Review 2 (Sept. 20)

1. **State classification: Recurrent and transient states**

    a. Communication and class
    b. Recurrent and transient states
        i. Recurrent state: starting from this state, with probability 1, the chain will come back in finite number of steps;
        ii. Transient state: starting from this state, with positive probability, the chain will never come back.
    c. Simple facts about recurrent and transient states:
        i. Starting from a recurrent state, the chain will revisit it for infinitely many times with probability 1.
        ii. Starting from a transient state, the number of visits the chain makes to this state is finite, following a geometric distribution.
        iii. A finite-state irreducible Markov chain is recurrent.
        iv. A state i is recurrent if and only if

$$\sum_{n=1}^{+\infty} p_{ii}^{n} = +\infty.$$

        v. Recurrence/Transience is a class property.

2. **Long-term behavior of Markov chains**

    a. In a positive recurrent aperiodic Markov chain, the following three concepts are equivalent:
        i. Long-run proportion;
        ii. Stationary probability distribution;
        iii. Long-term limits of transition probabilities.

    b. How to compute the stationary distribution:

$$\pi = \pi P \quad \text{and} \quad \pi \mathbf{1} = 1.$$

    c. Ergodic Theorem:

$$\lim_{n\to\infty} \frac{\sum_{t=1}^{n} r(X_t)}{n} = \sum_j r(j)\pi_j.$$

d. Some theoretical consideration: positive recurrent, null recurrent, aperiodic

More examples for discussion:

1. Each morning an individual leaves his house and goes for a run. He is equally likely to leave either from his front or back door. Upon leaving the house, he chooses a pair of running shoes (or goes running barefoot if there are no shoes at the door from which he departs). On his return he is equally likely to enter, and leave his running shoes, either by the front or back door. If he owns a total of 2 pairs of running shoes, what proportion of the time does he run barefooted?

2. Exercise 59, Ross's book, page 270.