T5_P1'	T6_end	
1	-1	0	0	0	0	Ps
0	1	-1	0	0	0	S1
0	0	1	-1	0	0	P2
0	0	0	1	-1	0	S2
0	0	0	0	1	-1	Pd

Considering different data sets for different destinations the Incidence matrix in Table 1 can be replaced with the one in Table 2 that shows the data sets with different destination. It may be noted that there is a token Tr that gets added to Ps on trigger of T1_arr while from all other places on triggers the data set comprising of Tr, d2 and d3 is removed and added which is for the passenger traffic intending to reach different destination. T2_Ps and T3_Ps' have addition and removal of MAX (Tr+d2+d3) while T4_P1 and T5_P1' will have addition and removal of MAX (Tr+d2) to state that the tokens of data set d2 will get removed at place P1 and hence will not be found further on other triggers. On the trigger of T6_end, the train will depart from the line as Pd is the last destination.

TABLE 2 INCIDENCE MATRIX FOR TRAIN MOVEMENT WITH DATA SETS

T1_arr	T2_Ps	T3_Ps'	T4_P1	T5_P1'	T6_end	
Tr	(-1) MAX(Tr+d2+d3)	0	0	0	0	Ps
0	MAX(Tr+d2+d3)	(-1) MAX(Tr+d2+d3)	0	0	0	S1
0	0	MAX(Tr+d2+d3)	(-1) MAX(Tr+d3)	0	0	P2
0	0	0	MAX(Tr+d3)	(-1) MAX(Tr+d3)	0	S2
0	0	0	0	MAX(Tr+d3)	(-1) Tr	Pd

H. Defining the tokens

The Passenger traffic is defined in terms of tokens for different destination based on O-D matrix.

I. Marking and Properties of Petri net with train movement

The train movement across the stations is purely a discrete system. If we do not consider the passenger movement in the train and consider the tokens exclusively for the movement of train and analyse, it has the following properties.

1) There cannot be more than one train in a platform or a section. The number of train tokens in a place cannot exceed one anytime.
2) The train leaves any of the platform places Ps, P1 only when there is no train in the respective next platform places P1 and Pd. This is modelled with inhibitors on transitions T2_Ps and T2_P1. A train at platform place Pn shall leave the place at time t, only when the train at platform place Pn+1 has left at t-.
3) The Petri net will always be live since the firing in the net continues as long as t1arr is a value which fires the token to P-s provided no token is in P-s.

1) Passenger movement in the train across the stations

The incidence matrix in Table 1 is good to model the train movement and comes from the Petri net basics. The incidence matrix also enables us to get all the reachable markings. It does not model the flow of passenger since it does not incorporate tokens of separate groups based on the destination. It can very well be modelled using the Colour Petri Nets with tokens representing colour sets to signify both the number of the passengers and the destination say, (n, dest). But the firing control, on the arcs T8_Ps_board, T10_P2_board, T13_P2_alight, T17_Pd_alight can be controlled using the actions like, if destination of the token is say Pn, then fire, otherwise not. But this remains as semantics as needed for the programming tool. But the disadvantage is that it may not exactly help us to build a mathematical equation. From this point of view, the channel arcs are defined as explained in earlier section that is part of the Model.

2) More details about the model

Fig 13 is a simplified form applied for lesser number of places and is simply a subset of any other complicated configurations used to explain the basic concepts behind.

a) Discrete events

At Ps, there are outgoing channel arcs with weight (1Tr):0 and (1)Tr : MAX d2 to the transitions T1_Ps_board an . Also, T1_Ps_board has an incoming arc from the place P_PlfEntry-1. The transition T1_Ps_board will be fired when there is a token with dataset Tr in Ps which represents train and a token available in P_PlfEntry-1. But the '0' in arc weight '(1Tr):0' represents that no token will be removed from P-s when T1_Ps_board is fired but the tokens will be removed from P_PlfEntry-1 at the rate of boardRate1. At P2, the arcs to and from T10_P2_board has the same significance as explained. At P2, there is an outgoing channel arc to T13_P2_alight which has arc weight MAX d2. This means, the tokens with data set d2 will be removed from P2 and added to P_PlfExit-2 since there is an outgoing arc from T13_P2_alight to P_PlfExit-2. The same explanation holds good for the channel arc from Pd.

b) Continuous events

The continuous transitions T9_Ps_Entry, T8_Ps_StnEntry, T12_P2_Entry, T11_P2_StnEntry, T14_P2_PlfExit, T13_P2_StnExit, T17_Pd_PlfExit, T16_Pd_StnExit all give rise to continuous flow of passenger with the flow rate of $v1en_1$, $v2en_1$, $v1en_2$, $v2en_2$, $v1ex_2$, $v2ex_2$, $v1ex_3$, *and* $v2ex_3$ respectively.

O-D matrix form: *StnEntryRate* =

$$\begin{bmatrix} 0 & \alpha_{1,2} & \alpha_{1,3} \\ 0 & 0 & \alpha_{2,3} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$$

StnEntryRate is a column matrix with each row is for a station.

$$v1en_1 = \alpha_{1,2}\, d_2 + \alpha_{1,3}\, d_3 \quad (22)$$

$$v1en_2 = \alpha_{2,3}\, d_3 \quad (23)$$

$$v1en_3 = 0 \quad (24)$$

$v1en_1$: Signifies the passenger flow into the Station Place into Platform place in Station 1. It is represented using the tokens of dataset d2 (destination is P-2) and d3 (destination P-d).

$v1en_2$**:** signifies the passenger flow from Station Place into the Platform place in Station 2. It is represented using the tokens of dataset

$v2en_1$ and $v2en_2$ are passenger flow rates in Station 1 and 2 respectively to reach the respective platform places.

$v1ex_2$, $v2ex_2$: are the passenger flow rates from platform place to station place and station place to exit respectively for station 2 while $v1ex_3$, $v2ex_3$ represent the same destination i.e. station 3.

c) Interaction between continuous and discrete parts

When the train has arrived it leads to two discrete trigger transitions one for the alight and other one for boarding. For station1, there will be only transition for boarding T7_board_Ps. When the train arrives in station2, the discrete transitions T10_board_P1 and T15_alight_P1 are triggered. When the train arrives in last destination, T20_alight_Pd is triggered and there will be only boarding.

Mathematically, the incidence matrix in Table 2 is considering train as well as the passenger flow for the model shown in

Figure 14. Number of places, in total is 13 and the number of triggers is 18. The incidence matrix is of dimension 13x18. 'MAX' within the matrix entries signifies that all the tokens from the places will be removed or added. The new marking of the place will be calculated based on Equation 21.

Passenger movement within a station

Station entry places: Ps_StnEntry, P2_StnEntry

Station exit places: P2_StnExit, Pd_StnExit

Platform entry places: Ps_PlfEntry, P2_PlfEntry

Platform exit places P2_PlfExit, Pd_PlfExit

Some example of marking changes on different transition firing is shown in TABLE 3.

With initial marking as all zeroes, the TABLE 3 shows the change in markings on different triggers. Values considered in the example are: $v1en_1 = 6d_2 + 10\, d_3$ passengers/10s with 6 to destination P2 and 10 to destination Pd, $v1en_2 = 8\, d_2$ passengers/10s, with all of them to destination Pd,

$v2en_1 = v2en_2 = 8$ passengers/10s,

boardRate1=*boardRate2*=10passengers/10s taking 10s as the unit time for the rates. T9_Ps_Entry-1, T12_P2_entry-2, T_plfentry-1, T_plfentry-2 are the triggers that can occur at the same time but the ordering of trigger is extremely important. The platform entry place and platform exit place are physically the same in real world since the arriving and boarding passengers share the common place but it is referred separately in the modelling to keep track of the passenger as separate entities. The station exit place is one from where the passenger exits the station. Please note that the starting station has P-s has only entries while the destination station P-d has only exits. Each railway station has a periodic operation which starts on the arrival of the train. The train is halted for a period of tstransit, t2transit and tdtransit in each platform after which the transition T1', T2' and T3' is fired respectively. The passengers board from PlfEntry places and arrive in PlfExit places.

VI. TRANSIT STATIONS WITH LINE INTERSECTION

We consider in this section the configuration of the transit station as shown in

Figure 4 as it needs addition modelling technique as compared to reference mode in section V.F Reference model description. On alighting from the train, the passengers will leave the platform place at a flow rate to enter the transit place. There is a walking time in the transit place that is modelled using pure delay of continuous flow as in Section 6.4.3 of [4].

