Simple Mathematical Facts used in Lecture 1

Conditional Probabilities

- Given an event F has occurred, the **conditional probability** that another event E occurs is defined as

$$P\left(E \mid F\right) = \frac{P(E \cap F)}{P(F)}.$$

- Example: Suppose that I throw a die. What is the conditional probability that I get 6, if you are told that I get an even number?

Independent Events

- Two events E and F are said to be **independent** if

$$P(E \cap F) = P(E)P(F).$$

- That implies, E and F are independent if and only if

$$P(E \mid F) = P(E).$$

- Example: Suppose that we toss two fair dice. Let E be the event that the sum of the two dice equals 7, and F the event that the first die equals 4. Is E independent of F ?

Conditionally Independent Events

- Two events E and F are said to be **independent, conditional on the occurrence** of event A, if

$$P(E \cap F \mid A) = P(E \mid A)P(F \mid A).$$

Complete Probability Formula

- $$P(E) = P(E \mid F)P(F) + P(E \mid F^c)(1 - P(F)).$$
- Example: Consider two urns. The first contains 2 white and 7 black balls, and the second contains 5 white and 6 black balls. We flip a coin and then draw a ball from the first or second urn depending on whether the outcome was head or tail. What is the probability that we draw a white ball?