SEG2430
Assignment 7 Solution

Question 6.7

(a)

The distribution is:

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	5/30 =1/6	4/30 =2/15	3/30 =1/10	2/30 =1/15	2/30 =1/15	2/30 =1/15	3/30 =1/10	4/30 =2/15	5/30 =1/6

The mean is:

$$\mu$$
$$=\sum x\cdot f(x)$$
$$=(-4)\times\frac{1}{6}+(-3)\times\frac{2}{15}+(-2)\times\frac{1}{10}+(-1)\times\frac{1}{15}+(0)\times\frac{1}{15}+(1)\times\frac{1}{15}+(2)\times\frac{1}{10}+(3)\times\frac{2}{15}+(4)\times\frac{1}{6}$$
$$=0$$

OR

$$\mu$$
$$=\sum x\cdot f(x)$$
$$=(-4+4)\times\frac{1}{6}+(-3+3)\times\frac{2}{15}+(-2+2)\times\frac{1}{10}+(-1+0+1)\times\frac{1}{15}$$
$$=0$$

[Indeed it is easy to see the mean equals 0.]

The variance is:

$$\sigma^2$$
$$=\sum(x-\mu)^2\cdot f(x)$$
$$=(-4-0)^2\times\frac{1}{6}+(-3-0)^2\times\frac{2}{15}+(-2-0)^2\times\frac{1}{10}+(-1-0)^2\times\frac{1}{15}+(0-0)^2\times$$
$$\frac{1}{15}+(1-0)^2\times\frac{1}{15}+(2-0)^2\times\frac{1}{10}+(3-0)^2\times\frac{2}{15}+(4-0)^2\times\frac{1}{6}$$
$$=8.667$$

OR

$$\sigma^2$$
$$=\sum(x-\mu)^2\cdot f(x)$$
$$=(-4-0)^2\times\frac{1}{6}\times2+(-3-0)^2\times\frac{2}{15}\times2+(-2-0)^2\times\frac{1}{10}\times2+(-1-0)^2\times\frac{1}{15}\times2+(0-0)^2$$
$$=8.667$$

[Thanks to the "square" the variance is not 0.]

(b)

In the 50 samples, the sample means are:

Set	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
Sample Mean $\overline{X}$	0.8	0.3	0.3	-0.9	-2.6	0.2	0.4	-0.8	0.2	1.3
Set	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
Sample Mean $\overline{X}$	0.4	0.1	0.5	0.7	0.4	0.7	0.1	0.3	0.2	0.5
Set	***21***	***22***	***23***	***24***	***25***	***26***	***27***	***28***	***29***	***30***
Sample Mean $\overline{X}$	0.3	0.1	-0.3	1	-1.5	-0.7	0.1	-0.1	-0.1	-1
Set	***31***	***32***	***33***	***34***	***35***	***36***	***37***	***38***	***39***	***40***
Sample Mean $\overline{X}$	0.3	-0.5	0.4	-0.3	-0.9	1.5	-0.7	-1.3	-0.6	1.2
Set	***41***	***42***	***43***	***44***	***45***	***46***	***47***	***48***	***49***	***50***
Sample Mean $\overline{X}$	-1	0	-0.7	0.6	-0.7	0.3	1.2	1.4	0	-1.4

[Now you can see the sample mean $\overline{X}$ is actually random, so it has a mean and variance!]
(c) The mean of $\overline{X}$ is -0.006, the variance of $\overline{X}$ is 0.6794.
(d)
By Theorem 6.1, expected mean of $\overline{X} = \mu_{\overline{X}} = \mu = 0$.
From the experiment the mean of $\overline{X}$ is -0.006, which is similar to the value of $\mu_{\overline{X}}$.
By Theorem 6.1, expected variance of $\overline{X} = \sigma_{\overline{X}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1} = \frac{8.667}{10} \cdot \frac{30-10}{30-1} = 0.5977$.
From the experiment the variance of $\overline{X}$ is 0.6794, which is similar to the value of $\sigma_{\overline{X}}^2$.