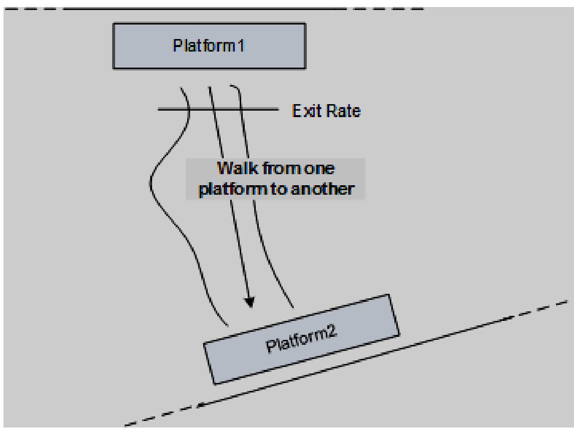


Figure 15: Example of walk to transit in a station

TABLE 3 INCIDENCE MATRIX

	T1_arr	T2_Ps	T3_Ps'	T4_P1	T5_P1'	T6_end	T7_Ps_board	T8_Ps_StnEntry	T9_Ps_Entry	T10_P2_board	T11_Ps_StnEntry	T12_Ps_Entry	T13_P2_Exit	T14_P2_PlfExit	T15_P2_alight	T16_Pd_Exit	T17_P2_PlfExit	T18_P2_alight
Ps	*Tr*	*(-1) MAX(Tr+d2+d3)*	*0*	*0*	*0*	*0*	*(boardRate1)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
S1	*0*	*MAX(Tr+d2+d3)*	*(-1) MAX(Tr+d2+d3)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
P2	*0*	*0*	*MAX(Tr+d2+d3)*	*(-1) MAX(Tr+d3)*	*0*	*0*	*0*	*0*	*0*	*(boardRate2)*	*0*	*0*	*0*	*0*	*(-1) MAX(d2)*	*0*	*0*	*0*
S2	*0*	*0*	*0*	*MAX(Tr+d3)*	*(-1) MAX(Tr+d3)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
Pd	*0*	*0*	*0*	*0*	*MAX(Tr+d3)*	*(-1) Tr*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-1) MAX(d3)*
Ps_PlfEntry	*0*	*0*	*0*	*0*	*0*	*0*	*(-boardRate1)*	*(v2en1)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
Ps_StnEntry	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-v2en1)*	*(v1en1)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
P2_PlfEntry	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-boardRate2)*	*(v2en2)*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
P2_StnEntry	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-v2en2)*	*(v1en2)*	*0*	*0*	*0*	*0*	*0*	*0*
P2_StnExit	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-v2ex2)*	*(v1ex2)*	*0*	*0*	*0*	*0*
P2_PlfExit	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-v1ex2)*	*MAX(d2)*	*0*	*0*	*0*
Pd_StnExit	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*(-v2ex2)*	*(v1ex2)*	*0*
Pd_PlfExit	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	(-v1ex2)	*MAX(d3)*

TABLE 4 PLACE MARKING FOR ENTRY PLACES ON TRAIN ARRIVAL

	Initial Marking	10s		10s		10s		10s	
		T9_Ps_Entry T12_Ps_Entry	T8_Ps_StnEntry, T11_P2_StnEntry,	T9_Ps_Entry T12_Ps_Entry	T1_arr, T8_Ps_StnEntry, T11_P2_StnEntry	T9_Ps_Entry T12_Ps_Entry	T7_Ps_board	T8_Ps_StnEntry, T11_P2_StnEntry	T9_Ps_Entry T12_Ps_Entry
Ps	0	0	0	0	Tr	Tr	Tr+10d2+10d3	Tr+10d2+10d3	Tr+10d2+10d3
S1	0	0	0	0	0	0	0	0	
P2	0	0	0	0	0	0	0	0	
S2	0	0	0	0	0	0	0	0	
Pd	0	0	0	0	0	0	0	0	
Ps_PlfEntry	0	0	5d2+5d3	5d2+5d3	10d2+10d3	10d2+10d3	0	5d2+5d3	5d2+5d3
Ps_StnEntry	0	6d2+10d3	d2+5d3	7d2+15d3	2d2+10d3	8d2+20d3	8d2+20d3	3d2+15d3	9d2+25d3
P2_PlfEntry	0	0	8d2	8d2	16d2	16d2	16d2	24d2	24d2
P2_StnEntry	0	8d2	0	8d2	0	16d2	16d2	8d2	16d2
P2_StnExit	0	0	0		0	0	0	0	
P2_PlfExit	0	0	0		0	0	0	0	
Pd_StnExit	0	0	0		0	0	0	0	
Pd_PlfExit	0	0	0		0	0	0	0	

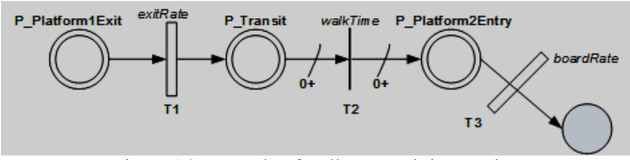


Figure 16: Example of walk to transit in a station

There is a fixed time considered for walking from platform 1 exit to Platform 2 entry and this corresponds to a pure delay. Passengers entering the transit place reach the Platform 2 entry after a constant time is elapsed. The pure delay of a continuous flow from a continuous place P_Transit to P_Platform2Entry is modelled using transition T2 between both, with arc weights 0^+ for arcs P_Transit->>T2 and T2->>P_Platform2Entry. This behaviour corresponds to the continuous firing of a D-transition. The weights 0_+ of arcs mean that as soon as an infinitely small quantity of part is available in P_Transit, transition T2 is enabled and this quantity will be placed at the end walking time at P_PlatformEntry when the time *walkTime* has elapsed.

VII. Behavioural Properties

The places in the Petri net are considered to be k-bounded. The number of train tokens does not exceed 1. This is been ensured with the inhibitor arcs. There cannot be more than one train in a station. The number of passenger in the train at any point of time must not exceed the value of considered 'k' value which depends on the capacity of the train and this can be used as a measure to find out whether the frequency and waiting times of train in each platform meets this requirement. The station entry and exit places, platform entry and exit places are also considered to be k-bounded. The number of passenger must not exceed the capacity of these places to determine whether it leads to an uncomfortable feeling to the passengers. This helps to design the platforms to suite the capacity but this is not assessed here as we do not have all the details.

VIII. Simulation and Output Analysis

A. Simulation scenario

We consider a configuration that has intersecting lines with a common station but people need to take time to transit from one platform to other platform in this station. From the Hong Kong route map, we consider the East Rail line and the Kwun Tong line that have common transit stations at Kowloon Tong. Taking this as the inspiration and with the characteristics of the train from the rolling stock data available from net and certain approximate inputs from publicly available data, the incidence matrix is calculated based on these rates. The names of the stations are mentioned generally as station 1, 2, 3... Etc. and platform 1, 2.. etc. instead of the exact names of stations in HK MTR map as it should not give an impression that we have the exact data available from MTR.

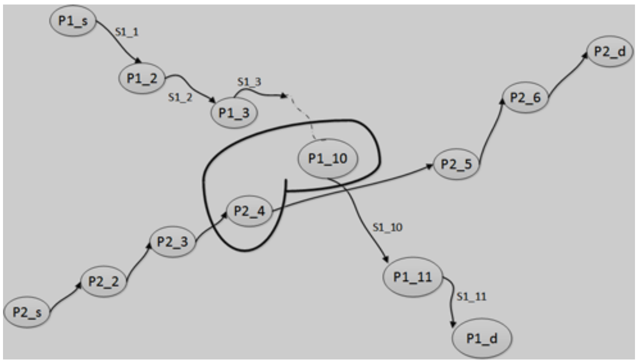


Figure 17 Two line topology with an intersection

B. Inputs to the simulation

The inputs to the model are: 1) Number of lines, Stations in each line and the common platforms across the lines 2) Scheduling of the train that includes frequency of the train at origin, distance between stations and dwell times in each platform 3) Inflow rate to enter the station and inflow rate to enter the platform via paid gates 4) Outflow rate to exit platform via paid gates and outflow rate to go out. The flow rates can be different in each station. TABLE 5 provides the inputs on the topology of the lines. TABLE 6 provides the characteristics of the trains in lines 1 and 2.

TABLE 7 and

TABLE 8 provides the train time tables of line 1 and line 2 respectively including the travel time and dwell times in different stations. TABLE 9 brings out the calculations for the boarding and alighting rate calculations.

The entry rates $v1en_n$ for each station goes according to TABLE 11 and TABLE 12 but a stochastic input is generated using randomisation with values going up to maximum for the passenger group pertaining to a destination based on the O-D matrix. The entry rates $v2en_n$ for each station to enter the platform goes according to the stochastic inputs is generated using randomisation as in TABLE 13 and TABLE 14. The model is generated for the simulation of the results with 144 transitions and 106 places to get all the necessary plots.

TABLE 5 TOPOLOGY OF THE SIMULATED STATION

	Values
Number of lines l	{2}
Number of Stations in each line n	{12, 7}
Change over main stations	(Transit of passengers from Station 8 of Line 1 to Station 5 of Line 2 and vice versa)
Cyclic frequency of trains in lines	{8, 5}
Speed of the train	80km/hr

TABLE 6 TRAIN CHARACTERISTICS TAKEN

Train on Line 1

Capacity	4548 passengers
Speed	80 km/hr
Acceleration	1 m/s/s
Deceleration	1 m/s/s
No. of vehicles or cars	12
No of doors in each vehicle	4
Train on Line 2	
Capacity	2504 passengers
Speed	80 km/hr
Acceleration	1 m/s/s
Deceleration	1 m/s/s
No. of vehicles or cars	12
No of doors in each vehicle	4

TABLE 7 TRAIN TIME TABLE ON LINE 1

Frequency of train	**6 minutes**	
Station no.	Dwell time (minutes)	Travel time to next station (minutes)
1	2	4
2	0.5	2
3	0.5	5
4	0.5	2
5	0.5	6
6	0.5	3
7	0.5	2
8	0.5	2
9	0.5	4
10	1	3
11	1	3
12	2	-
Duration from origin to last destination	46 minutes (including wait at destination)	

TABLE 8 TRAIN TIME TABLE ON LINE 2

Frequency of train	**6 minutes**	
Station no.	Dwell time (minutes)	Travel time to next station (minutes)
1	2	2
2	0.5	1
3	0.5	2
4	0.5	2
5	0.5	2
6	1	2
7	2	-
Duration from origin to last destination	18 minutes (including wait at destination)	

TABLE 9 BOARDING AND ALIGHTING RATE CALCULATION

Parameter	Symbol	Value	Remarks
Shared gate capacity	C_{door}	*Door width/Mean width* = 2	General Standards
Number of cars or vehicles	N_{vk}	12 for Line 1, 8 for Line 2	Rolling stock details from [24]
Number of doors per each vehicle	$N_{AB/v}$	5	Rolling stock details from [24]
Global capacity of train considering all doors	C_{Train}	$N_{vk} * N_{AB/v} * C_{door}$ 120 for Line1, 80 for Line 2	Estimation. [18] with adaptations.
Walking time through the doors	τ	*0.7s min, 3.3s max*	Approximations using [18]
Max Shared Boarding/Alighting Rates	$alightRate_{l,n}$ $boardRate_{1,n}$	C_{Train}/τ Trains in Line 1: min: 36 passengers/s max: 171 passengers/s Trains in Line 2: min: 24 passengers/s, max: 114 passengers/s	Estimations. These rates will be shared if boarding/rating takes place simultaneously.

