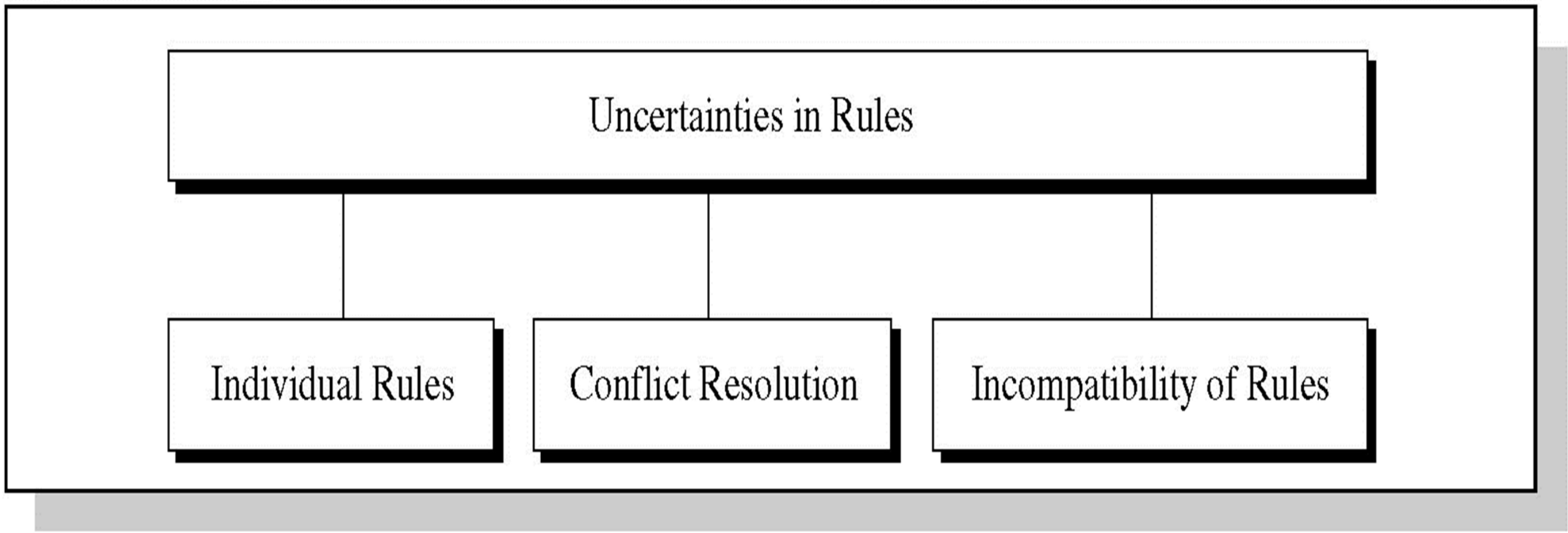
Certainty Factor Model

Introduction

- Probability theory has been called by mathematicians a theory of reproducible uncertainty
- Besides the subject probability theory, alternative theories were specifically developed to deal with human belief rather than the classic frequency interpretation of probability
  - All these theories are examples of inexact reasoning

