Q1. [5 Point] Prove the linearity of conditional expectation (Lemma 3 in the Slides of Lecture 1).

Q2. [15 Point] Prove the lower tail of the Chernoff bound (Theorem 12 in the Slides of Lecture 1).

Q3. [10 Point] Given a biased coin, we want to estimate the probability \( p \) that it shows the head. Thus, we make \( x \) different independent trials (by flipping the coin \( x \) times); let \( Y \) be the number of trials that shows the head among the \( x \) experiments and define \( \hat{p} = \frac{Y}{x} \). Assume that we know \( p \geq \tau \). Prove that it suffices to make \( x = \frac{3 \log (2n^\tau)}{\tau \delta^2} \) trials to guarantee that the following holds (\( 0 < \delta < 1 \)).

\[
P[|\hat{p} - p| \geq \delta \cdot p] \leq \frac{1}{n^\epsilon}
\]