TABLE 10 INPUT AS FLOW RATES

Parameter	Symbol	Value	Remarks
Entry Rates at Station *n* on line *l*	$v1en_{l,n}$	Stochastic using O-D matrix inputs as minimum	Simulated
Platform Entry Rate at Station *n* on line *l*	$v2en_{l,n}$	Stochastic with minimum rate	Simulated
Boarding Rate	$alightRate_{l,n}$	$alightRate_{l,n}$ + $boardRate_{l,n}$ = 171 passengers/s for line 1 and 114 passengers/s on Line 2	Since boarding and alighting is assumed to happen at the same time
Alighting Rate	$boardRate_{l,n}$		
Platform Exit Rate	$v2en_{l,n}$	30 per unit time of 30s	Simulated
Exit Rate from Station	$v2ex_{l,n}$	50 per unit time of 50s	

TABLE 11 O-D MATRIX FOR MAXIMUM FLOW RATE FOR LINE 1 STATIONS

Origin	Destination												Station Entry Rates
Station no.	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Transit to other line**	***v1en_n***
1	0	0	0	0	0	1	1	1	1	2	2	2	(0*d2+ 0*d3+ 0*d4 + 0*d5 + 0*d6 + 1*d7 + 1*d8 + (1+2)* d9 + 1 * d10 + 2 * d11 + 2 * d12)
2		2	1	2	2	2	1	1	1	1	2	2	(5 * d3+ 3 * d4 +4 * d5 + 4 * d6 + 4 * d7 + 2 * d8 + (2+5) * d9 + 3 * d10 + 3* d11 + 3 * d12)
3			2	2	2	2	1	1	1	2	2	2	(5 * d4 +4 * d5 + 4 * d6 + 4 * d7 + 3 * d8 + (3+5) * d9 + 3 * d10 + 3* d11 + 3 * d12)
4				2	2	2	2	1	1	2	1	2	(5 * d5 + 4 * d6 + 4 * d7 + 4 * d8 + (3+5) * d9 + 3 * d10 + 3* d11 + 3 * d12)
5					2	2	2	2	2	1	2	2	(4 * d6 + 3 * d7 + 3 * d8 + (4+5) * d9 + 4 * d10 + 3* d11 + 3 * d12)
6						2	1	2	2	2	2	3	(4 * d7 + 3 * d8 + (4+6)*d9 + 4*d10 + 3*d11 + 3 * d12)
7							3	2	2	2	2	3	(5 * d8 + (4+5) * d9 + 4 * d10 + 4* d11 + 4 * d12
8								3	2	2	2	2	(5+5) * d9 + 4 * d10 + 4* d11 + 4 * d12
9									3	2	2	2	5 * d10 + 4* d11 + 4 * d12
10										3	4	-	5* d11 + 4 * d12
11											10	-	10 * d12
12	No Entry in the last destination												

TABLE 12 O-D MATRIX FOR MAXIMUM FLOW RATE FOR LINE 2 STATIONS

Origin	Destination							Station Entry Rates
Station no.	**2**	**3**	**4**	**5**	**6**	**7**	**Transit to other line**	*v1en_n*
1	6	4	3	5	4	4	0	(6*d2+ 4* d3+ 3 * d4 + 5 * d5 + 4 * d6 + 4 * d7)
2		5	3	3	7	8	(3+2)	(5 * d3+ 3 * d4 +(3+3+2) * d5 + 7 * d6 +8 * d7)
3			10	6	6	7	(2+3)	(10 * d4 +(6+2+3) * d5 + 6 * d6 +7 * d7)
4				10	8	8	(5+2)	(10 * d4 + (6+5+2)* d5 + 6 * d6 + 7 * d7)
5					10	8	-	(10 * d6 + 8* d7)
6						10		(10*d7)
7	No Entry in the last destination							

TABLE 13 FLOW RATES IN EVERY STATION FOR ENTRY AND EXIT FOR UNIT TIME ON LINE 1

Station No.	**Station Entry Rate** *$v1en_n$*		**Platform Entry Rate** *$v2en_n$*		**Platform Exit Rate** *$v1ex_n$*	**Station Exit Rate** *$v2ex_n$*
	Min	**Max**	**Min**	**Max**		
1	0	10	30	30 + (5 *12)= 90	-	-
2	0	17	36	36+(5*11)=91	30	50
3	0	17	24	24+(5*10)=74	30	50
4	0	15	24	24+(5*9)=69	30	50
5	0	15	16	16+(5*8)=40	30	50
6	0	14	14	14+(5*7)=49	30	50
7	0	14	12	12+(5*6)=42	30	50
8	0	11	10	10+(5*5)=35	30	50
9	0	9	10	10+(5*4)=30	30	50
10	0	7	10	10+(5*2)=20	30	50
11	0	10	10	10+(5*1)=15	30	50
12	-	-	-	-	30	50
	O-D matrix being max flow rate, the passengers entering to different destination can be maximised depending on randomisation. For each group of passengers to a specific destination from a station, the following is applied: ((Number of passengers to a destination based on O-D Matrix) * ***rand)***		For each group of passengers to a specific destination from a station, the following is applied: (Number of passengers to a destination based on O-D Matrix) + (5 * ***rand)***			

TABLE 14 FLOW RATES IN EVERY STATION FOR ENTRY AND EXIT ON LINE 2

Station No.	**Station Entry Rate** *$v1en_n$*		**Platform Entry Rate** *$v2en_n$*		**Platform Exit Rate** *$v1ex_n$*	**Station Exit Rate** *$v2ex_n$*
	Min	**Max**	**Min**	**Max**		
1	0	26	10	10 + (5 * 6)= 40	-	-
2	0	31	10	10+(5* 5)=35	30	50
3	0	34	10	10+(5* 4)=30	30	50
4	0	33	10	10+(5* 3)=25	30	50
5	0	18	10	10+(5* 2)=20	30	50
6	0	10	10	10+(5*1)=15	30	50
7	-	-			30	50
	O-D matrix being max flow rate, the passengers entering to different destination can be maximized up-to 10 depending on randomisation. For each group of passengers to a specific destination from a station, the following is applied: (Number of passengers to a destination based on O-D Matrix) + (10 * ***rand)***		For each group of passengers to a specific destination from a station, the following is applied: (Number of passengers to a destination based on O-D Matrix) + (5 * ***rand)***			

C. *Outputs and inferences from the simulation*

The outputs are plotted for graphical representation and visualisation of different parameters for 120 minutes. Plots that are available are:

1) Station occupancy plot: This plot shows the station perspective about the train arrival, departure and dwell. Figure 18 and Figure 19 have these plots for line 1 and line 2 respectively.
2) Time space plot: This plot shows the movement of trains from origin to the last destination capturing the distance and time travelled between the stations. The plot is shown in
3) Figure 20 for line 1.
4) Passengers waiting plot: This plot shows the number of waiting passengers at platforms of different stations at different times. Plots in Figure 21and Figure 22 are for line 1 and line 2 respectively.
5) Passengers' entry plot – This plot shows the entry of passengers to the platforms in each unit time. Plots in Figure 24 and Figure 25 are for station 1 and 2 on line 1 respectively.
6) Train loading plot – This plot shows the number of passengers in the train at different times and hence gives a view of loading of train. Figure 26 shows the plot for line 1.
7) Accumulated waiting time plot – This is the plot of the total accumulated passenger waiting times. This parameter is quite important as it gives an idea of man hours spent waiting for the trains in total from all platforms on the line. The plots are in Figure 27 for line 1 and 2.
8) Instantaneous waiting plot – This is the plot of total passengers waiting considering all the platforms of a line. The plot in Figure 28 is for line 1 and 2.

Following can be observed from the plots available for the lines:

1) The first train in the time space plot shows the arrival, departure and dwell time as per the schedule and will be normal. If the dwell time of the later trains in the stations is longer, then there will be cascading effect that causes the delays in the trains. This also means, the inter-arrival time of those trains can be spaced little later so that it does not get stuck waiting for another train ahead to leave the platform.
2) The train on line 2 arrives in Station 5 and the passengers' transit to station 10 of line 1 and have to wait for a long time initially until the first train on line 1 arrives. Hence the passenger waiting plots for Station 10 (line 1) in Figure 23 shows an initial high value because of this. Similarly the transit of passengers on exit from platform 10 on line 1 to enter platform 5 on line 2 can be in observed but plot is not included here.
3) The train loading plot shows that the maximum is around 1100 passengers at the time elapse of 45 minutes in station 10. This is still a low loading as the capacity of train is more than 4000 passengers.
4) The accumulated waiting times at the end of 2 hours for line 1 and line 2 are 1676 man hours and 871 man hours respectively. This can be controlled by timetabling of train.

Following analysis can be derived from the output when we have fed the real data into the network:

1) The places in the Petri net are considered to be k-bounded. The number of train tokens does not exceed 1 with the selected parameters. There cannot be more than one train in a station. The number of passenger in the train at any point of time does not exceed the value of considered 'k' value which depends on the capacity of the train and the simulated output can be used as a measure to find out whether the frequency and waiting times of train in each platform meets this requirement.
2) The station entry and exit places, platform entry and exit places are also considered to be k-bounded. The number of passenger must not exceed the capacity of these places to determine whether it leads to an uncomfortable feeling to the passengers. This helps to design the platforms to suite the capacity but that is not accessed here as we do not have all the details.

The plots show two important parameters that are to be considered for the optimization:

a) Number of trains on line at any point of time.
b) Accumulated passenger waiting time, Instantaneous passenger waiting time.

