SEG2430
Solution of Assignment 6

Weiwu Jiang & Yi Yang

5.45 (10 points)

$$F(x)=\begin{cases} 0 & x\le\alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & \alpha<x<\beta \\ 1 & x\ge\beta \end{cases}$$

5.71 (20 points)

(a) Since X_1+X_2 is total number of defects, it should not be larger than 2.
The joint probability distribution should be

$$f(X_1,X_2)=\begin{cases} \dfrac{\binom{2}{X_1}\binom{1}{X_2}\binom{2}{2-X_1-X_2}}{\binom{5}{2}} & X_1=0,1,2,\ X_2=0,1\ and\ 0\le X_1+X_2\le 2; \\ 0 & others \end{cases}$$

(5 points)

(b) There total $\binom{2}{2}=1$ ways that 0 defect among the two selected. There total $\binom{3}{1}\cdot\binom{2}{1}=6$ ways that 1 defect among the two selected.

Therefore, the probability of 0 or 1 defect among the two selected should be $(1+6)/\binom{5}{2}=0.7$

(5 points)

(c) Since X_1 could only be 0,1,2

If X_1=0, $f_1(0)=\dfrac{\binom{3}{2}}{\binom{5}{2}}=0.3$

If X_1 =1, $f_1(1)=\dfrac{\binom{2}{1}\cdot\binom{3}{1}}{\binom{5}{2}}=0.6$

If X_1 =2, $f_1(2)=\dfrac{\binom{2}{2}\binom{3}{0}}{\binom{5}{2}}=0.1$ (5 points)

(d) Given X_2=0 and with X_1+X_2 no larger than 2, X_1 could be 0,1,2.
Therefore,

If X_1 =0, $f_1(0\,|\,0)=\dfrac{\binom{2}{2}}{\binom{4}{2}}=\dfrac{1}{6}$

If X_1 =1, $f_1(1\,|\,0)=\dfrac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}}=\dfrac{4}{6}$

If X_1 =2, $f_1(2\,|\,0)=\dfrac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}}=\dfrac{1}{6}$ (5 points)

5.72 (20 points)
(a)

Two random variables, say X_1 and X_2.

The probability distribution functions of $X_1: f_1(x_1)=\binom{2}{x_1}0.3^{x_1}0.7^{2-x_1}$

Similarly, the probability distribution function of $X_2: f_2(x_2)=\binom{2}{x_2}0.3^{x_2}0.7^{2-x_2}$ (5 points)

As X_1 and X_2 are independent, their joint probability distribution is:

$$f(x_1,x_2) = f_1(x_1)f_2(x_2) = \binom{2}{x_1}0.3^{x_1}0.7^{2-x_1}\binom{2}{x_2}0.3^{x_2}0.7^{2-x_2} = \binom{2}{x_1}\binom{2}{x_2}0.3^{x_1+x_2}0.7^{4-x_1-x_2}$$

(5 points)

(b)

For $X_2 > X_1$, the possible values of (x_1, x_2) are $(0,1)$, $(0,2)$, $(1,2)$.

Therefore, the probability for $X_2 > X_1$ is $f(0,1)+f(0,2)+f(1,2)$ (5 points)

$$\begin{aligned} & f(0,1)+f(0,2)+f(1,2) \\ &= \binom{2}{0}\binom{2}{1}0.3\cdot 0.7^3 + \binom{2}{0}\binom{2}{2}0.3^2\cdot 0.7^2 + \binom{2}{1}\binom{2}{2}0.3^3\cdot 0.7 \\ &= 0.2877 \end{aligned}$$ (5 points)

5.73 (20 points)

(a)

$$\begin{aligned} P(X_1<1, X_2<1) &= \int_{-\infty}^{1}\int_{-\infty}^{1} f(x_1,x_2)dx_1dx_2 \\ &= \int_{-\infty}^{1}\int_{-\infty}^{1} x_1x_2dx_1dx_2 \qquad \text{(5 points)} \\ &= \int_{-\infty}^{1} x_2\int_{-\infty}^{1} x_1dx_1dx_2 \\ &= \int_{-\infty}^{1} x_2(\frac{1}{2}-0)dx_2 \\ &= \frac{1}{2}\int_{0}^{1} x_2dx_2 \qquad \text{(5 points)} \\ &= \frac{1}{2}(\frac{1}{2}-0) = \frac{1}{4} \end{aligned}$$

(b)

$$\begin{aligned} P(X_1+X_2<1) &= \int_{-\infty}^{2}\int_{-\infty}^{1-x_2} x_1x_2dx_1dx_2 \\ &= \int_{-\infty}^{2} x_2\int_{-\infty}^{1-x_2} x_1dx_1dx_2 \qquad \text{(5 points)} \\ &= \int_{-\infty}^{2} x_2\int_{0}^{1-x_2} x_1dx_1dx_2 \end{aligned}$$

$$= \int_{-\infty}^{2} x_2 \cdot \frac{1}{2}(1-x_2)^2 dx_2$$

$$= \frac{1}{2}\int_{0}^{2} x_2(1-x_2)^2 dx_2$$

$$= \frac{1}{2}[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}]_0^1$$ (5 points)

$$= \frac{1}{24}$$

5.76 (15 points)

$$P(0.2 < X < 0.5, 0.4 < Y < 0.6)$$

$$= \int_{x=0.2}^{0.5} \int_{y=0.4}^{0.6} \frac{6}{5}(x+y^2)dxdy$$ (5 points)

$$= \frac{6}{5}\int_{y=0.4}^{0.6} [\frac{x^2}{2} + xy^2]_{x=0.2}^{x=0.5} dy$$

$$= \frac{6}{5}\int_{y=0.4}^{0.6} [\frac{0.5^2}{2} + 0.5y^2 - (\frac{0.2^2}{2} + 0.2y^2)]dy$$

$$= \frac{6}{5}\int_{y=0.4}^{0.6} 0.105 + 0.3y^2 dy$$ (5 points)

$$= \frac{6}{5}[0.105y + 0.1y^3]_{0.4}^{0.6}$$

$$= \frac{6}{5}[0.105 \times 0.6 + 0.1 \times 0.6^3 - (0.105 \times 0.4 + 0.1 \times 0.4^3)]$$

$$= \frac{6}{5}[0.0846 - 0.0484]$$ (5 points)

$$= 0.04344$$

5.108 (15 points)

(a) Since

$$\int_{-\infty}^{\infty} f(x)dx = \int_0^1 k(1-x^2)dx = k(x - \frac{1}{3}x^3)|_0^1 = 1$$

$k = 1.5$ (5 points)

The probability between 0.1 and 0.2 should be

$$\int_{0.1}^{0.2} f(x)dx = 1.5(x - \frac{1}{3}x^3)|_{0.1}^{0.2} = 0.1465$$ (5 points)

(b) The probability greater than 0.5 should be

$$\int_{0.5}^{1} f(x)dx = 1.5(x - \frac{1}{3}x^3)|_{0.5}^{1} = 0.3125$$ (5 points)