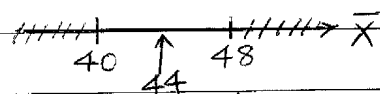


6.35 Pop mean $\mu=44$ $\sigma=8$

Random sample $n=25$ Let $\bar{X}$ denote sample mean

$$P(\bar{X} < 40 \text{ or } \bar{X} > 48) = ?$$



(a) by Chebyshev Thm. $P(|Y - \mu_Y| \geq k\sigma) \leq \frac{1}{k^2}$

$$\bar{X} \text{ is a RV} \quad E[\bar{X}] = \mu = 44 \quad V[\bar{X}] = \frac{\sigma}{\sqrt{n}} = \frac{8}{5}$$

$$P(\bar{X} < 40 \text{ or } \bar{X} > 48)$$

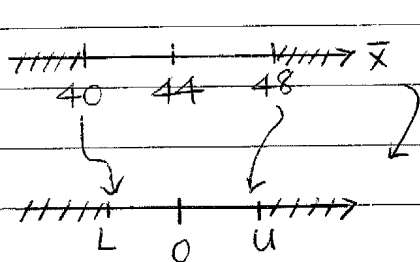
$$= P(|\bar{X} - \underbrace{\mu_{\bar{X}}}_{44}| \geq \underbrace{k\sigma_{\bar{X}}}_{4}) \leq \frac{1}{k^2}$$

$$k\left(\frac{8}{5}\right) = 4 \Rightarrow k = \frac{20}{8} = \frac{5}{2}$$

$$\Rightarrow \frac{1}{k^2} = 0.16$$

(b) by CLT

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{X} - 44}{\frac{8}{5}} \sim N(0,1)$$



$$L = \frac{40 - 44}{8/5} = -2.5$$

$$U = \frac{48 - 44}{8/5} = 2.5$$

$$P(Z < -2.5) + P(Z > 2.5)$$

$$= 2P(Z > 2.5) = 2(1 - F(2.5))$$

$$= 2(1 - 0.9938) = \boxed{0.0124}$$

6.43

(a) Total weight of the bag is $\sum_{i=1}^{30} X_i = Y$

Approx sampling distrib of $\bar{X}$

By CLT the RV $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow N(0, 1)$ when $n=30$ sufficiently large sample.

$$\bar{X} = \frac{Y}{n} \quad \therefore \quad \frac{\frac{Y}{n} - \mu}{\sigma/\sqrt{n}} \rightarrow N(0, 1)$$

(b) $Y = n\bar{X}$

$$E[Y] = nE[\bar{X}] = n\mu$$

$$V[Y] = n^2 V[\bar{X}] = n^2 \left[\frac{\sigma^2}{n} \right] = n\sigma^2$$

$$\therefore Y \sim N[\mu, n\sigma^2]$$

(c) $\mu = 0.2 \quad \sigma = 0.03$

$$P(Y \geq 6.2) = ?$$

$$E[Y] = 6 \quad V[Y] = 0.027$$

$$P(Y \geq 6.2) = P\left(\frac{Y - 0.2}{\sqrt{0.027}} \geq \frac{6.2 - 6}{\sqrt{0.027}}\right)$$

$$= P(Z \geq 1.217)$$

$$= 1 - F(1.217)$$

$$= \boxed{0.1118}$$

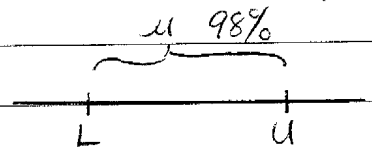
Example

Random sample of 50 univ students, measure their heights.

Sample mean is 174.5 cm & sample s.d is 6.9 cm

(a) What is the 98% conf interval for the mean height of all univ students?

$$n=50, \bar{X}=174.5, S=6.9,$$



$$\bar{X} - z_{\alpha/2} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + z_{\alpha/2} \frac{S}{\sqrt{n}}$$

$$174.5 - (2.33) \frac{6.9}{\sqrt{50}} \leq \mu \leq 174.5 + (2.33) \frac{6.9}{\sqrt{50}}$$

$$\boxed{172.8 < \mu < 176.8}$$

(b) What can we assert w 98% confidence about the possible size of our error, if we estimate μ to be 174.5 cm?

$$\max E = z_{\alpha/2} \frac{b}{\sqrt{n}}$$

$$\therefore \text{error} < (2.33) \frac{6.9}{\sqrt{50}} \Rightarrow \boxed{\text{error} \leq 2.27}$$

Example

It is claimed that cars are driven on the average more than 20,000 km per year. To test this claim, a random sample of 100 automobiles are sampled. The sample average is 23,500 km w sample s.d. of 3900 km. Would you agree that the average distance is greater than what is claimed?

$$\mu \leq \mu_0 \quad \mu = \mu_0$$

$$\mu > \mu_0$$

$$H_0: \mu_0 = 20,000$$

$$H_1: \mu_1 > 20,000$$

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{23500 - 20000}{3900/\sqrt{100}} = 8.97$$

large sample

$$p\text{-value is } P(Z \geq 8.97) \approx 0$$

$\therefore$ reject H_0

Example

It is hypothesized that the container of a brand of detergent contains 10 liters per bottle on average.