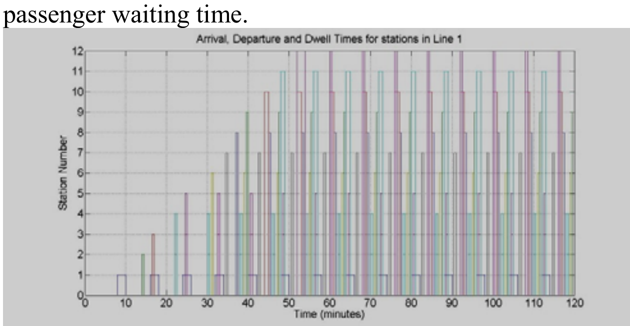


Figure 18: Station occupancy plot - train arrivals and departures in stations in line 1 (12 stations)

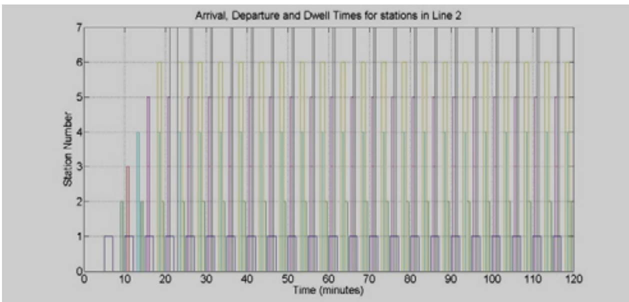


Figure 19: Station occupancy plot - train arrivals and departures in stations in line 2 (7 stations)

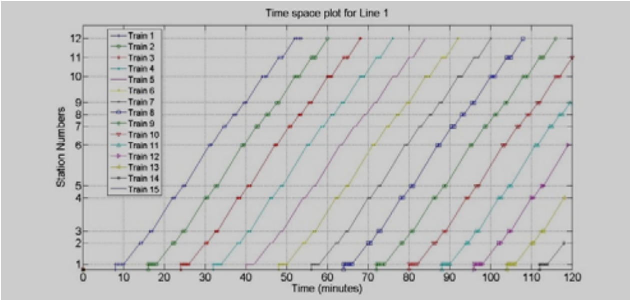


Figure 20: Time space plot for train on line 1 (12 stations)

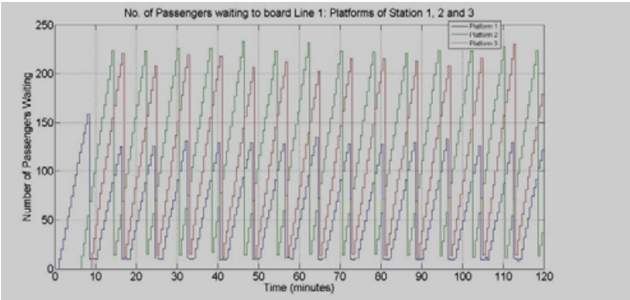


Figure 21: Passengers waiting plots - platforms to board in stations 1, 2 and 3 on line 1

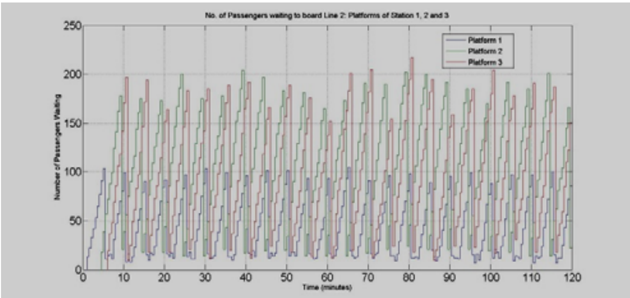


Figure 22: Passengers waiting plots - platforms to board in stations 1, 2 and 3 on line 2

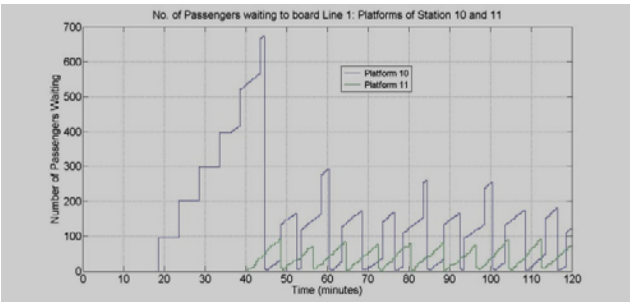


Figure 23: Passengers waiting plots - platforms to board in stations 10 and 11 on line 1

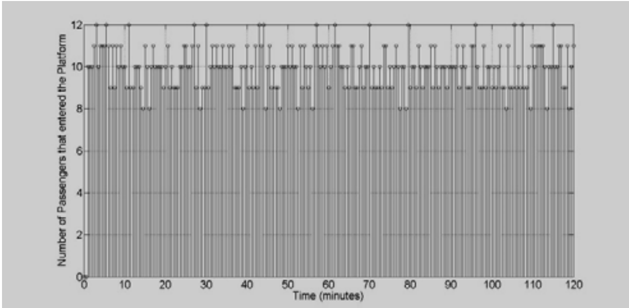


Figure 24: Passengers entry plot - platforms to board on line 1 - stations 1

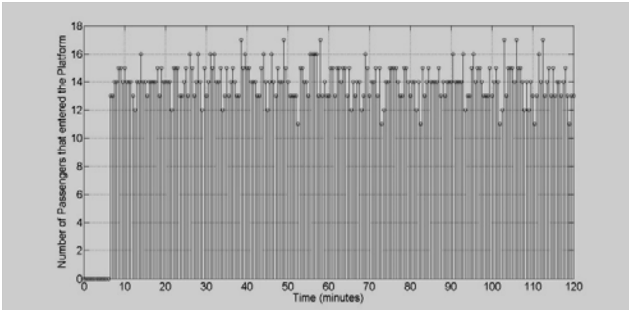


Figure 25: Passengers entry plot - platforms to board on line 1 - stations 2

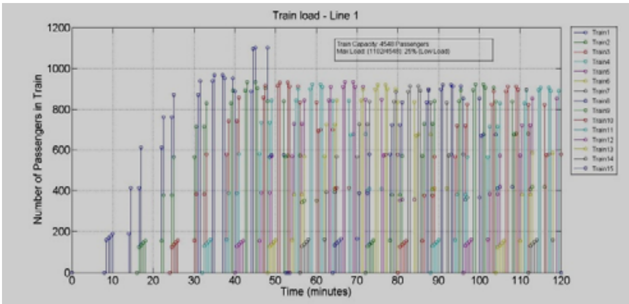


Figure 26 Train loading – line 1

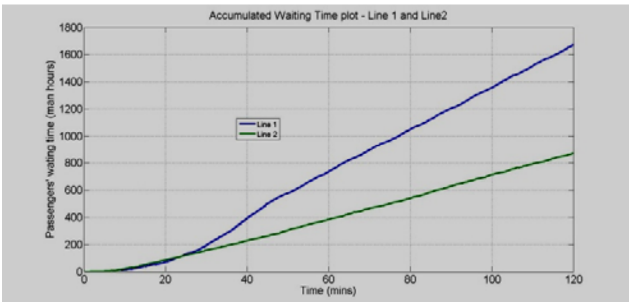


Figure 27 Accumulated passengers wait – lines 1 and 2

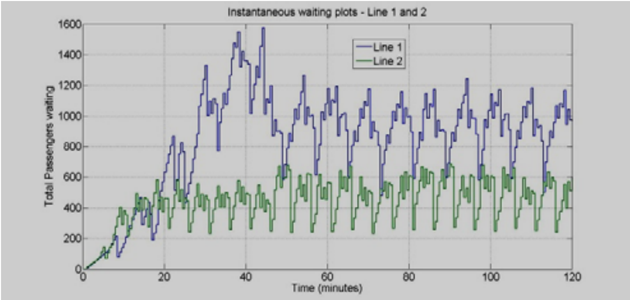


Figure 28 Instant passenger wait plots – lines 1 and 2

IX. Optimisation

After modelling the Railway and People traffic, it has been an interest in the research to work on optimization of the Train schedule in the railway network. This is not new in the field of transportation and operational research but the work is normally different considering different applications and systems. This section formulates a mathematical model for the optimization and work upon the algorithm needed to perform the same. An attempt is made to keep this section to be independent for the interested readers so that they could understand the case without reading the earlier sections.

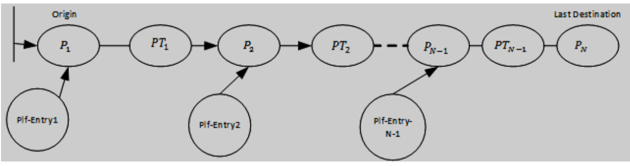


Figure 29 Representation of the train movement and platform

Train Entry trigger is the one that enables the train to go to the origin station according to the schedule of the train. The train waits for a certain time in each platform, goes to the next station after a travel time, waits in the platform for a certain dwell time and finally reaches the last destination. Passengers in the platform will have to wait for train arrival to board. The passenger flow rate entering the platform places may vary in each station depending on the flow model used based on peak traffic, non-peak traffic and average traffic times. The platforms are places P_1, P_2, P_3,….. P_{N-1}, P_N and the train will have to travel through intermediate places PT_1, PT_2, PT_3...$PT_{(N-1)}$, PT_N that are referred as transit places.

The optimization needed is on the schedule of the train

- Keeping the instantaneous train running cost to a certain minimum which alternatively means to limit the number of trains running at any point of time
- Keep the total cost (accumulated cost) of running for the entire operating in a day to be minimum
- People do not wait for a long time which leads to uncomfortable situation. To start with, we keep it as a fixed people flow rate but needs to be modelled further for different people flow models.

The train waiting time in each platform, travel duration between platforms etc. is considered fixed and will remain unchanged. The train cannot leave the current station as long as there is a train in the next station that is the next immediate destination. This is to ensure that there is a sufficient headway time for safety but it is considered as part of modelling and control and may not be too important for optimization as the passengers would have boarded the train. The timetables exist on the networks that consist of multiple track main lines in each direction with trains running in the same speed and there is no need to consider conflicts of different kind of trains running on line.

