

SEG 2430

Solution of Assignment 9

2010.04.15

8.11 (15 marks)

$n = 10, \mu = 71.3, \sigma = 1.32.$

Null hypothesis: $\sigma = 1.20$

Alternative hypothesis: $\sigma > 1.20$ (2)

Level of significance: $\alpha = 0.05$

Criterion: with degree of freedom 9. $\chi^2_{0.05} = 16.919$ (3)

Reject the null hypothesis if $\chi^2 > 16.919$

For the given sample sets, we have $\chi^2 = \frac{9(1.32)^2}{1.2^2} < 16.919$ (5)

So we cannot reject Null hypothesis.(5)

8.15 (20 marks)

Null hypothesis: $\sigma_1^2 = \sigma_2^2$

Alternative hypothesis: $\sigma_1^2 < \sigma_2^2$ (4)

Level of significance: $\alpha = 0.01$

Criterion:

Reject the null hypothesis if $F > F_{0.01}(20, 14) = 3.51$, where $F = S_2^2 / S_1^2$ (4)

For the given sample sets, we have $F = (4.2)^2 / (2.7)^2 = 2.42$ (4)

Since F does not exceed 3.51, the null hypothesis cannot be rejected; even though the sample standard deviation of method 2 is bigger than that of method 1, there is not sufficient evidence to conclude that the lighting in the second area is less uniform.(4)

Assumption: the two samples were independent random samples taken from some normal distributions..... (4)

8.20 (15 marks) $\chi^2_{(1-0.025)} \leq \frac{(n-1)S^2}{\sigma_0^2} \leq \chi^2_{0.025}$ (5)

$\Rightarrow \sigma_0^2 \in \left[\frac{(n-1)S^2}{\chi^2_{0.025}}, \frac{(n-1)S^2}{\chi^2_{1-0.025}} \right]$(5)

$\Rightarrow \sigma_0^2 \in \left[\frac{24 \times 4.9^2}{39.364}, \frac{24 \times 4.9^2}{12.401} \right]$

$\Rightarrow \sigma_0 \in [3.8261, 6.8167]$

So the 95% confidence interval for the corresponding population standard deviation is **[3.8261, 6.8167]** under the assumption that the population is normal.
(5)

9.1 (15 marks)

(a) $p = x/n = 84/200 = 0.42$ using table 9, $0.35 < p < 0.49$(5)

(b) $Z_{0.025} = 1.96$, $x/n = 84/200 = 0.42$.

By large-sample confidence interval

$$0.42 - 1.96\sqrt{0.42 \times 0.58 / 200} < p < 0.42 + 1.96\sqrt{0.42 \times 0.58 / 200}$$

$$\text{So } 0.352 < p < 0.488 \text{(10)}$$

9.18(15 marks)

Proportion $p = x/n = 834/2037 = 0.4094$ (5)

$$Z_{0.005} = 2.57 + 2.58 \cdot 2 = 2.575 \text{(5)}$$

The large-sample confidence interval is :

$$0.4094 - 2.575\sqrt{0.4094 \times 0.5906 / 2037} < p < 0.4094 + 2.575\sqrt{0.4094 \times 0.5906 / 2037}$$

$$\text{So } 0.381 < p < 0.438 \text{(5)}$$

9.29(20 marks)

Null hypothesis: $p_1 = p_2 = p_3$.

Alternative hypothesis: p_1, p_2, p_3 are not all equal.(2)

Level of significance: $\alpha = 0.01$

Criterion: Reject the null hypothesis if $\chi^2 > \chi^2_{0.01}$ with $3-1=2$ degree of freedom, which is 9.21.(3)

Calculations:

	Agency 1	Agency 2	Agency 3	Total
For	67	84	109	260
Against	33	66	41	140
Total	100	150	150	400

Expected frequencies for the first two cells of the rows:

$$e_{11} = \frac{260 \times 100}{400} = 65, e_{12} = \frac{260 \times 150}{400} = 97.5, e_{13} = \frac{260 \times 150}{400} = 97.5,$$

$$e_{21} = \frac{140 \times 100}{400} = 35, e_{22} = \frac{140 \times 150}{400} = 52.5, e_{23} = \frac{140 \times 150}{400} = 52.5. \text{ (10)}$$

So

$$\chi^2 = \frac{(67-65)^2}{65} + \frac{(84-97.5)^2}{97.5} + \frac{(109-97.5)^2}{97.5} + \frac{(33-35)^2}{35} + \frac{(66-52.5)^2}{52.5} + \frac{(41-52.5)^2}{52.5}$$

$$= 9.39.$$

Since $\chi^2 > \chi^2_{0.01}$, null hypothesis is rejected at 0.01 level of significance.(5)