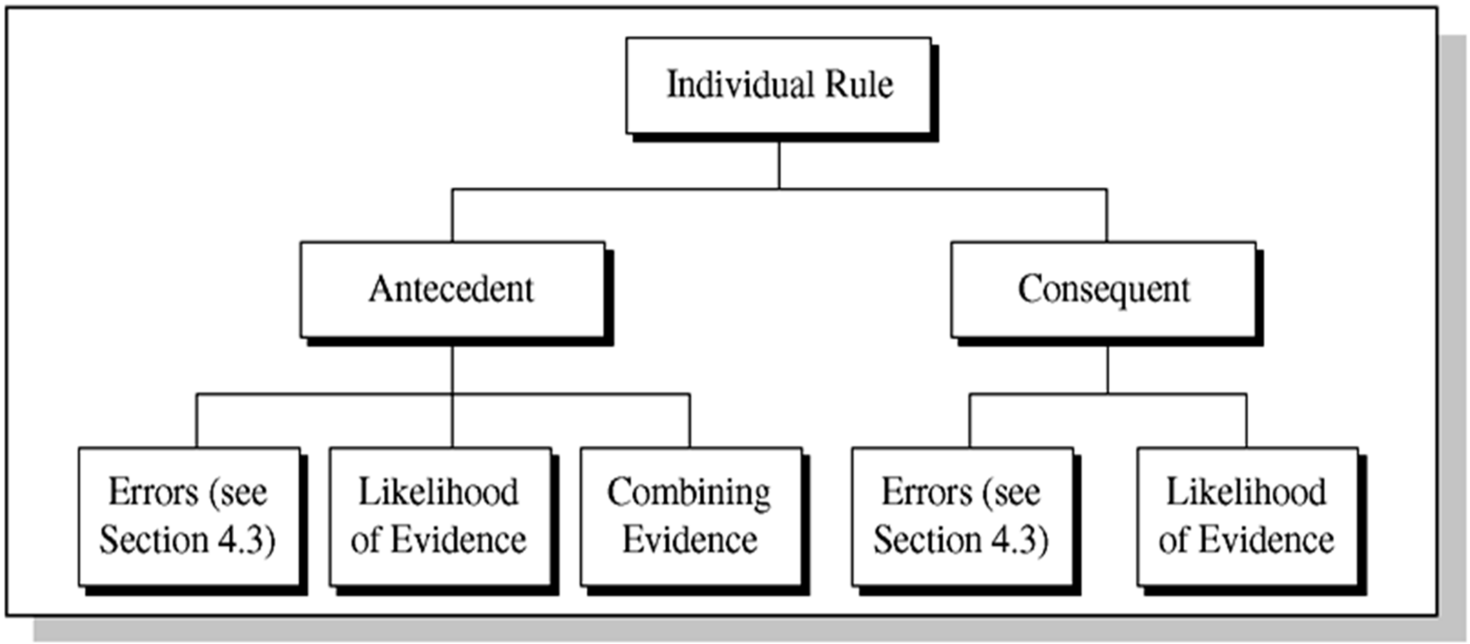
Uncertainty and rules

- Sources of Uncertainty in rules
  - The goal of the knowledge engineer is to minimize or eliminate these uncertainties



Sources of Uncertainty in rules

- Besides the possible errors involved in the creation of rules, there are uncertainties associated with the assignment of likelihood values.
  - For probabilistic reasoning, these uncertainties are with the sufficiency, LS, and necessity, LN, values



Sources of Uncertainty in rules

- Uncertainty with the likelihood of the consequent.
  - For probabilistic reasoning, written as
    - P(H I E) for certain evidence and
    - P(H I e) for uncertain evidence
- Another source of uncertainty is the combining of the evidence.
  - Should the evidence be combined. as in the following?

as E_1 AND E_2 AND E_3

or as E_1 AND E_2 OR E_3

or as E_1 AND NOT E_2 OR E_3

Certainty factor

- Another method of dealing with uncertainty uses certainty factors
- Difficulties with the Bayesian Method
  - Bayes' Theorem's accurate use depends on knowing many probabilities
    - For example, to determine the probability of a specific disease, given certain symptoms as:

$$P(D_i \mid E) = \frac{P(E \mid D_i)\,P(D_i)}{P(E)} = \frac{P(E \mid D_i)\,P(D_i)}{\sum_j P(E \mid D_j)\,P(D_j)}$$

    - where the sum over j extends to all diseases, and:
      - D_i is the i'th disease,
      - E is the evidence,
      - $P(D_i)$ is the prior probability of the patient having the Disease_i before any evidence is known
      - $P(E \mid D_i)$ is the conditional probability that the patient will exhibit evidence E, given that disease D_i is present

Difficulties with the Bayesian Method

- A convenient form of Bayes' Theorem that expresses the accumulation of incremental evidence like this is

$$P(D_i \mid E) = \frac{P(E_2 \mid D_i \cap E_1)\, P(D_i \mid E_1)}{\sum_j P(E_2 \mid D_j \cap E_1)\, P(D_j \mid E_1)}$$

- where E_2 is the new evidence added to yield the new augmented evidence:

$$E = E_1 \cap E_2$$

- Although this formula is exact, all these probabilities are not generally known

Belief and disbelief

- Another major problem was the relationship of belief and disbelief
  - the theory of probability states that:

    $P(H) + P(H') = 1$

    and so:

    $P(H) = 1 - P(H')$
  - For the case of a posterior hypothesis that relies on evidence, E:

    (1) $P(H \mid E) = 1 - P(H' \mid E)$
  - Experts were extremely reluctant to state their knowledge in the form of equation (1).