A. Formulation of Optimisation problem

1) Inputs

$\boldsymbol{d_1, d_2, ..., d_{n-1}, d_n}$ **:** Dwell times of train in each station's platforms $\boldsymbol{P_1, P_2, …, P_{n-1}, P_n}$ respectively. The waiting times in each platform enables the passengers to get in to the train and alight from the train as applicable for each platform.
$\boldsymbol{q_{1,2}, q_{2,3}, … q_{n-1,n}}$ **:** Time taken to travel from P_1 to P_2, P_2 to P_3….P_{n-1} to P_n respectively. These are the times for which the train will be in transit places between platforms – P_{1t}, P_t, P_{2t}, P_{3t} … $P_{(n-1)t}$ P_{nt}
Total duration of travel from origin to last destination is fixed as:
TRAIN_TRAVEL_DURATION :

$$(d_1 + q_{1,2})+ (d_2 + q_{2,3})+…+ (d_{(n-2)} + q_{(n-2),(n-1)})+ (d_{(n-1)} + q_{(n-1),n}) \quad (25)$$

This provides the maximum limit on the number of trains.
MAXPEOPLEWAIT: Total waits of the people considering all the people waiting in different platforms.
Assumptions:

1) The dwell time of train at every station is same. Hence the inter-arrival time and separation time between successive trains at every station is considered the same except for the first train in the opening hours.
2) For the separation time to be the same, the trains need to travel in the same planned durations between the stations that are very normal.

Remark: The last destination is of no importance as people do not wait to board the train there and hence not included in the discussions.

2) Objective

Finding the schedule for the train which includes the inter arrival time of the trains for the specified total duration for which the trains need to run and minimizing the number of trains running on the line:

$$\boldsymbol{\text{maximise } (t_k – t_{k-1}), \text{ minimise } noOfTrains} \quad (26)$$

$$\boldsymbol{T_{start} = 0 \le t_k \le T_{end} \quad \text{for } k=1,2…….M}$$

t_1, t_2, …. t_M - Represents the train arrival timings at the first station P_1 with respect to the start of Train operations $\boldsymbol{T_{start}}$

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and T_{end} represents the end of the Train operation. Units may be minute or seconds.

3) Constrains

a) Passenger flow

Considering the passenger flow with time Ts in each of the slots, the waiting time of the people can be formulated as follows:

Number of slots between the successive train arrivals $k= \{1, 2, 3 ....M\}$ at platforms of stations:

$$s_1 = (t_1 - t_0)/Ts,\ s_2 = (t_2-t_1)/Ts,\ ...\ s_M = t_M-t_{M-1}/Ts \quad (27)$$

As the people will not wait for long time knowing the timing of the first time arrival, we could consider the number of slots between the arrivals of train at each platform for which the people flow before each train arrives will be the same. The station operations start at different times because the first train arrival is different in each station. The differences in timings, say between 2nd train arrival and 1st one, 3rd train arrival and 2nd train arrival etc. and hence the frequency of the train will remain the same irrespective of the dwell times in stations. Thus, considering that the operation timings start at different timings for each station instead of same timings will simplify the relation.

Waiting time of passengers at platforms for 1st train arrival at time t_1:

$Pwait_{1,1} + Pwait_{2,1} + ... + Pwait_{N-1,1}$

Where

$Pwait_{1,1} = P_{1,1,1}\ s_1\,Ts + P_{1,1,2}\,(s_1-1)\,Ts + ... + P_{1,1,(s1-1)}\,(2)\,Ts + P_{1,1,s1}\,(1)\,Ts$

$Pwait_{2,1} = P_{2,1,1}\ s_1\,Ts + P_{2,1,2}\,(s_1-1)\,Ts + ... + P_{2,1,(s1-1)}\,(2)\,Ts + P_{2,1,2}\,(1)\,Ts$

….

$Pwait_{N-1,1} = P_{N-1,1,1}\ s_1\,Ts + P_{N-1,1,2}\,(s_1-1)\,Ts + ... + P_{N-1,1,(s1-1)}\,(2)\,Ts + P_{N-1,1,s1}\,(1)\,Ts$

(28)

$P_{j,1,i}$: Represents the number of passengers flow in the time slot i at platform j while waiting for 1st train arrival at t_1.

$i : 1$ to s_1

$j : 1$ to N

Similarly, total waiting time of passengers at stations for arrival of train t_2

$\boldsymbol{Pwait_{1,2} + Pwait_{2,2} + ... + Pwait_{N-1,2}}$

Where

$Pwait_{1,2} = P_{1,2,1}\ s_2\,Ts + P_{1,2,2}\,(s_2-1)\,Ts + ... + P_{1,2,(s2-1)}\,(2)\,Ts + P_{1,2,s2}\,(1)\,Ts$

$Pwait_{2,2} = P_{2,2,1}\ s_2\,Ts + P_{2,1,2}\,(s_1-1)\,Ts + ... + P_{2,1,(s1-1)}\,(2)\,Ts + P_{2,1,2}\,(1)\,Ts$

….

$Pwait_{N-1,2} = P_{N-1,2,1}\ s_2\,Ts + P_{N-1,2,2}\,(s_2-1)\,Ts + ... + P_{N-1,2,(s2-1)}\,(2)\,Ts + P_{N-1,2,s2}\,(1)\,Ts$

$P_{j,2,i}$: Represents the number of people flow in the time slot i at platform j while waiting for train arrival t_2

i: 1 to s_2

j: 1 to N

The above can be generalized as a constraint that for any kth train arrival at time t_k :

$${}_{k=fixed}\sum^{j=1..N} Pwait_{j,\,k} \leq MAXPEOPLEWAIT \text{ for every } k = 1 ...M \quad (29)$$

Where

$$Pwait_{j,k} = P_{j,k,1}\ s_k\,Ts + P_{j,k,2}\,(s_k-1)\,Ts + ... + P_{j,k,(sk-1)}\,(2)\,Ts + P_{i,k,sk}\,(1)\,Ts \quad (30)$$

$P_{j,k,i}$: Number of people entered the platform j at i th slot while waiting for train t_k. This depends on the passenger flow model that can be fixed, stochastic or whatever.

b) Headway Time

It must be ensured that there is a minimum time between the arrivals of the trains in every station and is more than a minimum value which is referred as the headway time. Assuming MIN_HEADWAY_TIME is the headway time, there must be a constraint for the same:

$$\boldsymbol{(t_k - t_{k-1}) \geq MIN\_HEADWAY\_TIME} \quad (31)$$

c) Number of trains

Since the cost incurred is directly proportional to the number of trains, there is a need to keep it limited which is formulated as following constrain:

At any point of time, there should not be more than certain trains running on the line:

$$(t_k - t_{k-noOfTrains}) \leq (d_1 + q_{1,2}) + (d_2 + q_{2,3}) + ... + (d_{(n-2)} + q_{(n-2),(n-1)}) + (d_{(n-1)} + q_{(n-1),n}) = TRAIN\_TRAVEL\_DURATION \quad (32)$$

Number of trains active on the line at any point of time: *noOfTrains*

B. Approaches and Optimisation

There are two approaches included as part of this paper. One approach is based considering different constant flow which can be average Passenger flow in each platform and making use of the standard optimization technique. This leads to a cyclic inter arrival time. The other approach is based an algorithm that runs on the samples of available data to make a proposal on the different inter arrival timings that means it is non-cyclic. Both are explained in the following sections.

1) Average passenger flow based

The passenger entry to the platform is always varied and data can be available in control rooms knowing how many enter the paid areas. However if the mathematical formulation is to be generally considered, it is complicated as the equations are considered for each inter-arrival time. A case of considering constant and average flow in all stations is considered here. If $PASSENGER_{1,k}$, $PASSENGER_{2,k}$, ...$PASSENGER_{N,k}$ are the average passenger entries in stations 1 to N respectively during the inter-arrival period in each slot, waiting for a train k and if we consider a cyclic timetable, then the formulations will be as follows:

$$Pwait_{1,1} = PASSENGER_{1,1}\ (s_1)(Ts) + PASSENGER_{1,1}(s_1-1)(Ts) + \ldots + PASSENGER_{1,1}\ (2)\ (Ts) + PASSENGER_{1,1}\ (1)\ (Ts)$$

$$Pwait_{2,1} = PASSENGER_{2,1}\ (s_2)\ (Ts) + PASSENGER_{2,1}\ (s_2-1)(Ts) + \ldots + PASSENGER_{2,1}\ (2)\ (Ts) + PASSENGER_{2,1}\ (1)\ (Ts)$$

.....

$$Pwait_{N,1} = PASSENGER_{N,1}\ (s_N)\ (Ts) + PASSENGER_{N,1}\ (s_N-1)(Ts) + \ldots + PASSENGER_{N,1}\ (2)\ (Ts) + PASSENGER_{N,1}\ (1)\ (Ts)$$

$$Pwait_{1,1} = [(PASSENGER_{1,1})\ (s_1)(s_1+1)/2].\ Ts$$
$$Pwait_{2,1} = [(PASSENGER_{1,1})\ (s_1)(s_1+1)/2].\ Ts$$
……
$$Pwait_{N,1} = [(PASSENGER_{1,1})\ (s_1)(s_1+1)/2].\ Ts$$

For the first arrival of the train:

$$\sum_{j=1,N} Pwait_{j,1} = [(PASSENGER_{1,1} + PASSENGER_{2,1} + \ldots + PASSENGER_{N-1,1})\ (s_1)\ (s_1+1)/2].Ts \le MAXPEOPLEWAIT \tag{33}$$

