SEG 2430 Applied Probability and Statistics

Solution to Homework 1

February 1, 2010

1. Solution to 3.20: (10 points)

   Clearly, this is a permutation counting. Thus, the total ways can be calculated by ${}_9P_3 = 504$.

2. Solution to 3.25: (16 points)

   First, we divide these 12 batteries into two different sets according to their quality. Let A be the set containing the defective battery and B be the set containing others. Then, the numbers of elements in A and B are 1 and 11, respectively.

   (a). That means that one element should be selected from set A and two elements be selected from set B. Clearly, these selections are "combinations". Thus, the total ways are ${}_1C_1 * {}_2C_{11} = 55$. .....(8)

   (b) That means that all three elements should be selected from set B. Thus, the total ways are ${}_3C_{11} = 165$. .....(8)

3. Solution to 3.34: (20 points)

   (a) $N(A) = N(A \cap B \cap C) + N(A \cap \bar{B} \cap C) + N(A \cap B \cap \bar{C}) + N(A \cap \bar{B} \cap \bar{C}) = 54 + 8 + 20 + 2 = 84$. .....(2)

   (b)Using the same idea of (a), we have $N(B) = 16 + 20 + 54 + 9 = 99$. .....(2)

   (c)Using the same idea of (a), we have $N(C) = 8 + 14 + 54 + 9 = 85$. .....(2)

   (d)$N(A \cap B) = N(A \cap B \cap C) + N(A \cap B \cap \bar{C}) = 54 + 20 = 74$. .....(2)

   (e)Using the same idea of (d), we have $N(A \cap C) = 8 + 54 = 62$. .....(2)

   (f)$N(A \cap B \cap C) = 54$. .....(2)

   (g)$N(A \cup B) = N(A) + N(B) - N(A \cap B) = 84 + 99 - 74 = 109$. .....(2)

   (h)$N(B \cup C) = N(B) + N(C) - N(B \cap C) = 99 + 85 - 63 = 121$. .....(2)

   (i)$N(\bar{A} \cup \bar{B} \cup C) = N(\bar{A}) + N(\bar{B}) + N(C) - N(\bar{A} \cap \bar{B}) - N(\bar{A} \cap C) - N(\bar{B} \cap C) + N(\bar{A} \cap \bar{B} \cap C) = 66 + 51 + 85 - 41 - 23 - 22 + 14 = 130$. .....(2)

   (j)$N[B \cap (A \cup C)] = N((B \cap A) \cup (B \cap C)) = N(B \cap A) + N(B \cap C) - N(B \cap A \cap C) = 74 + 63 - 54 = 83$. .....(2)

4. Solution to 3.63: (20 points)

   (a) $P(A|B) = P(A \cap B)/P(B) = 0.15/0.30 = 0.5 = P(A)$. .....(5)

   (b) $P(A|\bar{B}) = P(A \cap \bar{B})/P(\bar{B}) = (P(A) - P(A \cap B))/(1 - P(B)) = (0.5 - 0.15)/(1 - 0.3) = 0.5 = P(A)$. .....(5)

(c) $P(B|A) = P(A \cap B)/P(A) = 0.15/0.50 = 0.3 = P(B)$. .....(5)

(d) $P(B|\bar{A}) = P(B \cap \bar{A})/P(\bar{A}) = (P(B) - P(A \cap B))/(1 - P(A)) = (0.3 - 0.15)/(1 - 0.5) = 0.3 = P(B)$. .....(5)

PS: Actually, we can prove the following statement:

A and B are independent if and only if A and $\bar{B}$ are independent.

Proof: Because A and B are independent, $P(A \cap B) = P(A) * P(B)$. Hence, $P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)} = 1 - P(B) = P(\bar{B})$.

5. Solution to 3.70: (34 points)

(a). Let $A1$, $A2$, and $A3$ denote the events where the first, second, and third bullet is blank, respectively. Then, it is clear that $P(A1) = 3/6 = 1/2$, because three blank bullets can be chosen from six bullets. Then, there are totally 5 bullets left including 2 blank bullets. Hence, the probability of $A2$, supposing that $A1$ has happened, is $P(A2|A1) = \frac{1/5}{1/2} = 2/5$. Suppose the first and second bullets are blank, that is, two of three blank bullets have been chosen, then the probability that the third bullet is also blank is given by $1/4$, i.e., $P(A3|A1 \cap A2) = 1/4$.

Hence, $P(A1 \cap A2 \cap A3) = P(A1) * P(A2|A1) * P(A3|A1 \cap A2) = 1/2 * 2/5 * 1/4 = 1/20$. .....(8)

(b). Let $A1$ denote the event that the second day is a rainy day, $A2$ denote the event that the third day is a rainy day, $B3$ denote the event that the fourth day is a sunny day, and $A4$ denote the event that the fifth day is a rainy day. By the condition, $P(A1) = 0.8$, $P(A2|A1) = 0.8$, $P(B3|A2) = 1 - 0.8 = 0.2$, and $P(A4|B3) = 0.6$. Note that the wether on any given day only depends on that of the day before. Therefore, we have

$P(A1 \cap A2 \cap B3 \cap A4) = P(A1) * P(A2|A1) * P(B3|A2) * P(A4|B3) = 0.0768$. .....(8)

(c). Let $A1$, $A2$, and $A3$ denote that the customer do not pay promptly in the first, second, and third month, respectively. $P(A1) = 1 - 0.9 = 0.1$, $P(A2|A1) = P(A3|A2) = 0.5$.

Similar to (b), we have $P(A1 \cap A2 \cap A3) = P(A1) * P(A2|A1) * P(A3|A2) = 0.025$. .....(9)

(d) Let A_i be the events that the ith truck of 5 trucks does not meet emission standards. Clearly, $P(A_1) = 5/12$. Suppose the first truck does not meet emission standards, then there are totally 11 truck left and 4 of them do not meet emission standards. Hence, $P(A_2|A_1) = 4/11$. Similarly, we have $P(A_3|A_1 \cap A_2) = 3/10$ and $P(A_4|A_1 \cap A_2 \cap A_3) = 2/9$.

Therefore, we have $P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1) * P(A_2|A_1) * P(A_3|A_1 \cap A_2) * P(A_4|A_1 \cap A_2 \cap A_3) = 1/99$. .....(9)