Belief and disbelief

- For example, consider a MYCIN rule:

  IF 1) The stain of the organism is gram positive,

  and

  2) The morphology of the organism is coccus,

  and

  3) The growth conformation of the organism is chains

  THEN There is suggestive evidence (0.7) that the identity of the organism is streptococcus

- in terms of posterior probability as:

$$(2)\quad P(H \mid E_1 \cap E_2 \cap E_3) = 0.7$$

  where the E_i correspond to the three patterns of the antecedent

- An expert would agree to equation (2), they became uneasy and refused to agree with the probabilistic result:

$$(3)\quad P(H' \mid E_1 \cap E_2 \cap E_3) = 1 - 0.7 = 0.3$$

Belief and disbelief

- The fundamental problem is that
  - while P(H | E) implies a cause-and-effect relationship between E and H
  - there may be no cause-and-effect relationship between E and H'
  - Yet the equation:

$$P(H \mid E) = 1 - P(H' \mid E)$$

    - implies a cause-and-effect relationship between E and H' if there is a causeand-effect between E and H
- Certainty factors – representing uncertainty
  - ordinary probability
    - associated with the frequency of reproducible events
  - epistemic probability or the degree of confirmation
    - it confirms a hypothesis based on some evidence (another example of the degree of likelihood of a belief.)

Measures of belief and disbelief

- In MYCIN, the degree of confirmation was originally defined as the certainty factor
  - the difference between belief and disbelief:

$$CF(H,E) = MB(H,E) - MD(H,E)$$

    where

    CF is the certainty factor in the hypothesis H due to evidence E

    MB is the measure of increased belief in H due to E

    MD is the measure of increased disbelief in H due to E

- Certainty factor
  - is a way of combining belief and disbelief into a single number
  - can be used to rank hypotheses in order of importance

Measures of belief and disbelief

- The measures of belief and disbelief were defined in terms of probabilities by:

$$MB(H,E) = \begin{cases} 1 & \text{if } P(H) = 1 \\ \dfrac{\max[P(H \mid E),\ P(H)] - P(H)}{\max[1,0] - P(H)} & \text{otherwise} \end{cases}$$

$$MD(H,E) = \begin{cases} 1 & \text{if } P(H) = 0 \\ \dfrac{\min[P(H \mid E),\ P(H)] - P(H)}{\min[1,0] - P(H)} & \text{otherwise} \end{cases}$$

Measures of belief and disbelief

Some Characteristics of MB, MD, and CF

Characteristics	Values
Ranges	$0 \leq MB \leq 1$ $0 \leq MD \leq 1$ $-1 \leq CF \leq 1$
Certain True Hypothesis $P(H \mid E) = 1$	$MB = 1$ $MD = 0$ $CF = 1$
Certain False Hypothesis $P(H' \mid E) = 1$	$MB = 0$ $MD = 1$ $CF = -1$
Lack of Evidence $P(H \mid E) = P(H)$	$MB = 0$ $MD = 0$ $CF = 0$

Measures of belief and disbelief

- The certainty factor, CF, indicates the net belief in a hypothesis based on some evidence.
  - positive CF means the evidence supports the hypothesis since MB > MD
  - CF = 1 means that the evidence definitely proves the hypothesis.
  - CF = 0 means one of two possibilities
    1. CF = MB - MD = 0 could mean that both MB and MD are 0
       - that is, there is no evidence
    2. The second possibility is that MB = MD and both are nonzero
       - the belief is cancelled out by the disbelief
  - negative CF means that the evidence favors the negation of the hypothesis since MB < MD
    - there is more reason to disbelieve a hypothesis than to believe it
- With certainty factors there are no constraints on the individual values of MB and MD. Only the difference is important

$$
\begin{aligned}
CF &= 0.70 = 0.70 - 0 \\
&= 0.80 - 0.10
\end{aligned}
$$

Measures of belief and disbelief

- Certainty factors allow an expert to express a belief without committing a value to the disbelief

  $$CF(H,E) + CF(H',E) = 0$$

  - means that if evidence confirms a hypothesis by some value CF(H | E) the confirmation of the negation of the hypothesis is not 1 - CF(H | E) which would be expected under probability theory

  $$CF(H,E) + CF(H',E) \neq 1$$

  - For the example of the student graduating if an "A" is given in the course.
    - CF(H,E) = 0.70 CF (H', E) = - 0.70

    which means:
    - (6) I am 70 percent certain that I will graduate if I get an 'A' in this course
    - (7) I am -70 percent certain that I will not graduate if I get an 'A' in this course

Measures of belief and disbelief

- Certainty factors are defined on the interval:

$$-1 \leq CF(H,E) \leq +1$$

  where
  - 0 means no evidence.
  - values greater than 0 favor the hypothesis
  - less than 0 favor the negation of the hypothesis
- The above CF values might be elicited by asking:

  How much do you believe that getting an 'A' will help you graduate?
  - If the evidence is to confirm the hypothesis or:

  How much do you disbelieve that getting an 'A' will help you graduate?
  - An answer of 70 percent to each question will set
    - CF(H | E) = 0.70 and
    - CF (H' | E) = -0.70.