For the arrival of the train k:

$$\sum_{j=1,N} Pwait_{j,k} = [(PASSENGER_{1,k} + PASSENGER_{2,k} + .+ PASSENGER_{N-1,k})\ (s_N)\ (s_N+1)/2].Ts \le MAXPEOPLEWAIT \tag{34}$$

For line 1 of simulation scenario, following are the inputs
TRAIN_TRAVEL_DURATION: 43 minutes
MAXPEOPLEWAIT: 100*11*5*60 = 330000 seconds (100 people waiting in 11 stations for 5 minutes)
MIN_HEADWAY_TIME = 30s
Ts = 30s (Time Resolution)
Average passenger entry to platform in each station for each of time slots for the arrival of every train is taken from the stochastic inputs for which the simulation (**TABLE 15**) is run for 120 minutes:

$PASSENGER_{1,k} = 10$
$PASSENGER_{2,k} = 14$
$PASSENGER_{3,k} = 13$
$PASSENGER_{4,k} = 11$
$PASSENGER_{5,k} = 10$
$PASSENGER_{6,k} = 9$
$PASSENGER_{7,k} = 8$
$PASSENGER_{8,k} = 6$
$PASSENGER_{9,k} = 5$
$PASSENGER_{10,k} = 4$
$PASSENGER_{11,k} = 4$

The optimisation objective and constrains are summarized below for getting the inter-arrival time for 22 trains:

Objective:
Taking $t_k - t_{k-1} = z$
maximise z, minimise $noOfTrains$

$$\Rightarrow \textbf{minimise } (-z + noOfTrains) \tag{35}$$

Decision Variables: (22 inter-arrival times)

$$z, s_1, s_2 \ldots s_{22}, noOfTrains \tag{36}$$

In fact all the decision variables are related.

Constrains:

1) $z \ge MIN\_HEADWAY\_TIME$ (37)

2) $s_1 \cdot (Ts) \ge MIN\_HEADWAY\_TIME$
$s_2 \cdot (Ts) \ge MIN\_HEADWAY\_TIME$
$s_3 \cdot (Ts) \ge MIN\_HEADWAY\_TIME$
…
$s_{22} \cdot (Ts) \ge MIN\_HEADWAY\_TIME$

3) For the arrival of train k = {1, 2,…M}. Below relation leads to M equations.

$$\sum_{j=1,N} Pwait_{j,k} = [(PASSENGER_{1,k} + PASSENGER_{2,k} + \ldots + PASSENGER_{N-1,k})\ (s_N)\ (s_N+1)/2].Ts \le MAXPEOPLEWAIT \tag{38}$$

4) $t_k - t_{k\text{-}noOfTrains} \le TRAIN\_TRAVEL\_DURATION$ (39)

Knowing that the inter arrival time is cyclic this formulation is equivalent to having

$$s_1.(Ts) \ge TRAIN\_TRAVEL\_DURATION/noOfTrains$$
$$s_2.(Ts) \ge TRAIN\_TRAVEL\_DURATION/noOfTrains$$
………………..
$$s_{22}.(Ts) \ge TRAIN\_TRAVEL\_DURATION/noOfTrains \tag{40}$$

Using MIQCQP (Mixed Integer Quadratically Constrained Quadratic Program) from MATLAB based OPTI Toolbox from [25], solvers are available. The inter-arrival time for the 22 trains was found as follows:

s_1	10 slots, $(t_1 - t_0)$ =300s
s_2	10 slots, $(t_2 - t_1)$ = 300s
s_3	10 slots, $(t_3 - t_2)$ = 300s
...	….
...	……
s_{22}	10 slots, $(t_{22} - t_{21})$ = 300s
z	300 s
noOfTrains	9

Similarly, for line 2 with the following data
TRAIN_TRAVEL_DURATION: 16 minutes
MAXPEOPLEWAIT: 100*6*5*60 = 180000 seconds (100 people waiting in 6 stations for 5 minutes)
MIN_HEADWAY_TIME = 30s
Ts = 30s (Time Resolution)
Average Passenger entry to platform in each station is taken from the stochastic inputs given to the simulation:
$PASSENGER_{1,k} = 14$
$PASSENGER_{2,k} = 18$
$PASSENGER_{3,k} = 18$
$PASSENGER_{4,k} = 18$
$PASSENGER_{5,k} = 9$
$PASSENGER_{6,k} = 5$

The following inter-arrival times for the 22 trains were found as follows and this is applicable for Line2:

s1	8 slots $(t_1 - t_0)$ = 240 s
s2	8 slots $(t_2 - t_1)$ = 240 s
s3	8 slots $(t_3 - t_2)$ = 240 s
...	….
...	……
s22	8 slots $(t_{22} - t_{21})$ = 240 s
z	240 s
noOfTrains	5

The number of slots is actually an integer since it is the multiple of Unit time for inter arrival time. Normally, the railways run with time tabling of resolution of 30s or in the order of minutes while with Japanese railway systems it goes to the extent of using 10s resolution. Since the averaging out and cyclic time table is used in this approach, even though not many people are waiting in the beginning of the operation, trains will be scheduled at the same frequency and hence aggressively keeping the people waiting time lower. With the application of optimisation, at the end of 2 hours, the number of waiting man hours waiting saved is 623 and 163 for line 1 and 2 respectively. This accounts for reduction of 37% and 18% of People waiting times. This is achieved with the increase of trains from 6 to 9 and 4 to 5 running on Lines 1 and 2 respectively. The train running times is increased from 8.5 to 13.5 train hours on line 1 and from 5.7 to 7.3 hours on line 2 with this and hence the cost increase. Plots in Figure 30 to Figure 33 show the impact of optimisation in terms of the accumulated passenger waiting time and also the train running times.

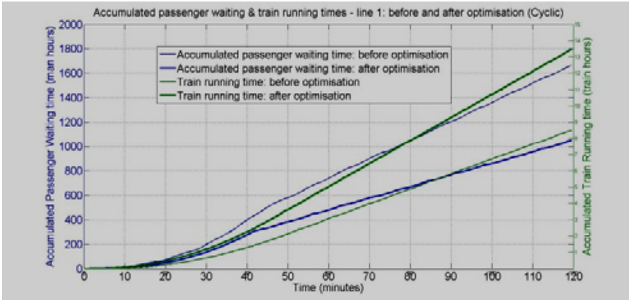

Figure 30 Accumulated passenger wait and train run times – optimisation for line 1 with cyclic timetable

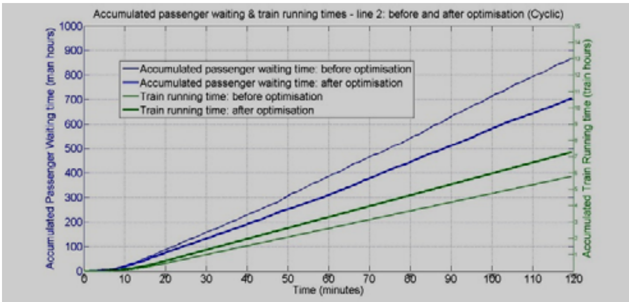

Figure 31 Accumulated passenger wait and train run times – optimisation for line 2 with cyclic timetable

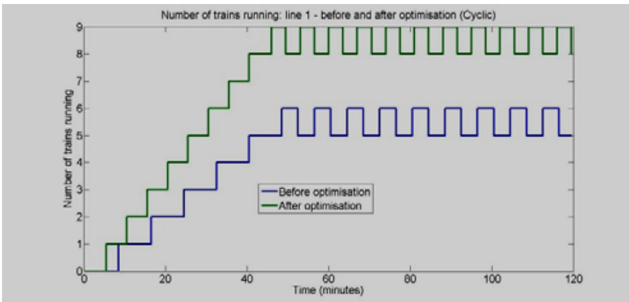

Figure 32 Number of operating trains for line 1; before and after optimisation with cyclic Timetable

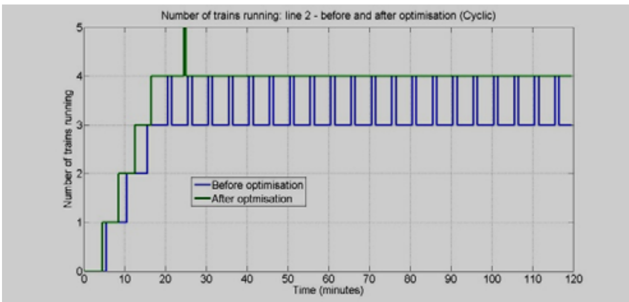

Figure 33 Number of operating trains for line 2; before and after optimisation with cyclic Timetable

TABLE 15 PASEENGERS ENTERING PLATFORMS IN 11 STATIONS (UNIT TIME: 30s)