A random sample of 10 bottles show that they contain: 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, 9.8 liters

Assume the distribution of contents is normal. Test w $\alpha = 0.05$

$$H_0: \mu = 10, H_1: \mu \neq 10$$

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{10.06 - 10}{0.246/\sqrt{10}} = 0.77 \quad \text{d.f.} = 9$$

$$t_{\alpha/2} = -3.25, t_{\alpha/2} = 3.25 \quad \text{Reject if } t < -t_{\alpha/2} \text{ OR } t > t_{\alpha/2}$$

Decision: Not reject H_0

Example

15 students were asked to complete a computer game in pre-fatigue and post-fatigue conditions. The time periods required are:

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Prefatigue	158	92	65	98	33	89	148	58	142	117	74	66	109	57	85
Postfatigue	91	59	215	226	223	91	92	177	134	116	153	219	143	164	100

It is claimed that fatigue increases completion time.

Test this claim. $\alpha = 0.05$

$$H_0: \mu = \mu_0$$

$$H_1: \mu > \mu_0$$

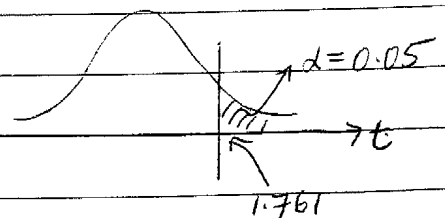
$$d = t_{\text{post}} - t_{\text{pre}}$$

$$\bar{d} = 54.13 \quad s_d = 83.002$$

$$t = \frac{54.13 - 0}{83.002 / \sqrt{15}} = 2.53 \quad \text{d.f. } 14$$

$$t_{0.05} = 1.761$$

$\therefore$ Reject H_0



Problem
7.95

7.95

$$H_0: \mu_1 - \mu_2 = 53.2$$

$$\alpha = 0.01$$

$$H_1: \mu_1 - \mu_2 > 53.2$$

$$\sigma_1 = 10.8 \quad \sigma_2 = 14.4$$

$$n_1 = n_2 = n$$

$$\text{test stat: } \mu_1 - \mu_2 = 66.7$$

$$\beta = 0.09$$

$$d = \frac{|\delta_1 - \delta_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} = \frac{|66.7 - 53.2|}{\sqrt{(10.8)^2 + (14.4)^2}} = 0.75$$

$$\alpha = 0.01 \rightarrow \text{Table 8(b) gives } n \approx 25$$

$$\beta = 0.09$$

Problem
7.96

Sample 1, alloy 1, $n_1 = 58$, $\bar{X}_1 = 70.7$, $S_1 = 1.8$

Sample 2, alloy 2, $n_2 = 27$, $\bar{X}_2 = 76.13$, $S_2 = 2.42$

90% conf interval

$$(a) \text{ Pooled: } \bar{X}_1 - \bar{X}_2 \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

$$df (n_1 + n_2 - 2) \quad t_{0.05}$$

$$\text{Sub values } (70.7 - 76.13) \pm (1.645) \sqrt{\frac{(57)(1.8)^2 + (26)(2.42)^2}{83}} \sqrt{\frac{1}{58} + \frac{1}{27}}$$

$$= -5.43 \pm 0.77, \text{ i.e. } -6.2 < \mu_1 - \mu_2 < -4.66$$

$$(b) \bar{X}_1 - \bar{X}_2 \pm z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\text{Sub. } (70.7 - 76.13) \pm (1.645) \sqrt{\frac{(1.8)^2}{58} + \frac{(2.42)^2}{27}} = -5.43 \pm 0.86$$

$$\text{i.e. } -6.29 < \mu_1 - \mu_2 < -4.57$$

Problem
9.59

$$H_0: p_1 = p_2$$

$$H_1: p_1 > p_2$$

$$\alpha = 0.05$$

$$\text{method 1 } \frac{57}{150}$$

$$\text{method 2 } \frac{33}{100}$$

$$Z_{0.05} = 1.645 \quad \text{Critical region } Z > Z_\alpha$$

Large sample

$$Z = \frac{\frac{X_1}{n_1} - \frac{X_2}{n_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{57 + 33}{150 + 100} = 0.36$$

$$Z = \frac{\frac{57}{150} - \frac{33}{100}}{\sqrt{(0.36)(0.64)\left(\frac{1}{150} + \frac{1}{100}\right)}} = 0.807 \quad (\text{does not lie in critical region})$$

Conclusion: cannot reject H_0

9.67 χ^2 test for independence

$$d.f. = 4$$

$$\alpha = 0.05$$

$\therefore$ Critical region $\chi^2 > \chi^2_{0.05} = 9.488$

Recall
$$e_{ij} = \frac{(\text{row total}) \times (\text{col total})}{\text{grand total}}$$

E.g.
$$e_{11} = \frac{(30)(40)}{150} = \frac{1200}{150} = 8$$

	20	7	3	30
	(8)	(14)	(8)	
	13	51	16	80
	(21.33)	(37.33)	(21.33)	
	7	12	21	40
	(10.67)	(18.67)	(10.67)	
	40	70	40	

$$\begin{aligned}\chi^2 &= \frac{(20-8)^2}{8} + \frac{(7-14)^2}{14} + \frac{(3-8)^2}{8} + \frac{(13-21.33)^2}{21.33} + \frac{(51-37.33)^2}{37.33} \\ &\quad + \frac{(16-21.33)^2}{21.33} + \frac{(7-10.67)^2}{10.67} + \frac{(12-18.67)^2}{18.67} + \frac{(21-10.67)^2}{10.67} \\ &= 47.862\end{aligned}$$

Conclusion: Reject the indep between the 2 methods