Calculating with certainty factors

- There were difficulties with the definition CF=MB-MD
  - E.g., 10 pieces of evidence might produce a MB = 0.999 and one disconfirming piece with MD = 0.799 could then give:

    CF = 0.999 - 0.799 = 0.200

    - In MYCIN, CF > 0.2 for the antecedent to activate the rule
- **Threshold value** is an ad hoc way of minimizing the activation of rules which only weakly suggest a hypothesis

Calculating with certainty factors

- The definition of CF was changed in MYCIN in *1977* to be:

$$CF = \frac{MB - MD}{1 - \min(MB, MD)}$$

- To soften the effects of a single piece of disconfirming evidence

$$CF = \frac{0.999 - 0.799}{1 - \min(0.999, 0.799)} = \frac{0.200}{1 - 0.799} = 0.995$$

MYCIN Rules for Combining Antecedent Evidence of Elementary Expressions

Evidence, E	Antecedent Certainty
E_1 AND E_2	min [CF(H,E_1), CF(H,E_2)]
E_1 OR E_2	max [CF(H,E_1), CF(H,E_2)]
NOT E	– CF(H,E)

Calculating with certainty factors

- For example, given a logical expression for combining evidence such as:

$$E = (E_1 \text{ AND } E_2 \text{ AND } E_3) \text{ OR } (E_4 \text{ AND NOT } E_5)$$

  - The evidence E would be computed as:

$$E = \max [\min (E_1, E_2, E_3) , \min(E_4, -E_5)]$$

  - For values:

$$E_1 = 0.9 \quad E_2 = 0.8 \quad E_3 = 03$$
$$E_4 = -0.5 \quad E_5 = -0.4$$

  - the result:

$$E = \max[\min(0.9,0.8,0.3), \min(-0.5,-(-0.4)]$$
$$= \max[0.3, -0.5]$$
$$= 0.3$$

Calculating with certainty factors

- The fundamental formula for the CF of a rule:

  IF E THEN H

  is given by the formula:

  (8) CF(H,e) = CF(E,e) CF(H,E)

  where:

  CF(E,e) is the certainty factor of the evidence E making up the antecedent of the rule based on uncertain evidence e.

  CF(H,E) is the certainty factor of the hypothesis assuming that the evidence is known with certainty, when CF(E,e) = 1.

  CF(H,e) is the certainty factor of the hypothesis based on uncertain evidence *e*.

  - If all the evidence in the antecedent is known with certainty (CF(E,e) = 1)

    CF(H,e) = CF(H,E)

Calculating with certainty factors

- As an example:

  IF 1) The stain of the organism is gram positive,

  and

  2) The morphology of the organism is ooccus, and

  3) The growth conformation of the organism is chains

  THEN There is suggestive evidence (0.7) that the identity of the organism is streptococcus

  where the certainty factor of the hypothesis under certain evidence is :

  $$CF(H,E) = CF(H, E_1 \cap E_2 \cap E_3) = 0.7$$

  and is also called the **attenuation factor.**

- The attenuation factor
  - is based on the assumption that all the evidence E_1, E_2 and E_3 is known with certainty $CF(E_1,e) = CF(E_2,e) = CF(E_3,e) = 1$
  - expresses the degree of certainty of the hypothesis, given certain evidence

Calculating with certainty factors

- A complication occurs if all the evidence is not known with certainty
  - For example

    $CF(E_1, e) = 0.5$

    $CF(E_2, e) = 0.6$

    $CF(E_3, e) = 0.3$
  - then:

    $$\begin{aligned} CF(E,e) &= CF(E_1 \cap E_2 \cap E_3, e) \\ &= \min[CF(E_1,e), CF(E_2,e), CF(E_3,e)] \\ &= \min[0.5, 0.6, 0.3] \\ &= 0.3 \end{aligned}$$
  - The certainty factor of the conclusion is :

    $$\begin{aligned} CF(H,e) &= CF(E,e)\, CF(H,E) \\ &= 0.3 * 0.7 \\ &= 0.21 \end{aligned}$$