Unit Time: 30 s	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Station 1	0	0	10	10	10	11	12	10	11	11	11	12	11	9	11	9	11	10	11	10	10	9	12	9	9	10	10	10	9	8	11	10	8	10	11	10	10	10	10	9	10	11	9	10	9	9	9	10	10	11	11	10	10	11	12	9	11	8	9	9	12
Station 2	0	0	0	0	0	0	0	0	0	0	0	0	0	13	13	14	14	15	15	14	15	14	14	14	13	12	14	14	16	14	14	13	14	14	14	14	15	13	15	14	14	14	14	12	15	15	15	13	13	14	15	13	16	15	13	14	16	14	12	15	13
Station 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	15	14	16	16	11	14	12	13	14	13	13	16	14	12	12	12	13	14	14	14	15	12	11	13	13	13	13	13	13	13	12	13	13	14	14	15	12	13	16	14	13	14
Station 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	11	14	12	12	12	11	11	14	11	11	14	11	13	11	11	11	13	12	12	10	11	13	11	10	12	11	15	11	13	13	13
Station 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	11	12	12	12	11	13	10	12	11	12	12	13	11	12	10	10	10	13	11	12	10	11	13	13	11	11
Station 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	13	9	12	10	12	11	12	12	11	12	10	13	12
Station 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11	12	10	10	8	11	12
Station 8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	7
Station 9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Station 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Station 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Unit Time: 30 s	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Station 1	9	10	11	10	11	10	10	11	10	10	11	10	11	9	9	9	10	8	10	11	9	10	9	11	9	12	11	12	8	10	11	9	10	9	9	8	10	10	10	11	10	11	10	10	8	11	9	11	10	9	10	8	11	12	11	10	11	10	11	9
Station 2	14	16	15	16	14	14	15	12	13	15	13	14	15	14	13	13	17	15	16	15	15	14	14	14	13	13	15	16	14	12	14	16	14	13	15	14	14	17	15	14	13	13	13	13	11	15	15	14	14	13	16	16	16	16	13	17	13	14	13	13
Station 3	12	15	14	14	13	15	15	15	16	14	12	12	16	13	13	14	13	14	12	13	11	12	14	12	12	13	11	15	15	12	13	15	12	11	12	13	14	12	12	15	14	11	13	15	14	13	13	13	13	13	14	13	14	11	13	10	13	14	11	13
Station 4	13	12	13	13	10	13	12	14	13	13	12	13	12	14	15	12	9	14	14	13	12	11	10	12	13	11	14	13	14	14	11	12	14	11	13	14	12	11	12	15	14	13	13	14	10	12	10	12	12	12	15	12	11	13	13	14	12	11	12	11
Station 5	13	11	12	11	11	13	14	13	12	11	12	13	10	9	12	13	11	13	14	12	13	12	12	11	11	11	12	12	15	10	8	11	10	13	10	11	12	11	13	14	9	11	11	12	11	11	11	11	12	13	11	11	13	10	12	15	11	13	11	10
Station 6	9	12	11	9	12	13	9	11	10	10	11	8	11	10	9	11	10	13	11	10	13	10	9	9	11	11	13	11	11	12	11	12	9	10	11	10	8	9	11	11	11	13	13	12	11	10	8	11	8	11	12	10	13	10	12	10	10	9	10	12
Station 7	10	12	12	11	11	10	10	11	12	12	10	9	12	11	12	10	13	10	10	12	9	10	12	11	13	8	8	11	12	9	11	8	11	9	8	11	9	12	9	10	9	11	12	7	10	10	9	11	9	11	10	11	10	10	9	11	10	11	8	11
Station 8	8	8	9	8	9	7	8	6	8	8	8	7	9	11	7	6	7	6	9	9	8	8	8	9	10	6	10	9	7	7	7	9	10	7	8	9	8	7	8	7	6	8	9	8	9	7	7	8	8	7	8	9	8	8	10	9	8	8	9	8
Station 9	0	0	0	5	7	7	6	7	8	7	8	6	8	6	8	9	4	5	8	5	6	6	7	7	8	8	6	7	5	6	6	7	7	8	6	5	6	4	6	6	6	6	7	5	7	4	6	6	5	7	7	4	7	4	7	6	8	5	7	6
Station 10	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	5	5	2	6	6	4	6	5	2	6	5	7	5	6	3	5	5	4	3	7	4	4	5	7	2	5	4	3	5	6	7	3	5	6	5	5	4	4	5	3	6	5	6	6	3
Station 11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	1	3	8	10	8	1	7	5	9	7	7	2	6	1	9	10	3	7	9	2	2	7	1	1	4	7	5	7	9	2	3	7	8	7	2	3	10	6	4

Unit Time: 30 s	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
Station 1	10	11	12	11	11	10	10	9	10	9	11	11	9	9	10	11	9	10	10	12	9	10	9	9	10	9	11	11	9	10	9	10	11	10	8	10	10	8	12	9	10	10	9	11	10	9	10	10	10	11	9	11	10	9	10	10	10	9	10	10
Station 2	15	15	13	15	15	14	15	15	13	15	12	14	13	14	14	12	13	16	15	13	14	14	15	13	15	11	12	14	14	15	15	15	15	14	13	13	15	15	14	13	14	14	12	14	11	15	13	14	15	14	14	14	14	15	13	14	13	14	14	14
Station 3	14	14	14	12	12	13	11	14	13	13	14	14	12	13	14	13	16	14	13	11	15	14	13	13	13	15	14	14	12	12	16	14	14	12	12	13	16	16	11	12	12	13	12	11	15	15	13	13	10	11	13	14	15	15	14	14	15	11	15	13
Station 4	13	12	14	13	12	14	11	11	14	13	13	12	11	13	12	10	12	14	13	13	11	12	11	9	13	11	12	12	12	12	13	13	12	11	10	11	15	13	11	11	12	10	13	15	14	12	14	10	12	12	12	14	14	13	12	13	11	12	11	12
Station 5	13	11	12	11	14	15	11	11	12	11	13	13	12	9	11	12	13	11	12	13	9	11	11	13	11	10	14	9	10	11	11	11	10	12	13	13	11	11	12	13	10	14	10	13	12	13	10	12	11	9	8	13	12	11	10	12	13	10	10	11
Station 6	13	10	13	10	13	10	10	10	11	10	10	12	9	10	9	11	11	11	10	9	11	9	9	8	12	9	11	9	10	11	11	9	11	10	12	9	8	10	10	11	11	12	12	10	12	12	8	11	9	10	11	10	12	11	13	11	9	11	9	12
Station 7	13	9	11	7	12	13	9	11	8	9	9	12	10	8	10	13	12	13	9	10	11	11	10	11	11	7	10	9	9	11	8	9	10	12	12	8	11	10	11	12	7	8	12	9	8	11	9	10	12	11	13	7	10	8	9	10	10	8	10	11
Station 8	10	8	8	8	7	8	7	9	9	8	8	8	6	8	7	9	6	7	8	8	10	9	10	7	6	8	8	8	7	7	5	6	8	9	10	9	6	7	8	7	9	8	7	9	7	9	8	6	6	9	8	8	7	9	9	8	8	9	9	8
Station 9	7	7	7	7	6	7	7	7	7	8	7	6	6	6	6	7	6	7	6	7	6	5	6	7	6	7	6	7	5	6	8	8	7	7	7	6	5	4	6	5	9	8	5	6	7	4	6	8	7	7	7	6	9	8	6	8	7	9	6	8
Station 10	5	3	5	6	5	7	6	5	7	4	5	6	4	2	6	4	5	5	2	4	3	4	4	4	5	4	5	3	6	7	4	5	4	4	4	3	6	4	6	5	5	3	6	3	5	4	5	4	6	4	4	7	3	5	3	6	4	5	5	4
Station 11	6	2	10	4	4	8	2	7	3	4	5	9	4	2	5	9	5	3	1	8	4	8	1	8	9	1	7	7	6	5	1	10	7	9	3	6	4	3	1	10	4	3	10	9	4	2	6	6	2	1	6	9	4	5	6	9	2	6	4	8

Unit Time: 30 s	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
Station 1	10	10	11	9	10	11	10	9	9	11	11	12	10	9	9	11	9	10	10	9	10	9	10	11	9	9	8	11	9	9	12	9	10	9	12	9	9	10	8	11	10	11	11	11	11	10	8	10	10	12	10	10	11	11	9	9	11	8	10	11
Station 2	16	14	14	14	14	16	12	13	13	15	16	14	15	15	14	13	13	13	15	13	14	15	14	11	12	17	13	15	15	15	14	17	15	15	14	12	14	12	14	13	11	13	16	14	17	13	14	15	13	15	13	14	14	13	14	15	13	11	13	13
Station 3	14	14	13	13	16	14	11	13	10	14	11	13	13	13	12	14	12	13	12	14	12	14	13	14	15	14	11	16	15	14	14	14	15	15	14	14	15	15	14	13	14	12	16	16	15	14	15	15	12	12	13	13	11	13	14	13	11	11	12	15
Station 4	12	12	13	12	10	14	11	11	11	12	14	13	13	12	14	10	10	12	11	11	10	11	13	12	12	11	11	11	12	11	11	11	14	11	14	12	12	14	11	12	12	11	13	13	13	12	11	12	12	11	12	12	11	10	11	14	11	10	12	12
Station 5	12	10	10	11	12	12	11	11	11	12	12	10	12	11	12	10	12	12	13	11	11	11	11	12	13	10	11	13	13	12	10	12	10	13	13	12	12	14	12	11	12	13	14	12	12	10	11	13	10	13	11	11	10	11	11	11	12	12	11	10
Station 6	9	8	10	10	10	11	10	11	10	12	11	11	9	11	13	13	10	11	12	12	11	10	9	11	11	11	11	11	12	9	12	10	10	11	12	11	10	10	11	10	13	10	10	12	9	9	13	11	13	8	8	12	12	8	11	12	11	11	13	12
Station 7	9	12	9	9	9	9	11	10	9	12	12	11	6	12	10	11	8	10	9	10	9	12	10	10	10	12	8	9	12	9	10	10	9	10	9	8	8	9	9	9	11	11	11	7	11	10	10	10	11	11	12	8	10	10	10	10	8	7	11	10
Station 8	7	8	8	9	9	9	8	7	10	9	7	9	7	6	10	8	9	8	8	9	7	6	8	5	10	8	11	8	8	7	10	8	7	10	8	8	6	6	8	10	8	7	7	10	8	10	7	7	7	10	6	8	6	9	8	7	7	8	8	11
Station 9	7	8	6	6	4	5	8	9	8	7	5	6	7	7	7	5	8	7	7	6	6	5	7	6	8	6	5	4	6	5	8	6	9	6	5	7	5	6	9	5	6	5	5	7	7	8	7	5	7	8	6	9	4	7	7	7	6	8	7	6
Station 10	4	5	3	6	7	4	2	3	5	3	4	6	2	4	3	6	5	6	5	5	6	4	4	6	6	2	6	4	4	6	3	7	4	3	6	3	4	4	6	3	7	6	6	4	5	6	6	6	7	5	5	5	4	5	3	4	3	6	3	2
Station 11	1	2	2	8	7	3	1	7	4	8	7	7	10	6	3	5	3	7	9	6	2	9	8	7	3	8	10	4	7	4	7	3	9	4	3	4	9	7	10	6	3	7	10	5	10	8	5	7	1	7	2	7	5	5	6	7	1	8	3	2