Question 6.15

$\mu_{\overline{X}} = \mu = 176$

$\sigma_{\overline{X}}^2 = \frac{\sigma^2}{n} = \frac{256}{100} = 2.56$

As the sample size n = 100 ≥ 30, by the Central Limit Theorem, the required probability is:

$$F\left(\frac{178-176}{\sqrt{2.56}}\right) - F\left(\frac{175-176}{\sqrt{2.56}}\right)$$
$$= F(1.25) - F(-0.625)$$
$$= 0.8944 - \frac{0.2676 + 0.2643}{2}$$
$$= 0.6285$$

Accept [0.6268,0.6301]

Question 6.21

Since σ is unknown, we assume the distribution of the times is normal, and we have:

$$\overline{X} = \frac{\sum x}{n} = \frac{27+15+20+32+18+26}{6} = \frac{138}{6} = 23$$
$$S^2$$
$$= \sum \frac{\left(x - \overline{X}\right)^2}{n-1}$$
$$= \frac{(27-23)^2 + (15-23)^2 + (20-23)^2 + (32-23)^2 + (18-23)^2 + (26-23)^2}{6-1}$$
$$= 40.8$$

Then we use the t distribution with the degree of freedom υ = 6-1 = 5:

$$t = \frac{\overline{X} - \mu}{\sqrt{S^2 / n}} = \frac{23-20}{\sqrt{40.8/6}} = 1.150$$

From the t table (Table 4, Page 523):

$t_{0.1,5} = 1.476$

The probability of t < 1.476 is 0.9. Thus we can say the ambulance service's claim is reasonable, or we fail to reject the claim.

Question 6.34

(a)

$\mu_{\overline{X}} = \mu = 63$

$\sigma_{\overline{X}}^2 = \frac{\sigma^2}{n} = \frac{81}{36} = \frac{9}{4}$

Assuming the distribution is symmetric, and by the Chebyshev's Theorem:

$$P(\overline{X} > 66.75)$$
$$= P(\overline{X} > 63 + 2.5 \times \sqrt{\frac{9}{4}})$$
$$= \frac{1}{2} P(\overline{X} > 63 + 2.5 \times \sqrt{\frac{9}{4}}) + P(\overline{X} < 63 - 2.5 \times \sqrt{\frac{9}{4}})$$
$$= \frac{1}{2} P(| \overline{X} - 63 | > 2.5 \times \sqrt{\frac{9}{4}})$$
$$= \frac{1}{2} P(| \overline{X} - \mu_{\overline{X}} | > 2.5 \times \sigma_{\overline{X}})$$
$$< \frac{1}{2} (\frac{1}{2.5^2})$$
$$= 0.08$$

(b)

By the Central Limit Theorem, the required probability is:

$$1 - F\left(\frac{66.75 - \mu}{\sqrt{\sigma^2 / n}} \right)$$
$$= 1 - F\left(\frac{66.75 - 63}{\sqrt{81/36}} \right)$$
$$= 1 - F(2.5)$$
$$= 1 - 0.9938$$
$$= 0.0062$$

[We can see from the two parts that, again similar to the midterm, Chebyshev's Thm. gives a loose bound]

Question 7.1

(a) [There are $C_3^5 = 10$ combinations]	(b) $\overline{x}$	Probability: $f(\overline{x})$
3,6,9	6	1/10
3,6,15	8	1/10
3,6,27	12	1/10
3,9,15	9	1/10
3,9,27	13	1/10
3,15,27	15	1/10
6,9,15	10	1/10
6,9,27	14	1/10
6,15,27	16	1/10
9,15,27	17	1/10

(b) <continued> The mean of the $\overline{x}$'s is:

$$\mu_{\overline{x}} = \sum \overline{x} f(\overline{x}) = (6+8+12+9+13+15+10+14+16+17) \times \frac{1}{10} = 12,$$

which is the mean of the population μ $[\mu = \frac{\sum x}{N} = \frac{(3+6+9+15+27)}{5} = 12]$.

Question 7.12

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{z_{0.1/2} \cdot 60}{10} \right)^2 = (1.645 \times 6)^2 = 97.4169$$

We need a sample of size 98.