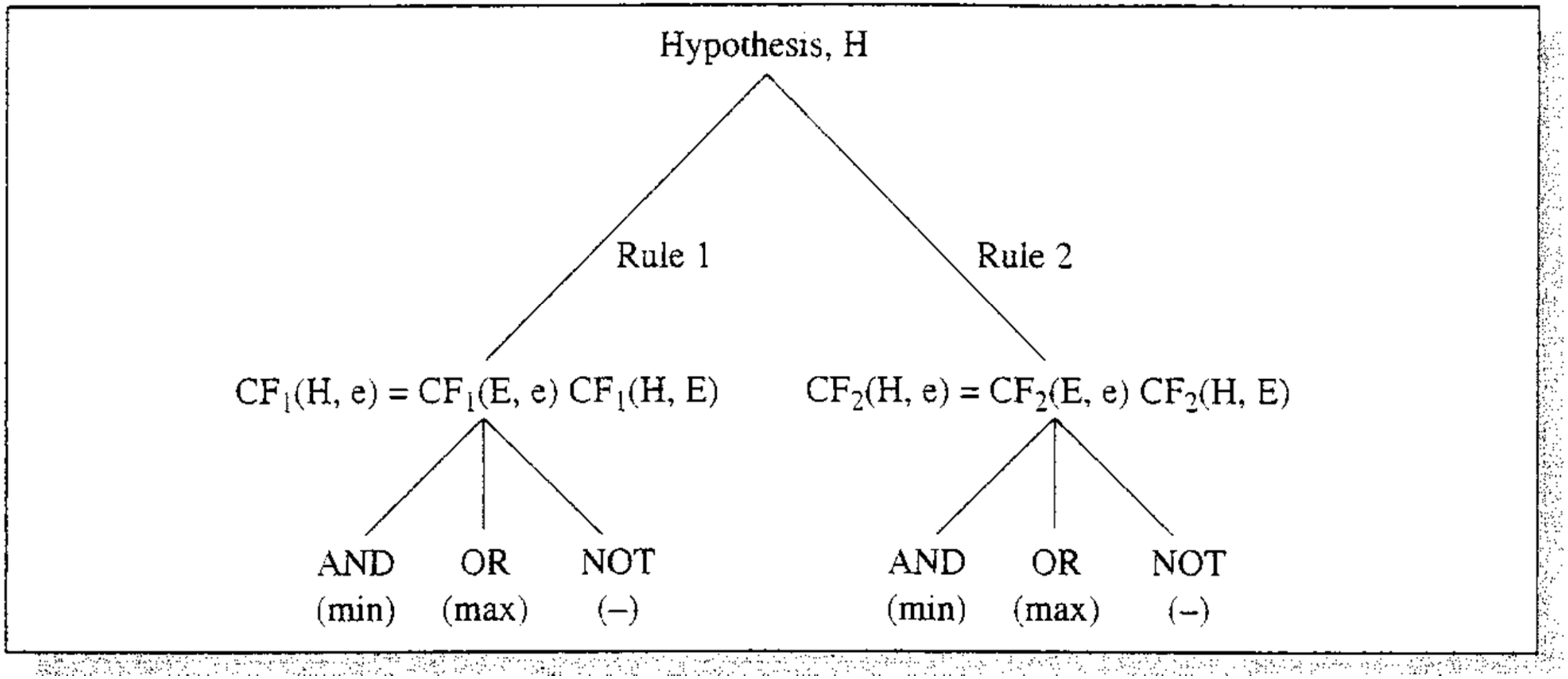
Calculating with certainty factors

- Suppose another rule also concludes the same hypothesis, but with a different certainty factor.
  - The certainty factors of rules concluding the same hypothesis are calculated from the **combining function**

$$(9)\quad CF_{COMBINE}(CF_1, CF_2,) = \begin{cases} CF_1 + CF_2\ (1 - CF_1) & \text{both} > 0 \\ \dfrac{CF_1 + CF_2}{1 - \min(|CF_1|, |CF_2|)} & \text{one} < 0 \\ CF_1 + CF_2\ (1 + CF_1) & \text{both} < 0 \end{cases}$$

Calculating with certainty factors

CF of Two Rules with the Same Hypothesis Based on Uncertain Evidence



Calculating with certainty factors

- if another rule concludes streptococcus with certainty factor CF, = 0.5, then the combined certainty

  $CF_{COMBINE}$ (0.21,0.5)= 0.21 + 0.5 (1 - 0.21) = 0.605
- Suppose a third rule also has the same conclusion, but with a CF_3 = - 0.4.

$$CF_{COMBINE}(0.605, -0.4) = \frac{0.605 - 0.4}{1 - \min(|0.605|, |-0.4|)}$$

$$= \frac{0.205}{1 - 0.4} = 0.34$$

- The $CF_{COMBINE}$ formula preserves the commutativity of evidence. That is

  $CF_{COMBINE}(X,Y) = CF_{COMBINE}(Y,X)$
- MYCIN stored the current $CF_{COMBINE}$ with each hypothesis and combined it with new evidence as it became available