2) Processing Entry inputs

Non-cyclic timetable is not commonly used as it is inconvenient to the passengers and also for the railway management to remember the schedule. But it is clear that a better trade-off can be achieved for operating costs versus passenger waiting time more efficiently with such acyclic timetable. The flow chart in Figure 38 proposes a very simple mechanism to build the time table based on the historical data available from control room on the passenger traffic flow. Once the historical data is analysed and visualised using the simulation, the passenger waiting time can be analysed. The passengers waiting in every station is summed to calculate the total waiting time and is compared against a tolerable limit. The waiting time is measured over the whole line and not just at origin or specific stations as such waits in any station causes discomfort to the passengers if it is for long time. The waiting time is expressed in terms of people waiting time in minutes. For example, one could set that 100 passengers waiting in each station for 10 minutes is tolerable and from operational cost point of view, it would be a good trade-off. Thus, this is defined as MAXPEOPLEWAIT = 100*5*11*60 seconds = 3300s for line 1 that comprises of 11 stations. The stochastic inputs of People entry and waiting times that are used for simulator are also used to generate the time tabling as an example using the Algorithm. The algorithm takes the following inputs:

1) MAXPEOPLEWAIT: Allowed waiting time of passengers
2) MIN_HEADWAY_TIME: Minimum time elapse between the arrivals of successive train.
3) UNIT TIME: This is the sample time for which every Passenger entry sample is been considered. The scheduling will be based on multiple of such Unit times.
4) PERIOD_OF_OPERATION: This is the duration that we consider for scheduling overall.
5) *TRAIN_TRAVEL_DURATION:* This is the duration of the train to travel from origin to last destination including the dwell time. This is required if there is a need to ensure that there are only limited trains MAX_TRAINS_ON_LINE at a time on the line, taking the rolling stock into consideration.

Taking two hour data (TABLE 15**)** for the inputs of the simulation and using the algorithm from for line 1, the optimised non-cyclic schedule is as shown in TABLE 16.

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TABLE 16 OPTIMISED NON-CYCLIC TRAIN SCHEDULE FOR LINES 1 and 2

Line1

Train Number	1	2	3	4	5	6	7	8	9	10	11
Train Arrival Time (multiple of 30s)	16	30	44	58	72	86	100	114	128	142	156
Train Arrival Time (minutes)	8	15	22	29	36	43	50	57	64	71	78
Inter-arrival time (minutes)	8	7	7	7	7	7	7	7	7	7	7

Line2

Train Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Train Arrival Time (multiple of 30s)	14	26	38	50	62	74	86	98	110	122	134	146	158	170	182	194	206
Train Arrival Time (minutes)	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91	97	103
Inter-arrival time (minutes)	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

With the application of the non-cyclic time table, it may be observed from plot in Figure 34, the accumulated passenger time for line 1 reduces by 291 man hours with the train running times increased by 2.2 hours. In Figure 35, the plot for line 2 shows an increase in accumulated waiting time by 162 hours while the reduction in running train by 0.55 hours. The non-cyclic timetable considers all the passenger entries unlike the previous approach of cyclic timetabling which takes the average of passenger entries. The comparison of the initial simulation scenario and the optimisation using cyclic and non-cyclic time table is available in

.

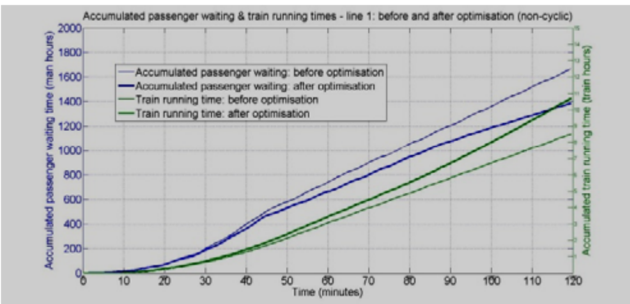

Figure 34 Accumulated passenger wait and train run times – optimisation with non-cyclic frequency for line 1

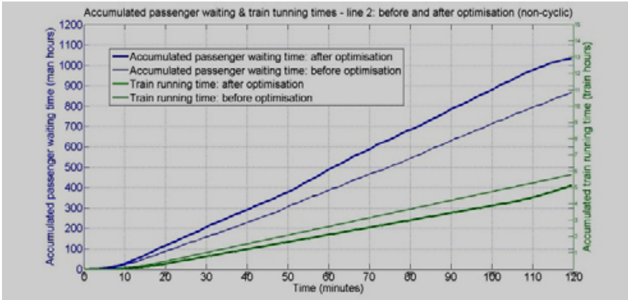

Figure 35 Accumulated passenger wait and train run times – optimisation with non-cyclic frequency for line 2

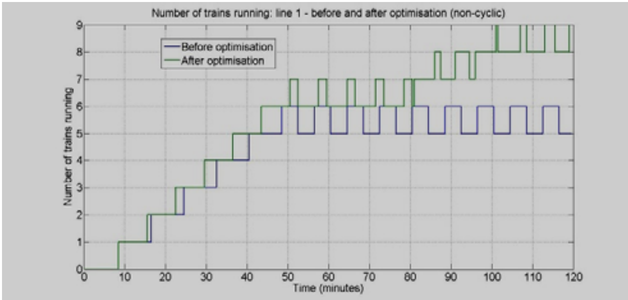

Figure 36 Number of operating trains – before and after optimisation with non-cyclic timetable line 1

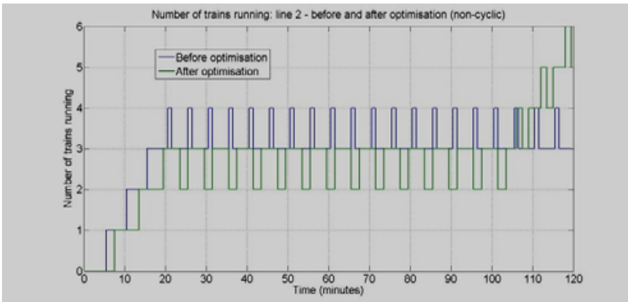

Figure 37 Number of Operating trains – before and after optimisation with non-cyclic timetable line 2

TABLE 17 COMPARISON OF TIMES FOR INITIAL SIMULATION AND AFTER OPTIMISATION AT THE END OF 2 HOURS FOR LINES 1 and 2

Line 1	**Initial one (no optimisation)**	**Optimised with cyclic frequency**	**Optimised with non-cyclic frequency**
Accumulated passenger waiting time	1677 man hours	1054 man hours	1386 man hours
Train running time	8.51 train hours	13.5 train hours	10.7 train hours
Max number of trains on line	6	9	9

Line 2	**Initial one (no optimisation)**	**Optimised with cyclic frequency**	**Optimised with non-cyclic frequency**
Accumulated passenger waiting time	871 man hours	708 man hours	1033 man hours
Train running time	5.7 train hours	7.3 train hours	5.15 train hours
Max number of trains on line	4	5	6

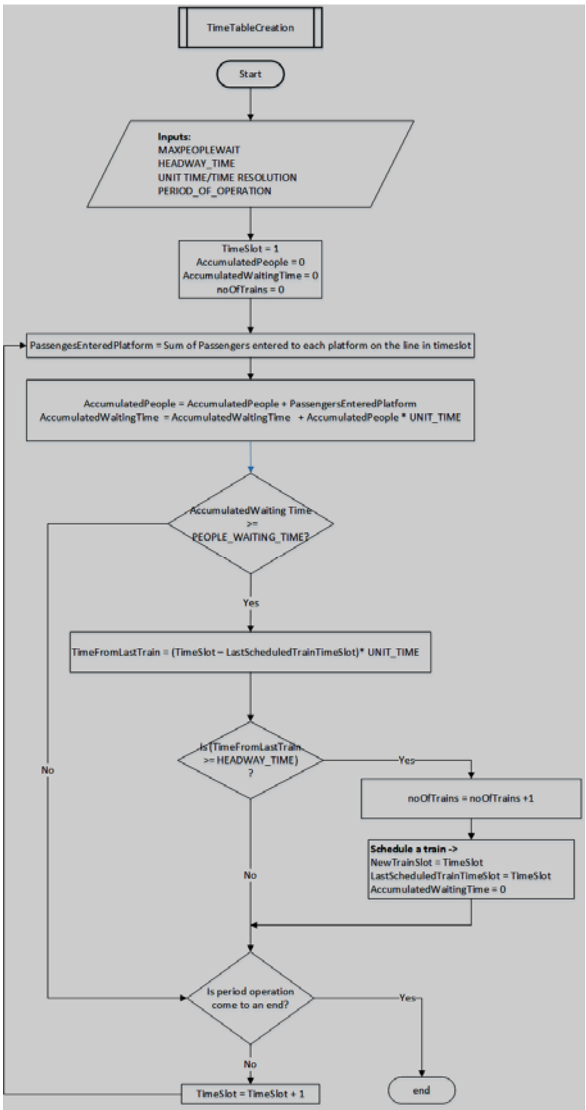

Figure 38 Algorithm for optimised train table based on passenger waiting

X. CONCLUSIONS

This paper is an attempt to give a framework for modelling and simulation of different topologies of metro single unidirectional lines. It also shows different mathematical equations and matrices that are used for the model. The simulation enables the user to provide O-D matrix for each stations, different flow rates and enable the plots to clearly visualize different parameters for timetabling. The paper at the end discusses the possibilities of optimisation using the trade-off for the trains running on the line and the passenger waiting time with both cyclic and non-cyclic time tables. The future work is to include the model for spatial distribution of passengers in stations, include track side elements for modelling and enhance simulation framework to make it more generic for any system.

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