SEG 2430 Applied Probability and Statistics

2010 Spring Assignment 3 Solutions

Due Feb 26, 2010

1. Solution to 4.60: (20 points)

A company rents time on a computer for periods of t hours, for which it receives \$600 an hour. The number of times the computer breaks down during t hours is a random variable having the Poisson distribution with $\gamma = (0.8)t$, and if the computer breaks down x times during t hours, it costs $50x^2$ dollars to fix it. How should the company select t in order to maximize its expected profit?

Suppose $E(t)$ is the expected profit, which can be expressed as:

$$E(t) = 600t - \sum_{x=0}^{\infty} f(x, 0.8t) 50x^2$$

$$= 600t - \sum_{x=0}^{\infty} \frac{(0.8t)^x e^{-0.8t}}{x!} 50x^2$$

$$= 600t - 50 \sum_{x=0}^{\infty} x^2 \frac{(0.8t)^x e^{-0.8t}}{x!}$$

$$= 600t - 50E(x^2)$$
$$= 600t - 50(var(x) - E(x)^2)$$
$$= 600t - 50(0.8t - (0.8t)^2)$$
$$= 560t - 32t^2$$

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$$\frac{dE(t)}{dt} = 560 - 64t = 0$$

$$t = 8.75$$

$$\frac{d^2E(t)}{dt^2} = -64 < 0 \rightarrow maximum\ point$$

Therefore the expected profit if maximized when $t = 8.75$hours

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2. Solution to 4.67: (15 points)

The number of flaws in a fiber optic cable follows a Poisson process with an average of 0.6 per 100 feet.

(a) Find the probability of exactly 2 flaws in a 200-foot cable.

Denote *x* be the number of flaws in the fiber optic cable.

Exactly 2 flaws in a 200-foot cable means λ = 2*0.6 =1.2

$$P(x = 2) = f(2,1.2) = 0.2169$$

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(b) Find the probability of exactly 1 flaw in the first 100 feet and exactly 1 flaw in the second 100 feet.

$$P = f(1,0.6) * f(1,0.6) = 0.1084$$

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3. Solution to 4.68: (20 points)

Differentiating with respect to p on both sides of the equation

$$\sum_{x=1}^{\propto} p(1-p)^{x-1} = 1$$

Show that the geometric distribution

$$f(x) = p(1-p)^{x-1} \quad for\ x = 1,2,3, \dots \dots$$

has the mean 1/p.

According to the definition of mean:

$$\mu = \sum_{x=1}^{\infty} xf(x) = \sum_{x=1}^{\infty} xp(1-p)^{x-1}$$

Differentiating with respect to p on both sides of the equation

$$\sum_{x=1}^{\propto} p(1-p)^{x-1} = 1 \rightarrow \sum_{x=1}^{\infty}(1-p)^{x-1} - \sum_{x=2}^{\infty}(x-1)p(1-p)^{x-2} = 0$$

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$$\frac{1}{p} - \sum_{x=2}^{\infty}(x-1)p(1-p)^{x-2} = 0$$

Substitute x-1 with t, then the equation will become:

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$$\frac{1}{p} - \sum_{t=1}^{\infty} tp(1-p)^{t-1} = 0$$

According to the definition of mean,$\mu = \sum_{t=1}^{\infty} tp(1-p)^{t-1}$, therefore:

$$\mu = \frac{1}{p}$$

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4. Solution to 4.73: (15 points)

As can easily be shown, the probability of getting 0,1,or 2 heads with a pair of balanced coins are $\frac{1}{4},\frac{1}{2},\frac{1}{4}$. What is the probability of getting 2 tails twice, 1 head and 1 tail 3 times, and 2 heads once in 6 tosses of a pair of balanced coins?

Since the 6 tosses are independent,

$$P(\text{2 tails twice, 1 head and 1 tail 3 times, and 2 heads once}) = C_6^2 C_4^3 \frac{1}{4}^2 \frac{1}{2}^3 \frac{1}{4}$$

$$= 0.117$$

5. Solution to 4.90: (15 points)

As can be easily verified by means of the formula for the binomial distribution, the probabilities of getting 0,1,2, or 3 heads in 3 flips of a coin whose probability of heads is 0.4 are 0.216,0.432,0.288 and 0.064. Find the mean of this probability distribution using

(a) The formula that defines μ;

$$\mu = \sum_{x=0}^{3} x c_3^x\, 0.4^x 0.6^{3-x} = 0 * 0.216 + 1 * 0.432 + 2 * 0.288 + 3 * 0.064 = 1.2$$

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(b) The special formula for the mean of a binomial distribution.

$$\mu = np = 3 * 0.4 = 1.2$$

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6. Solution to 4.95: (15 points)

The daily number of orders filled by the parts department of a repair shop is a random variable with $\mu = 142$ and $\sigma = 12$. According to Chebyshev's theorem, with what probability can we assert that on any one day it will fill between 82 and 202 orders?

Let X be the number of customers.

$$k = \frac{142 - 82}{12} = \frac{202 - 142}{12} = 5$$

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$$P(|X - \mu|) < k\sigma) \geq 1 - \frac{1}{k^2} \; and \; P(82 < X < 202) \geq 1 - \frac{1}{5^2} = \frac{24}{25}$$

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